News from Nordic mathematics education

In this issue of Nomad, the NoRME column reports on some activities within the field of mathematics education research in the Nordic region. Readers having information relevant for this column or for the NoRME web page (www.norme.me) are encouraged to contact Mette Andresen, the chair of NoRME, by e-mail: mette.andresen@uib.no

MatRIC

MatRIC continues its activities, amongst others one colloquium on mathematical simulation and visualisation May 26–27, 2015, one colloquium on mathematical modelling May 28–29, 2015 and one colloquium on computer aided assessment in mathematics June 1–3, 2015. For information on events, publications and workgroups etc., see www.matric.no

The Agder project

University of Stavanger and University of Agder collaborate within *Agder prosjekt*, a research and development project which aims at the development of knowledge about key content for five year old kids in kindergarden. See www.uis.no/forskning/skole-og-barnehage/agderprosjektet/om-agder-prosjektet (in Norwegian).

Nordic & Baltic GeoGebra network

The *Nordic & Baltic GeoGebra network* continues its activities; next conference will take place in Karlstad University, Sweden, October 23–25, 2015. Read more here: nordic.geogebra.no

Besides arranging the conferences, the network project group is obliged to survey the network project's influence on the theachers' use of GeoGebra in school mathematics. Therefore, we invite everybody to answer the Nordic questionnaires on the project's website or to contact the survey team with relevant information. The survey team consists of: Morten Misfeldt (Denmark), Maria Fahlgren (Sweden), Anna Kairema (Finland), Sirje Pihlap (Estonia) and Mette Andresen (Norway).

Mette Andresen
University of Bergen

Andresen, M. (2015). News from Nordic mathematics education. *Nordic Studies in Mathematics Education*, 20 (2), 103–114.

NORSMA 8

NORSMA 8: Connecting research and practice. The 8th Nordic research conference on special needs education in mathematics will be held at Kristianstad University, Sweden, in November 2015. Read more here: www.hkr.se/ruck-norsma8

Ph.D course

Ph.D course at Copenhagen University: *Educational design in mathematics and science: The collective aspect*. Read more here: www.ind.ku.dk/english/course_overview/PhD_Courses/collective

NORDIMA

NORDIMA is a joint Nordic master's programme in didactics of mathematics. University of Copenhagen is part of a consortium with 5 universities from Norway, Denmark and Finland offering this new master programme in the didactics of mathematics called NORDIMA. The programme results in a so called "double degree". At University of Copenhagen the programme is rooted at the Departement of Science Education. Professor Carl Winsløw is coordinator for the programme. Read more here: www.ind.ku.dk/english/course_overview/nordima

NORMA 17

IPC NORMA17 is now established. NORMA 17 will take place May 30–June 2, 2017 at Stockholm University. The IPC members are:

Sweden
Eva Noren, Chair (eva.noren@mnd.su.se), Hanna Palmer (hanna.palmer@lnu.se), Johan Prytz (johan.prytz@edu.uu.se) and Paul Andrews (paul.andrews@mnd.su.se)
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Freya Hreinsdóttir (freyjah@hi.is)

New dissertations in mathematics education

Sweden

Niclas Larson (2014), *Matematikämnet och stadiebytet mellan grundskolan och gymnasieskolan. En enkät- och klassrumsstudie.*
Linköping university
English abstract: The aim of the study was to explore mathematics education and the role of mathematics in the transition from lower to upper secondary school. The study was guided by two research questions about the affection of mathematics while students apply for a study programme at upper secondary school and what characterises mathematics education in last year of lower secondary and first year of upper secondary school. To answer the first question a survey was given to all students in one municipality in their last year of lower secondary school, while the second question was investigated through videotaped lesson observations at both school levels, complemented by student interviews. The outcomes show that future options and occupation were important factors for the choice of study programme. About 36 % of the students claimed they had not been influenced by mathematics in their choice of programme while 35 % expressed they had, mainly in a positive sense although there were differences between the programmes. The analysis of the classroom observations and interviews was supported by concepts from Bernstein's theory of pedagogic discourse and the anthropological theory of the didactic, and indicated strong similarities between the levels regarding the lesson structure and students' freedom to select tasks to work on. However, at the upper level front teaching was more frequent as well as extended in time and the students reported an increased study pace. The study highlights several aspects of and connections between students' choice of programme and characteristics of the mathematics education during the transition, which should be of interest to career choice advisors and the organisation and design of education. Potential problems related to the invisible pedagogy observed regarding knowledge criteria and selection of tasks need to be raised in current debates on school mathematics.

Yvonne Liljekvist (2014), *Lärande i matematik – om resonemang och matematikuppgifters egenskaper.*
Karlstad University
English abstract: Since mathematical tasks are central to the teaching of mathematics, it is crucial to extend our knowledge of the characteristic features of tasks that are conducive to student development of problem-solving and reasoning abilities as well as conceptual understanding.The

aim of the dissertation is to investigate how different types of mathematical tasks affect student learning and choice of learning strategies. This is done through a twofold approach: 1) to test the hypothesis that tasks affording students the opportunity and responsibility for constructing knowledge are more effective learning tools than tasks for which the solution is presented, and 2) to analyse the educational message embedded in the teacher's formulation of the mathematical tasks on the Internet. The main conclusion is that the type of task students engage with is important for their learning of new things. The participants who were engaged in creating their own solutions were less successful during practice but performed better on the tests in comparison with the participants who were involved in solving the tasks with a given method. The results of the sub-studies indicate that in a learning situation consisting of repeated practice of a solution method, the results are closely related to the students' cognitive ability. The investigation shows that tasks inviting the opportunity to be solved through creative reasoning, to a certain extent serve a compensatory function in relation to students' cognitive resources. This means that the participants need not put in so much effort in the test situation if they have practiced creative reasoning. One conclusion to be drawn from the study of the educational message in Internet documents, when it comes to teachers' formulation of tasks, is that there are many teachers who design tasks that encourage young students' creative reasoning. However, the educational message in the documents shows that the teachers demand relatively little of the students in the majority of the tasks. The result indicates that there is some uncertainty about how to formulate and use tasks to support the older student's mathematical development. The way the tasks are formulated indicates a lack of discursive tools to clarify the intended educational situation. Thus, the qualities in the tasks are hidden resources.

Kicki Skog (2014), *Power, positionings and mathematics – discursive practices in mathematics teacher education.*
Stockholm university
Abstract: This is an ethnographic study from within mathematics teacher education in Sweden. A methodological insider approach enabled to view teacher education from the students' perspectives, by focusing how discursive power-relations affected what becoming mathematics teachers brought forward as concerning during two years of education. I took a socio-political theoretical perspective and understood discourse, power and positioning as dynamically interrelated concepts, which allowed the analysis to foreground several aspects simultaneously and to illustrate elusive phenomena as they occurred and disappeared.The results

show that the mathematics education and mathematics discourses are open and multifaceted and reveal empowered positionings, whereas the language/culture and institutional discourses both are narrower and more constraining. These constraints, in turn, affect students' possibilities to enact empowered positionings within the more open discourses. The core of education, that is mathematics and mathematics education, may therefore be obscured by discourses of "truths".The study shows a need for further research on how to strengthen students' possibilities to influence their education, and to ask questions like why education is organised this way, and who benefits from that.

Olov Viirman (2014), *The function concept and university mathematics teaching.*
Karlstad University
Abstract: This thesis concerns the teaching of mathematics at university level, with a particular focus on the teaching of the function concept. The main aim of the thesis is describing and analysing the teaching practices of university mathematics teachers regarding the function concept, and how this concept is constituted through these practices. To this end, video recordings of lectures by seven mathematics teachers at three Swedish universities were analysed using a discursive perspective, Sfard's commognitive framework. The observed teaching was traditional in form, with teachers using "chalk talk" – simultaneously talking and writing on the board. The results show that the teaching practices of the teachers belong to two distinct but intertwined discourses – a mathematical discourse, and a discourse of mathematics teaching. Classifications of important aspects of these discourses are presented, and it is found that the teachers' discursive practices, while sharing overall form, still display considerable differences. Other results include an analysis of the levels of objectification displayed by the teachers in their discursive constitution of the function concept. The study contributes to a small but growing body of empirical research on university mathematics teaching practice.

Björn Palmberg (2014), *The influence of national curricula and national assessments on teachers' beliefs about the goals of school mathematics.*
Umeå University
Abstract: What students should learn in school and therefore also what teachers should teach is an important issue worldwide. Mathematics teaching (and teaching in other subjects) is often regulated by some form of governing text in a written curriculum communcating a set of standards. Another common mean through which policy is communicated is assessments, which for example can convey policy by communicating

desirable outcomes in student learning. A common problem with regulating what teachers do through policy means is that it often is difficult to achieve intended changes.

This study investigates the impact of a national reform in Sweden initiated in 1994, introducing mathematical competency goals by communicating them through the national curriculum and national assessments. The study is based on analysis of data obtained from the Swedish Schools Inspectorate (SSI), which conducted a quality review of upper secondary school mathematics teaching. During this quality review, the SSI collected data on a representative sample of 145 upper secondary mathematics teachers through interviews, observations, and surveys. This was done in 2009 and 2010, which means that the reform from a time perspective has had ample time to exert influence on teachers. In the study the data obtained from the SSI was analyzed in order to answer two questions:

1. have teachers changed their beliefs about the goals of upper secondary school mathematics in line with the intentions of the reform, and

2. why have, or have they not, changed their beliefs about the goals of upper secondary school mathematics in line with the intentions of the reform?

In research on teachers' reception of policy messages, similar to the one introduced in Sweden, it has been found that a common response to these messages is that teachers are positive to the message. However, although positive, teachers have often been found to only adopt superficial properties of the reform while still maintaining a highly traditional view of teaching and the goals of teaching, not consistent with the intentions of the reform. Therefore, the questions in this study were examined by using a model that can explain why teachers, when confronted with a reform message, change their beliefs in profound or superficial ways, or not at all. Through analysis of the SSI-data, measures on constructs of the model were obtained, and with statistical means it was examined whether the model can account for the changes in teachers' beliefs about the goals of upper secondary school mathematics.

The results of the study suggest that the Swedish reform has had a relatively small impact, and that the model can give an explanation to why some Swedish upper secondary teachers of mathematics have changed their beliefs in line with the reform, some have changed them in superficial ways, and some have not changed them in any discernable way. Whether teachers perceive the reform as entailing an important and nontrivial change for them seems to be of utmost importance. The results

of this study suggest that if teachers do not perceive this, they will not process the message deeply, which by the results of this study suggest that there is little chance for them to change their beliefs in a profound way. If they however do perceive the message as entailing an important and non-trivial change, this study suggests that chances are greater that teachers will change their beliefs in line with the reform. Teachers' interest in the subject and their perceptions of the usefulness of the documents communicating the message are then in this study suggested to be important factors influencing whether teachers will process the reform message systematically, which in turn heavily influences whether they will change their beliefs in a profound way. One practical implication, suggested by this study is that when policy communicates a new and non-trivial message with the intention of influencing teachers, it is important that the message is communicated clearly. Such clarity makes it more difficult for a teacher to superficially interpret the message as being in accordance with the teacher's earlier beliefs, and thus not entail any need for change. However, to attain such clarity of a complex message is not an easy task to accomplish.

Norway

Anette Jahnke (2014), *Insegel till dialog. Skolans matematikutbildning – en studie i fyra praktiker.*
University of Nordland
English abstract: Mathematics education in Swedish schools is the subject of constant debate which seeks to improve and change school practice. This is, however, often undertaken without sufficient attention to or understanding of the practices that shape mathematics education on a daily basis. The purpose of this thesis is to reach a better understanding of how school mathematics education is formed by examining four different practices; the curriculum developer's, the mathematician's, the teacher's and the school principal's. The investigation involves formulating knowledge – often tacit and unformulated – developed in the four practices, three of which make up my own professional experience. The work is grounded in the epistemological claim that practical knowledge denotes an unique species of knowing which cannot, without further ado, be reduced to applied theory, but must be developed and communicated in its own way. The work is also based on an insight drawn from discussions in the philosophy of language perspective; namely, that it is through our actions that language receives its meaning, and that even theoretically formulated knowledge is constituted by the unsaid as much as by the said. The Dialogue Seminar Method, developed at KTH Royal

Institute of Technology in the subject of Skill and Technology, has been used, with the help of the philosophy of science and art, to find a language to convey practical knowledge. The method has been used partly on my own professional experience, and partly on the professional experience of a group of school principals. Furthermore, the method has resulted in texts produced between 2004 and 2011. These texts have been interpreted in light of the phenomenological and hermeneutical traditions, in accordance with the tradition of the subject Studies of Professional Practice at the University of Nordland. In addition, an epistemological investigation has been made of the concept of knowledge that the Swedish school has been based on since 1994. The thesis provides three contributions. The first is a theoretical contribution where three shortcomings are highlighted in the concept of knowledge that the Swedish school rests on, where knowledge is seen as consisting of facts, understanding, skills and familiarity: i) tacit knowledge is not only found in familiarity but cuts across all four knowledge forms; ii) understanding is not only associated with facts but also with skills; iii) the language that understanding can be conveyed through is not only in terms of defined concepts, but also in the form of metaphors, analogies and stories. The second contribution is a portrayal of the four practices in the form of stories. The hermeneutical and phenomenological work on the stories provides the third contribution in the form of a deepened understanding of six phenomena that practitioners manage in their work life: i) problem and solution; ii) nonexistence, solitude and community; iii) concern about the unforeseen and human ignorance; iv) the relationship between school mathematics and usefulness; v) the curriculum – text, interpretation and action; vi) reactions from pupils on their activities. The constitution of these phenomena is analysed and their mutual influence on each other is discussed, as well as how a view on knowledge that equates practical knowledge with applied theory affects the phenomena. The three contributions of the thesis show that it is a fundamentally practical knowledge to understand and use mathematics. This gives a different and deeper understanding of how mathematics education is formed than the one that dominates public debate. Based on the three contributions conclusions are drawn for the further development of mathematics education. Practical knowledge and its tacit elements in mathematics education exist and need to be addressed. The habit of discussing mathematics education in the form of problem and solution, where metaphorically this is seen as a puzzle to be solved, means that we are concentrating too much on trying to manage an unpredictable reality by predicting – because mathematics education can metaphorically also be regarded as a living being that is dependent on its heredity and environment, and on recurring natural needs. The thesis is a seal to enhance the quality and continuity of the dialogue we

conduct on mathematics education, by making visible the complexity of knowledge, and our disagreements, thus going from many monologues to a genuine dialogue – from school for Bildung to the Bildung of the school.

Hans Kristian Nilsen (2015), *Learning and teaching functions and the transition from lower secondary to upper secondary school.*
University of Agder
Nilsen has followed eight students during their final semester at lower secondary school and into their first semester at upper secondary school in order to obtain insight into how the concept of function develops in the transition from lower to upper secondary school. In his study he investigates both how the students express their conception of functions, and also related topics such as gradients, and how these concepts are presented at the two levels of schooling. In Norway students in upper secondary school choose either the "general studies programme" (GS) or the "vocational studies programme" (VS) and both programmes include mathematics. Since Nilsen decided to follow the same students in the transition he would have to enter the study programme at upper secondary that each of the students happened to choose. This had as a result that the study includes transition both from lower secondary to upper secondary VS and to upper secondary GS. Both aim and scope of mathematics in GS is quite different from that in VS but the concept of function is present in both programmes. However, when it comes to concepts following on from the function concept the situation is quite different. In first year of GS the derivative is introduced but this is not the case in VS. This is the reason why Nilsen is referring to the broader concept of gradient instead of to the derivative.

The study is characterised as a longitudinal case study within an interpretative paradigm. The main data for the project come from video and voice recordings, including classroom observations, interviews with students and interviews with teachers. To describe how functions and related concepts are perceived and presented a process of coding and categorisation has been used. In his analysis Nilsen draws heavily on the epistemological theory by Heinz Steinbring, where the Epistemological Triangle plays an important role. Nilsen also makes use of semiotic chaining to show how a concept develops from a rather concrete starting points into more abstract ideas using more advanced semiotic representations. An example of this is how the derivative as a limit develops from the gradient, which in the representation $y = ax + b$ is referred to as "one unit right and a up or down". The study shows that in all schools involved the function is usually identified with its representations and there are very few traces of work with the general idea of a function. The uniqueness property of functions was only implicitly dealt with. For gradients the

prevailing strategy was the "one unit right and a up or down" approach, combined with a physical movement on the graph.

This study adds nicely to the literature on transition issues, which seems to gain increased interest. Nilsen comments on the need for teachers in both lower and upper secondary school to have knowledge of both levels. Similar comments could be made also at other transition points, such as between upper secondary school and higher education.

Ida Friestad Pedersen (2014). *Insights from TIMSS Advanced on critical aspects of the advanced mathematics program in Norwegian upper secondary school, content, competence, and motivation.*
University of Oslo
English abstract: Concerns have previously been raised regarding recruitment to tertiary studies in the science, technology, engineering, and mathematics (STEM) fields. This includes (1) insufficient participation and female underrepresentation in this field, and (2) students entering tertiary studies with an inadequate mathematical background. With this as a backdrop, I aim to offer insights on the mathematical content of courses offered to upper secondary school students opting to specialize in mathematics, on the mathematical competence of these students, and on their motivations for participating in mathematics.

This thesis utilizes data from the large-scale international achievement study TIMSS Advanced, and presents analyses of student responses to assessment items and background questionnaires, as well as descriptions of the mathematical content of the TIMSS Advanced assessment instruments and the Norwegian curricula for the advanced mathematics courses.

Briefly, my analyses reveal that the curriculum documents appear to place the greatest emphasis on applying procedures, while the more advanced cognitive processes (analyzing, evaluating, and creating) seem virtually absent. I also find that there is a moderate agreement between the curriculum and the TIMSS Advanced mathematics tests, indicating that there are differences which should be taken into account in order to draw valid conclusions from this large-scale survey. Turning to student competence, my results show that although the curricular emphasis is on applying procedures and methods, the students tend to perform weakly when items require the use of algebraic procedures and stronger when they are required to formulate mathematical models or use mathematics in applied contexts. Finally, the utility value of mathematics for university admission was an important motivation for choosing mathematics, alongside interest in mathematics as a school subject. These findings may

inform the development of teaching approaches, or interventions aimed at encouraging student participation in STEM.

This dissertation is a contribution to the field of mathematics education research, and has been conducted at the Department of Teacher Education and School Research, University of Oslo.

Janne Fauskanger (2015), *Å måle og studere matematikklæreres undervisningskunnskap.*
University of Stavanger
English abstract: The notion of "Mathematical knowledge for teaching" (MKT) describes the special and particular knowledge required by teachers to do the work of teaching mathematics. This notion, developed by researchers in the USA, connects mathematical content knowledge with knowledge of teaching and learning. As such, teachers' MKT is widely acknowledged by researchers and educators as an important factor in determining the quality of teaching and of students' learning. Having accepted the notion of MKT, the issue under consideration in this work is how to access and assess teachers' personal mathematical knowledge for teaching. The founding scholars in the USA developed a conceptual framework and based on this framework an instrument to access and assess teachers' MKT. A growing body of research from inside and outside of the USA supports the usefulness of both the framework and the instrument in measuring situated knowledge and in monitoring the development of MKT over time. The instrument is arguably of high quality and the multiple-choice format allows for large-scale use. More interestingly, studies have found significant correlations between MKT as measured by the instrument and both the quality of teaching and students' results. This doctoral work is constituted of six studies involving in total 177 teachers from Norwegian schools. The main aim of the work is: to develop knowledge about the efficacy of the MKT instrument in accessing and assessing mathematical knowledge for teaching and in this process to identify associated challenges and potentials in the employment of the instrument. In line with the MKT notion, knowledge, in this work, is viewed as a personal construction and teachers' knowledge is studied through analyses of its expression in oral and written utterances. A mixed methods design is used in the study. This choice is based on the premise that a combination of data creation methods would illuminate different aspects of the research question(s) and thus provide a better understanding and deeper insight into a complex research question. The MKT instrument was developed in another country for another teaching context. Studies 1 and 2 focus on issues related to translating and adapting the instrument for the Norwegian context. Though mathematics is

often described as a universal language it was found that the naming of mathematical concepts is not so easily translated. Study 3 focuses especially on the challenges and potential of the multiple-choice format of the MKT items. The contribution of the first study has been the elaboration of several categories in the documentation of translation work. Study 2 concluded that the adaptation process is a dynamic and ongoing process, not complete after one round of translation, revision and quality control. Study 3 provided a deeper understanding of the complexities related to the use of multiple-choice format in the MKT items – referring particularly to the suggested solutions in the MKT multiple-choice items. Studies 4 and 5 considered more deeply and directly which information on teachers' MKT is accessed by the MKT instrument. By asking teachers to elaborate on their answers to the MKT multiple-choice items, and analyzing these responses the study indicated that teachers drew on aspects of MKT not intended by the items. These findings may suggest inconsistencies between the MKT framework and the MKT items. The multiple-choice response "I'm not sure" proved especially challenging to interpret. The study (number 6) did not find a clear correlation between the MKT as assessed by response to the multiple-choice and the MKT as assessed by the corresponding written reflection on the item. The work in its entity illuminates several challenges in using the MKT instrument. In the Norwegian context the issues of language translation, cultural adaptation, differing situations, unfamiliar format and so on are all found complex and further complicated by the illusive nature of mathematical concepts. All these aspects influence the interpretation of the MKT measures. The results indicate that the MKT framework and instrument, in its present design, should be used with caution in the Norwegian context. More importantly, the study opens up, explores and challenges the concept of MKT providing useful knowledge for a research based teacher education in mathematics teaching.

Denmark:

Latest dissertations will be announced in the next issue (if I get the relevant information from you out there, please!). We can expect more dissertations in future: five ph.d. students are working on it, and two more will start in the autumn.

Kind regards, Mette