Incorporating the Practice of Arguing in Stein et al.'s Model for Helping Teachers Plan and Conduct Productive Whole-Class Discussions

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How can pedagogical models support in-service and pre-service teachers in the complexity of orchestrating productive mathematical whole-class discussions? The overarching aim of this paper is to elaborate on a newly developed model to make it an even more useful tool for teachers to manage the challenging task of conducting productive whole-class discussions. Analyses of audio-recorded interviews and video-recorded whole-class discussions with a proficient mathematics teacher result in principles for how student solutions can be sequenced in order to take into account argumentation as well as connection-making in whole-class discussions. The findings suggest broadening the last practice in the five practices model to also incorporate the practice of arguing.

Introduction

Mathematical discussions that focus on important relationships between mathematical ideas in students' different solutions to demanding problems can be seen as a significant ingredient in high-quality or ambitious mathematics teaching (Cobb & Jackson, 2011; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010) that aims at developing students' mathematical competencies (NCTM, 2000; NRC, 2001). For teachers to learn the challenging task (Brodie, 2010) of orchestrating such productive whole-class discussions that take both students' participation and important mathematical content into consideration (cf. Ryve, Larsson and Nilsson, 2011), there is need for supportive routines of practice (Franke, Kazemi, & Battey, 2007) or instructional practices (Cobb & Jackson, 2011). Stein, Engle, Smith and Hughes' (2008) model of the five practices anticipating, monitoring, selecting, sequencing and connecting aims at helping teachers plan the orchestration of productive whole-class discussions that both build on student ideas and highlight and advance important mathematical ideas and relationships. Stein et al.'s (2008) model is designed to be used in in-service and pre-service teacher education as a tool for mathematics teachers at all school levels to learn to conduct productive mathematical discussions that focus on connections between different student ideas and between student ideas and key ideas. However, both arguing and connecting constitute the keys for creating opportunities in discussions for extending student thinking (Cengiz, Kline, & Grant, 2011). The overarching aim of this paper is to further elaborate on Stein et al.'s (2008) five practices model in order for teachers to manage to conduct productive whole-class discussions that focus on argumentation as well as connection-making. Stein et al. (2008) emphasize that much more research is needed on how to sequence student solutions and a particular aim of this paper is to contribute to that area of research.

Conceptual framework

Stein et al.'s (2008) five practices model for helping teachers plan the orchestration of productive mathematical discussions is central in my analysis and the model itself is also analyzed. The five practices in Stein et al.'s (2008) model are: anticipating student responses to cognitively demanding tasks, monitoring student responses during the explore phase, selecting student responses for whole-class discussion, purposefully sequencing student responses and connecting different student responses to each other and to key mathematical ideas. Each practice builds on and benefits from the practices that precede it. The five practices have clear connections to teaching practices in Japan, where teachers often organize a complete lesson around students' various solutions to a single problem in a whole class setting (Shimizu, 1999). Crucial Japanese instuctional practices include anticipating student approaches and observing or monitoring students' work, looking for good ideas "with the intention of calling on those students – in a certain order – in the subsequent discussion" (Shimizu, 1999, p. 109). The order is critical for making connections among student ideas.

The basic assumptions underlying the five practices model as articulated by Smith and Stein (2011) are that we learn through using others as resources in social interaction, sharing our ideas and participating in co-construction of knowledge (cf. Cobb, 2000; Cobb, Stephan, McClain, & Gravemeijer, 2001). To support student learning, Smith and Stein (2011) accentuate the importance of encouraging students to evaluate their own and other students' mathematical ideas. However, Stein et al.'s model provides no explicit support for teachers regarding this aspect. I will operationalize this aspect in my elaboration of their model to take into account argumentation as well as connection-making.

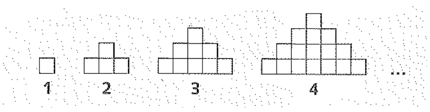
Methodology

The primary data source for this paper comes from a project which I conducted in collaboration with a very experienced and proficient teacher regarding problem solving discussions. I observed the teacher during eight days in one school year without making interventions. I had a particular focus on the teacher's orchestration of whole-class discussions based on students' different solutions to challenging mathematical problems. Data consists of video-recorded lessons focusing on the teacher during whole-class discussions, audio-recorded teacher interviews before and after every lesson, audio-recorded student interviews, audio-recorded teacher meetings and collected student solutions. Stein et al.'s (2008) model serves as the primary framework for analyzing the data. Data from this project will feed into my ongoing work on suggesting elaborations of Stein et al.'s (2008) framework, together with data from several intervention projects that I have conducted (Larsson & Ryve, 2011; 2012). In these intervention projects I collaborated with teachers learning to conduct whole-class discussions of students' different ideas. One project involved all mathematics teachers in grade 6-9 at one school during the course of two years.

Analysis and results

As an illustration of how the proficient teacher reasons when she sequences student solutions, I will now go into a whole-class discussion in 6th grade of students' different solutions to the problem Winners' stands. I will relate this particular discussion to principles for sequencing student solutions to take into account argumentative aspects as well as connection-making aspects.

Winners' stands



How large perimeter and area has winners' stand number:

a) 15 b) 20 c) n

In Table 1, you find the student solutions for area in the sequential order that they were brought up in whole-class discussion. In fact, the solutions correspond to Mason's (1996) three major approaches for how algebraic formulas are constructed: (1) finding a recursive rule of how to construct the next term from the preceding terms (Edward and Anna), (2) manipulating the figure to make counting easier (Anders and Pia, Fredrika and Carl), and (3) finding a pattern which leads to a direct formula (majority of the students).

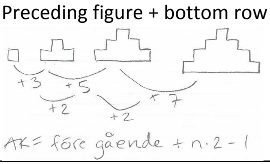
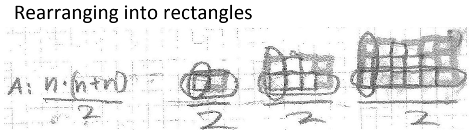
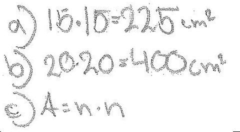
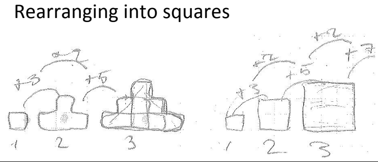
Edward's and Anna's solution Preceding figure + bottom row 	Anders' and Pia's solution Rearranging into rectangles 
Majority of the students' solution Seeing number pattern in a table 	Fredrika's and Carl's solution Rearranging into squares 

Table 1: Student solutions for area of winners' stands in sequential order.

In the following excerpt, we enter the discussion from the start when Edward explains his and Anna's formula for the area of the winners' stands (see Table 1).

1	Teacher:	Let's start with area. This is one solution. Eeh then we have (.) let's look at winners' stand number 1, number 2, number 3, number 4 [points at the figures one at a time] and shown that the difference is 3, 5, 7 [points at the differences one at a time] and that it then increases with 2 and 2 [points at the twos one at a time]. And then your formula is (.) could you just explain your formula.
2	Edward:	So the number of squares equals the preceding figure before, because it's them you can- you can see that they sit and then you have just added a bottom. And the bottom equals the number of the figure times 2 minus 1.
3	Teacher:	The number of the figure times 2 minus 1. So for example in figure number 2, no number 3 it is 1, 2, 3, 4, 5
4	Edward:	And 3 times 2 equals 6, minus 1 is 5.
5	Teacher:	Does anybody understand what kind of formula they've written here?
6	Students:	Yes.
7	Teacher:	You do understand?
8	Fredrika:	Yes.
9	Teacher:	Fredrika, could you explain the formula to see if we underst- if we all understand.
10	Fredrika:	So look, it's like this. Eeh, if-
11	Teacher:	Edward, listen to see if, if Fredrika understands what you mean.
12	Fredrika:	If we deal with, if we say that we're on figure number 3
13	Teacher:	There [points at figure number 3]
14	Fredrika:	Yes. Eeh (.) okay you (.) if you look at the preceding number 2, before [teacher points at figure number 2] it looks like that. And the difference between that and number 3, it's that you have added a bottom in it, a new floor farthest beneath. If you see that. Yes.
15	Teacher:	Mm.
16	Fredrika:	So then it's the preceding figure
17	Teacher:	The one up here [points]

18	Fredrika:	Yes, exactly. And then plus this bottom which is (.) so n so the figure times 2 minus 1.
19	Teacher:	So 3 times 2 is 6, 6 minus 1 is 5. 1, 2, 3, 4, 5. Now I understand. Does anybody else than I understand?
20	Students:	Yes. Mm. I understand.
21	Teacher:	Sanna, do you understand?
22	Sanna:	Yes.
23	Teacher:	Hannes understands?
24	Hannes:	Yes, but I don't get how- how would you find out the preceding figure?
25	Student:	No.
26	Edward:	I know, that's our little problem, that if you don't know that then you can't really use this one.

To begin the discussion with Edward's and Anna's recursive formula in which the area for one winners' stand builds on the area for the preceding winners' stand serves as a springboard for the rest of the discussion since the limitations with the solution are made explicit by Edward himself in [26] after Hannes' question in [24]. The teacher chose to begin with Edward's and Anna's solution because "there was still a problem to solve" (interview after discussion). The teacher does not authoritatively evaluate Edward's and Anna's solution, but instead facilitates for the students to evaluate each other's solutions which is salient for a dialogic approach that takes different points of views into account (Ruthven, Hofmann, & Mercer, 2011). The teacher first lets Edward explain his and Anna's solution ([2] and [4]). Then the teacher repeatedly asks if anybody understands ([5], [19]), after which she follows up with asking if specific students understand ([7], [21] [23]) and asking Fredrika to actually explain how she understands Edward's solution ([9]), emphasizing the importance that Edward listens carefully to see if Fredrika understands what he means ([11]). When the teacher asks if Hannes understands he raises the question of how you can find out the area for the preceding figure ([24]), which is a clear limitation to the solution that Edward already seems aware of ([26]). The teacher confirms in the interview after the discussion that Edward was in fact aware of this limitation before the discussion but that "Edward was completely convinced that, when he presented, that certainly all of them had that problem" and "that was why he was so sure and could explain that yes, if I only knew what the preceding is".

After this exchange, Anders' and Pia's solution of rearranging the winners' stands into rectangles (see Table 1) is discussed. According to the teacher "they realized later that there was an easier method, but they were exceedingly happy when they drew their rectangle". Their solution is evaluated by the students to be a smart solution that resembles a solution to another problem that they have previously worked with, but that there exist easier solutions to this problem. A majority of the students have seen from the number pattern in a table that the formula for the area of the winners' stands can be expressed as n·n. This is the next solution to be discussed very shortly (see Table 1) and Anders states that he regards it as much easier than his own solution. Finally, the teacher highlights Fredrika's and Carl's rearrangement of the winners' stands into squares to find out the formula n·n (see Table 1).

If we step back from these solutions for a moment, we can see how the recursive solution serves as a springboard for argumentation. We can also imagine how an early introduction of the solution from the majority of the students could have affected the quality of the argumentative aspects of the whole-class discussion. If a majority of the students have already received confirmation in the beginning of the discussion that their own solution is correct, there is a considerable risk that they do not listen as carefully to the other student contributions and that they do not contribute by putting forward arguments for or against the validity of different solutions. I will now go further into how the first four practices, in particular sequencing, are critical for argumentation as well as for connection-making.

Anticipating, monitoring, selecting and sequencing to promote argumentation as well as connection-making

Clearly, the first four practices in Stein et al.'s model are crucial in order to create opportunities to connect student solutions to each other and to key mathematical ideas (cf. breadth and depth connections in Ma, 1999). However, anticipating, monitoring, selecting and sequencing students' solutions are also crucial for argumentation during the whole-class discussion. When anticipating student solutions, in particular misconceptions, an important aspect for the teacher is to prepare for the kind of arguments that students are likely to present during whole-class discussion. When monitoring student ideas, my findings suggest that it is critical that the teacher does not disclose to the students whether their solution is correct or not. The reasons are both related to the problem-solving process and to the quality of the argumentation during the subsequent whole-class discussion. The proficient teacher states that "It's quite hard but it's extremely important that you don't tell if it's right or wrong because then you have removed what's the problem in the problem" (interview, Oct 27, 2011). This important aspect of the monitoring practice needs to be emphasized in Stein et al.'s model. If the students ask if their answers are correct during the problem-solving process, the proficient teacher asks questions to activate the students as owners of their own learning (e.g. "What do you think, is it right or wrong?") or as instructional resources for one another (e.g. "I don't know, discuss it with your friend."). (cf. Wiliam, 2007).

When selecting and sequencing student solutions, Stein et al. (2008) suggest that you start with either: a strategy based on a common misconception, a strategy that is particularly easy to understand or a strategy that a majority of the students have used. The first two suggestions are in line with my findings. Starting the discussion with a strategy based on a common misconception give the students the opportunity to straighten out their misconceptions before going deeper into the discussion of different correct strategies. Starting the discussion with a strategy that is particularly easy to understand resonances with the goal of accessibility (Stein et al., 2008) so that as many students as possible are able to follow and contribute to the discussion.

However, my findings suggest that there are some problems with the third suggestion. Starting the whole-class discussion with a solution that a majority of the students recognize as their own, or very close to their own, may compromise argumentation during the discussion. Instead of starting with a solution that many of the students have made, the proficient teacher places a common type of solution among the last ones in the sequence (see Table 1), or even skips it totally if it is very well-represented in the class. Analysis of whole-class discussions and interviews with the proficient teacher result in the following principles for sequencing student solutions:

1. an incorrect solution that seems reasonable that gives rise to argumentation (cf. common misconception in Stein et al., 2008)
2. a correct solution that is well structured with each step written where you can easily follow the whole line of thought (cf. goal of accessibility in Stein et al., 2008)
3. different solutions that show variety among solution strategies and representations with the potential to generalize to key mathematical ideas carefully considered, sequenced as more and more difficult to understand
4. (a solution that a majority of the students have made)
5. an elegant solution that makes the problem appear easy

The suggestion that teachers should not only discuss the students' correct solutions but also their incorrect solutions builds on the view that errors and misconceptions are "a normal part of coming to a correct conception" (Brodie, 2010, p. 14). The importance of giving students the opportunity to correct their own mistakes in front of the class is emphasized by the proficient teacher in my study, in line with Boaler and Humphreys (2005). The teacher states that "they get a chance to say to the whole class: Ah, I made a mistake here, but I should have done like this instead". A 7th grade student in her class expresses herself like this: "While you explain, some understand that they have made a mistake, so they learn while they explain".

My findings indicate that the first four practices are crucial not only for the practice of connecting but also for the practice of arguing which needs to be properly addressed within Stein et al.'s model. Therefore I suggest broadening the last practice in Stein et al.'s model to incorporate the practice of arguing.

The last practice in the model: Extending by arguing and connecting

Extending student thinking has to do with further development and challenging of student thinking (Cengiz et al., 2011). Arguing and connecting actually constitute the main part of creating possibilities in discussions for extending student thinking. Cengiz et al. (2011) state that "recognizing moments for building new connections or addressing misconceptions seems to be key in creating opportunities for extending student thinking" (p. 362). Misconceptions can be addressed by challenging them with mathematical arguments during discussions. In order to incorporate both arguing and connecting as being at the heart of mathematical discussions and to highlight extending student thinking as an overarching umbrella, I propose that the Connecting practice is elaborated into the *Extending by arguing and connecting* practice.

The teacher's role in whole-class discussions is to build upon students' reasoning about their ideas and to help them advance key mathematical ideas and connections in order to create opportunities for them to extend their thinking. In this, the teacher needs to promote further reflection and arguments from the students (Ruthven et al., 2011). To be able to recognize moments in whole-class discussions that create possibilities for extending student thinking by arguing and connecting, teachers need to be well-prepared. With the powerful help of working with the preceding practices of Stein et al.'s model the teacher can prepare for the arguing aspect to a certain extent in advance, as is also the case for connecting. Thus, the practice of *Extending by arguing and connecting* is in line with the strong emphasis on planning in Stein et al.'s model.

Discussion

From my collaboration projects with in-service teachers, I have found three dimensions along which to elaborate on Stein et al.'s model: breadth, depth and length. The suggestion to broaden the last practice to also include arguing falls into the first dimension. A suggestion that falls into the second dimension is to deepen the last practice to distinguish between different kinds of connections. Connections can for example be made between representations (Cengiz et al., 2011), especially between different forms of representations, between solution strategies (Stein et al., 2008) and between lessons or units (Cengiz et al., 2011; Lampert, 2001). Ma (1999) distinguishes between connections to basic ideas (concepts and principles) and connections between multiple approaches of an idea. The suggestion to deepen the connecting practice is based on my observations during several intervention projects that teachers who are new to the approach of teaching mathematics through problem-solving may make limited connections (Larsson & Ryve, 2011; 2012). Finally, a suggestion that falls into the third dimension, made by teachers in my two-year intervention project, is to lengthen the model with a launching practice in which the teacher leads a whole-class discussion to introduce the problem in order to address the issue of equity properly (Jackson & Cobb, 2010).

I conclude with discussing the practical implications for in-service and pre-service teacher education. Working with a tool such as Stein et al.'s model has the potential of helping teachers over time to conduct mathematical discussions that focus on important relationships between mathematical ideas, which is a key ingredient in high-quality teaching that aims at developing students' mathematical competencies. I have used Stein et al.'s model extensively in both in-service and pre-service education and many teachers express that their whole-class discussions are raised to a new level with the help of the model. My suggestions to elaborate on Stein et al.'s model to also incorporate the practice of arguing and to refine the sequencing practice can make the model even more useful to teachers. My work will continue with elaborating on Stein et al.'s model and also to further explore the moment-to-moment decisions during the classroom interaction that a mathematics teacher need to take in order to promote students' further reflection and arguments.

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