# Preschool Teachers' Awareness of Mathematics

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Preschool teachers' expectations about what mathematics they should engage children in are generally centred about numbers and counting. However, the Swedish preschool curriculum and research into young children's development of mathematical understanding suggest that children can be offered a much richer set of ideas. In this paper, we examine material from a professional development course which indicates that discussing Bishop's six universal mathematical activities provided preschool teachers with a wider perspective for discussing the mathematics, which children in their preschools engaged with.

#### Introduction

Previous research on mathematics in early childhood education indicates a focus on counting and this may be related to the emphasis that it has in the school curriculum (Johansson, Lange, Meaney, Riesbeck, & Wernberg, 2012). However, the idea that mathematics in preschool is a watered-down version of school mathematics is problematic as it does not acknowledge young children's exploration of mathematical ideas in the same way that they explore other aspects of their world. Therefore, an alternative to seeing mathematics in preschool as a precursor to school mathematics, it can be considered closer to the experimenting and discovery type done by mathematicians (Devlin, 1999).

Not only do researchers seem to have a limited view on what mathematics in preschool could, but many preschool teachers share a similar or even more restricted view (Ginsburg, Lee, & Boyd, 2008). In Sweden, Björklund and Barendregt (submitted) asked 116 preschool teachers about what they focused on in mathematics in their preschools. They found that most teachers focused on numerical and spatial aspects.

The absence of working with mathematical patterns is notable. Still, there is a tradition in Swedish preschools of working with beads, pearls or sorting games and play, but this may not be seen as a means for working with mathematical relationship and is thereby not problematized and scrutinized as a learning object and content within the goal-oriented education. One reasonable explanation for this could be that teachers have not reflected on the variety of

aspects that mathematics consist of and thereby do not regard such activities as being part of mathematics. (Björklund and Barendregt submitted)

Even though the Swedish preschool curriculum, implemented in 1998 (Skolverket, 1998), included mathematical topics, such as measurement, shape, space and time, there is some uncertainty about the extent that Swedish preschool teachers introduce children to these ideas (Doverborg, 2006). This paper investigates preschool teachers' descriptions of mathematical activities that children engage with, after they have participated in a professional development course, and in the light of the revised Swedish curriculum. Using Bishop's (1988a; 1988b) six mathematical activities we analyse data provided by preschool teachers as a response to a prompt about the about the mathematics they present to children or consider that children engage in.

## **Bishop's six mathematical activities**

Although not formally acknowledged, the mathematical objectives highlighted in the revised Swedish preschool curriculum can be traced back to Bishop's (1988a; 1988b) six mathematical activities (see Utbildningsdepartementet, 2010). In this background document is written:

One way to concretely approach the objectives of the curriculum is to start from six historically and culturally founded activities. These activities may function as a structure in different context where mathematics can be discerned, explored and experienced. The activities provide opportunity to work with all objectives in mathematics in the preschool. They point out in which situations children and adults may need to use mathematics among other things. This entails that these activities not just connect to all objectives but also to the motives for the objectives. (Utbildningsdepartementet, 2010, p. 11; our translation)

Bishop (1988a) argued that the six activities were universal for any culture and labelled them as mathematics, with a small "m". The discipline of academic Mathematics, which he capitalised, included specific versions of the six activities. Bishop (1988b) summarised the six activities as:

**Playing**. Devising, and engaging in, games and pastimes, with more or less formalised rules that all players must abide by.

**Explaining**. Finding ways to account for the existence of phenomena, be they religious, animistic or scientific.

**Measuring**. Quantifying qualities for the purposes of comparison and ordering, using objects or tokens as measuring devices with associated units or 'measure-words'.

**Designing**. Creating a shape or design for an object or for any part of one's spatial environment. It may involve making the object, as a 'mental template', or symbolising it in some conventionalised way.

**Counting**. The use of a systematic way to compare and order discrete phenomena. It may involve tallying, or using objects or string to record, or special number words or names.

**Locating**. Exploring one's spatial environment and conceptualising and symbolising that environment, with models, diagrams, drawings, words or other means. (p. 182)

In an analysis of video recorded data from one preschool in Sweden, we found that Bishop's six activities were all represented either through explicit interactions or incidentally through the provision of physical resources in preschools, (Johansson et al., 2012). However, this analysis was based on our interpretation of what we, as researchers, saw in the data. We were not sure that the teachers would have produced a similar analysis.

In this paper, we use Bishop's activities as an analytical tool for two reasons. One is the connection to the preschool curriculum (Utbildningsdepartementet, 2010). The second reason is that the objectives in the curriculum are not learning objectives for the children to reach and be assessed upon, but objectives for the preschools in regard to the learning opportunities they provide to children. Thus, we needed an analytic tool that ensured that we did not tacitly and inappropriately import school views on what counts as mathematics. By introducing teachers to the idea of Bishop's six activities and then asking them to describe what occurred in their own preschools from this perspective, we wanted to determine whether there was an even distribution of activities in the teachers' descriptions and to find out how the teachers reflected on using such a classification.

## Collecting and analysing the data

The data consist of the final written assignment of 84 preschool teachers who had attended an in-service course focused on mathematics in preschool. Although not explicitly stated in the course syllabus, the course was based on Bishop's six mathematical activities. At the end of the course all participants were asked to answer the following three questions as a writing task.

- Vilka insikter har du gjort om dig själv, barnen och din praktik? (What have you learned about yourself, the children and your practice?)
- Vilka kunskaper har du utvecklat i och om matematik? (What knowledge have you developed in and about mathematics?)

 Beskriv hur du relaterar dessa kunskaper till hur barn l\u00e4r och anv\u00e4nder matematik. (Describe how you relate this knowledge to how children learn and use mathematics.)

Sometime after the course had finished, the teachers were asked if their assignment could be used as data. Of the 147 participants contacted, 84 responded favourably. When responding to these questions, teachers were expected to quote from the course literature and the preschool curriculum. Consequently, in the analysis of the data, statements about the curriculum or quotes from the literature were ignored. Instead we categorised the examples the teacher gave as examples of the mathematics, on which they were working or had begun to pay attention to, according to Bishops six mathematical activities.

## Categorising the mathematical activities

Each teacher's response was read and examples were classified based on Bishop's (1988a, 1988b) descriptions of the six activities. When Macmillan (1998) used Bishop's six activities to classify preschool children's play, each example was labelled as only one kind of activity. However, in our data it was common that the teachers' examples could be classified as several activities simultaneously. Bishop (1988b) indicated that both kinds of categorisations were possible "the activities can either be performed in a mutually exclusive way or, perhaps more significantly, by interacting together, as in 'playing with numbers'" (p. 183). An example from our data is:

Not to forget the winter which we are approaching, where one can build in snow and experience the concept of high and then on your own get to the top of the large snow pile and to experience it with your own body how difficult it actually is to climb that high

The part "one can build in snow" is categorised as Designing while "experience it with your own body how difficult it actually is to climb that high" is categorised as Locating. Thus, some examples could be in several categories while others were categorised as only belonging to one activity.



Figure 1: The distribution of preschool teacher's examples

Figure 1 provides an overview of how the examples were classified. The vertical scale is number of times each category appears. We can see that Designing, Measuring and Counting more often featured in the teachers' examples than the activities of Locating, Playing and Explaining. Playing and Explaining never occurred in isolation but always in relationship to one of the other four activities. In the next sections, we discuss why this can be and also give examples of each activity, while acknowledging that some examples could exemplify several activities.

#### Playing

Bishop's (1988b) mathematical activity Playing has similarities with the Swedish word "lek", but also some differences. According to Bishop, Playing consist of rules, which are more or less formalised. This has a connection to playing games, which in Swedish would be "spela", but could also be role-play, playing families and other kinds of play where the children imitate the real world in same way. This kind of play is "leka", but could include making decisions about the rules of the play (who is going to be the mother, father or dog, for example).

In the data, almost all of the preschool teachers used the Swedish word "lek" in connection to building play, movement play (bygglek, rörelseslek) etc. These were not counted as indicating the activity Playing, because it was not clear if the teachers were discerning the children's modelling, abstraction or hypothetical thinking which Bishop means is what makes Playing an mathematical activity. Rather it seemed that the teachers' conceptions of play were tightly connected to the curriculum which suggests that learning occurs through play.

Children's play and creative activity cannot be separated from their learning because it is the same thought process which is activated when children express themselves in, for example drama play or drawing, as when children try to create understanding and solve a mathematical problem or inversely use mathematics or technology to make a stable construction in creative activity and building play. In that way mathematics becomes both a goal and a means [to achieve other goals]. (Utbildningsdepartementet, 2010, p. 5; our translation)

Play, as in "lek", is central in the curriculum and this seems to be reflected in preschool teachers' views on what should happen in the preschool. Therefore, it is not so surprising to see it mentioned but with a limited connection to the mathematical activity Playing. However, it is clear that if Playing is to be taken seriously as a mathematical activity and not just as a pedagogical practice, then future pre-service and in-service education needs to support teachers to gain a more comprehensive view of what Playing can and should be as a mathematical activity in preschools.

## Explaining

According to Bishop (1988a), the mathematical activity Explaining answers the question "why". Nevertheless, preschool children's explanations often have a different form to those of adults or older children and so teachers may not always recognise them. In the following example, Sara describes a little boy playing and exploring with some sticks:

One morning at preschool, I saw how little Emil from the toddlers group went around with a bunch of short sticks in one hand and a long stick in the other. "What do you have there?" I asked. "Many sticks," he replied. "What do you have in the other hand then?" "Not many!" said Emil. "Yes, that's right," I said, "because you only have a stick." He went to show his sticks to some of the older kids who were involved in building a hut from long branches. They had pushed one of the branches down in a snowdrift. Emil stabbed his long stick in the snow and looked alternately at it and the even longer branch, and said, "Mine is small!" At another time the same morning he sat on the ground and had lined up his sticks, two of them had the same length, which he had placed next to each other. He had the sticks in his hand throughout the morning before finally putting them in his pocket to go to lunch. In the afternoon he went out with the sticks in his hand! During the morning Emil explored a lot. He noted that the sticks were similar but at the same time different in shape and size. He distinguished and grouped parts into a whole, he categorised, formed pairs and more. He met adults who saw and put into words what he experienced and adults who had the ability to take his point. Teachers from the toddlers section had seen how important the sticks were for him. Sara

In this example, we can see from Sara's description that Emil does clearly not use a verbal explanation but rather provides a form of explaining through categorising. Bishop (1988a) suggests categorising is one kind of explaining because it involves identifying a relevant attribute by which to make distinctions between items. Thus, there is an implicit explanation in deciding that an item belongs to one group rather than another. However, it would seem that the teacher identified this child's actions as examples of the mathematical activities Measuring and Counting. All the examples which we categorised as the mathematical activity Explaining would perhaps not be recognised by the teachers as such but rather as other mathematical activities. Although one of the goals of the curriculum is that preschools should offer children opportunities to "develop their mathematical skill in putting forward and following reasoning" (Skolverket, 2011, p. 10), if the teachers do not recognise classification as a form of Explaining, they perhaps will miss opportunities to develop this activity.

## Locating

Locating as a mathematical activity is about children locating themselves and other things in space. In the data, the examples included drawing, following maps and exploring the environment. Often position words were mentioned by the teachers. An example is the following:

For example if the child should go on the slide, then I give the terms for what they are doing right then - now you climb up the ladder, then you should go down the slide. Another example - look the toy car went under the table, can you crawl under the table to retrieve it? *Marcus* 

It was somewhat surprising that examples of Locating appeared relatively rarely in the examples that the teachers gave. Connections to space were mentioned in the 1998 version of the curriculum (Skolverket, 1998) as well as the revised curriculum (Skolverket, 2010). From our previous investigation (Johansson et al., 2012), we also had identified many examples that we could classify as Locating. It may be that exploring space and giving labels to children's experiences are so built into teacher's practices that they fail to recognise them as mathematical activities. However, it is clear that more research is needed to better understand why Locating, Explaining and Playing are not so well represented in teachers' examples.

# Designing

Designing uses the image of a structure, often based on something in the environment to design an artefact. This design can be used to construct the artefact, but Bishop (1988a) is careful to point out that it is the mental actions of designing that makes Designing a mathematical activity. However, the focus of the preschool teachers seemed to be not so much on the designing of artefacts as of naming shapes and their particular features. The following is an example of this.

When children do a puzzle, they must look at the shape, colour and image simultaneously. *Klara* 

Being able to imagine the features needed in building is not mentioned, for example. Rather, the preschool teachers consider preschool children's choice of shapes in the construction of artefacts to be connected to the mathematical activities of Counting, as in the example below, or as Measuring.

For example, at the lego table, the counting and calculating - I need a red narrow six door. *Fredrik* 

This example was included as Counting because the teacher seemed to focus on the six. However, as the child seemed to focus on the features of the block needed for completing the building it was also classified as Designing. As the case with Locating, it seemed that the teachers did not recognise situations, in which the children engaged, that had links to other mathematical activities than Counting and Measuring. Clements and Sarama (2011) indicated that "geometry and spatial thinking are often ignored or minimized in ... early education" (p. 133). However, we would suggest that it is not a case of ignoring or minimising the situations, but rather not recognising that they and the children were in engaged with Locating and Designing. The examples that teachers gave, which we categorised as Designing, fitted the more traditional view that preschool children should learn the names of two dimensional shapes. This raises questions about whether teachers need to be introduced more explicitly to Bishop's six activities in order for them to be able to recognise them in their own practice and to be able to provide opportunities for the children to engage in all of the mathematical activities put forward in the curriculum.

## Measuring

There were more examples that were classified as Measuring than any other mathematical activity. Almost a third of these examples were about sorting or comparing in terms of size. Almost all of these were about length as was the case in the first example and in the example below.

On the first occasion, they measured one child's length using pencils. Then they started making their own tapes which became too tedious after a while. Then they came up with the idea to take the bead jars to help them to measure the remaining children. This was not completed all the way when one of the children ran off to fetch blocks. One problem that arose for the children on the first occasion was that the kids realized that the boy was seven and a half pencils long were in fact the longest. Two of the other boys were equally long, but shorter than the boy who was measured using pencils. They were thirty jars and blocks long and the girl who became the shortest was twenty-eight. How could that be? *Lena* 

The examples of the mathematical activity Measuring is not dominated by measuring with a specific tool but rather measuring or comparing with different kinds of objects. However, research on a six/seven year old child's out-of-school experiences (Meaney, 2011) suggest that there would be many other kinds of measuring than just length that children engage with. Consequently, it may be that the teachers need some more understanding of how to recognise potential situations in which to engage children in Measuring activities.

## Counting

Counting was also a mathematical activity with many examples. The examples in this category include counting objects, sharing, determining how many remain after something is removed and pairing. The examples are from the everyday life

in the preschool whereas the examples for the other activities were from playful or planned situations. The following example is typical in that sense:

For example, at mealtime, setting the table, the children count how many children are going to eat, set the table the appropriate number of plates glasses and cutlery for the number of children. We share the fruit in halves quarters etc. *Agneta* 

Ginsburg et al. (2008) and Björklund and Barendregt (submitted) suggested that preschool teachers' predominant view of mathematics revolves around numbers and shape names. It was therefore interesting to find that there were more Measuring examples than any other activity. It also seems that Ginsburg et al. (2008) concerns that US preschool teachers "generally do little to encourage counting or estimation, and seldom use proper mathematics terminology" (p. 6) were not relevant in regard to these Swedish preschool teachers who by introducing fractions involved children in a wider range of Counting activities. Nevertheless, it also seemed that some of the variety of activities that was documented was a result of the teachers attending the professional development. In the following quote, a teacher described how she had previously equated mathematics with the mathematical activity Counting.

For example I have not used the word mathematics instead replacing it with "let's count".

Still there did remain some confusion over what mathematics could be developed from engaging in different situations:

Finger Chants do not necessarily have a mathematical content, but it encourages mathematical thinking.

There seems to be a contradiction in this quote which suggests that some more research about what preschool teachers learn from engaging in professional development and how it affects their practice is needed.

## **Discussion and conclusion**

This paper identified that although preschool teachers focused more on counting and measuring in their writing task, other mathematical activities were exemplified as well. Although some teachers recognised all the mathematical activities, it was apparent that most provided examples of Playing, Explaining and Locating but, by not explicitly labelling them as mathematical, may not have recognised them as such. It is interesting to note that sometimes teachers were aware that this was the case for the children, but not necessarily the case for themselves:

The children 'talk' about mathematics without knowing it.

Nevertheless, although Bishop's (1988a) six activities were not explicitly described in the professional development, the teachers considered as beneficial having an alternative way of viewing mathematics was considered:

Even to me, because I work with preschool, and work a lot with school, I somehow slipped into the school's working too much. Instead of transferring the preschool approach to school so it has become the opposite. Maybe because I have not had the right argument to advance preschool practices.

It is likely that the twelve years that teachers had engaged in school mathematics will have affected their perceptions of what and how they can engage children in mathematical situations in preschool. From our research, it seems that providing an alternative way of conceptualising mathematics may help preschool teachers take a broader view of what they should offer children. However, it also seems that changes will take time and a more explicit discussion of Bishop's six activities could be beneficial for future professional development programmes.

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