The relevance of qualities of theories in mathematics education research

Mathematics education research has developed significantly during the last three decades, not only measured by the number of researchers, PhD students, publications, research conferences and other types of research related activities but also in terms of the quantity and quality of the theoretical production. Mathematics education research is an interdisciplinary field of research with complex relations to a number of supporting sciences. The ultimate aim of the research is to improve the practices of mathematics teaching and learning. These features of the nature, the growth and development of mathematics education as a research field call for reflections on the relevance and the qualities of the theoretical products developed.

In the Nordic countries mathematics education research has developed even more dramatically – in a shorter time span. During the last two decades the number of PhDs graduated in the Nordic countries has increased with a factor 5 to an average of around 10 per year. In the Nordic community it is, therefore, of particular relevance to keep the meta-reflections on our field of research alive and vivid. In general, metadiscussions and reflections can have at least four important functions. They can (1) strengthen the identity and foster reflections and critique among researchers, (2) play an important role in the education and enculturation of new researchers, (3) strengthen the interplay with supporting research disciplines, and (4) contribute to an explicit and transparent basis for discussing quality issues.

This issue

With this thematic issue of NOMAD we seek to contribute to keeping the meta-discussions alive on the Nordic scene of mathematics education research. Taken together, in fact, the papers published in this issue deal with all the four functions of meta-discussions mentioned. However, we do not intend to limit the meta-discussions to this issue of NOMAD.

On the contrary, papers and commentaries addressing meta-questions concerning our field are very welcome for coming issues of NOMAD.

The issue at hand contains four papers, a commentary and furthermore some information from NoRME, the Nordic Society for Research in Mathematics Education, that takes over from NoGSME on the continuous reporting of activities related to Nordic collaboration in mathematics education research.

In the first paper *What is quality in a PhD dissertation?* Mogens Niss shares with the readers his comprehensive experiences and deep reflections about criteria for scientific quality in PhD dissertations and, in particular, what he finds to be causes for rejecting dissertations. The paper is based on the many talks that Mogens has been giving at PhD-courses and conferences internationally and in the Nordic community, and on reflections deepened through many years of interaction and scholarly debate within the mathematics education research environment. However, it is one of the points of the paper that mathematics education research is a multi-disciplinary field of research in which there is no general agreement on what constitutes sufficient quality of a PhD dissertation. Therefore it is underlined that the paper presents the author's personal position on the issue. In accordance with this characterisation of the discipline it is argued that every PhD project is unique, and that every graduate student must strive for independence and originality in his or her research.

The paper concludes that as a PhD student in mathematics education you can not escape "thinking from scratch" in establishing the relevance and "worthwhileness" of the research, even if you are supervised by one of the most experienced, renowned and respected supervisors.

The second paper Theorising in mathematics education research: differences in modes and quality by Eva Jablonka and Christer Bergsten addresses precisely the question of the nature and quality of the theoretical constructs that mathematics education research creates. As indicated already in the title one of the main points in the paper is that not every framework or construct that is labelled theoretical posses sufficient qualities to deserve that label and that we should be cautious about what we promote as theories in mathematics education research. The authors discuss different frequently used notions such as theoretical framework, theory, theoretical model, theorising, theoretical object and construct, and illustrate with examples how such notions are used with different meanings in the research literature. Four cases of theorising are presented and discussed more thoroughly, namely the PISA framework, the notion of authentic tasks, the APOS theory, and ATD. Modes and quality of the last two cases of theorising are analysed and discussed using Bernstein's internal/external languages of description and Dowling's grammatical modes. The authors make the case that consistency and coherence of "home grown" theorising should be put into focus in mathematics education research.

For the first time we bring in this issue a commentary to a paper of the issue. Bharath Sriraman, who has edited together with Lyn English the newly published volume *Theories of mathematics education: seeking new frontiers* in the *Advances in mathematics education* series from Springer science, is commenting on the paper by Jablonka and Bergsten. Although Sriraman is generally in line with the analyses presented in the paper, he argues for the need of taking the analysis one step further in order to include the ideological basis for our judgements about what counts as theories in our field and also to analyse the ideological functioning of strong theories such as ADT in the forming of research communities in our field.

In the fourth paper Connecting theories in mathematics education: from bricolage to professionalism Tine Wedege in a sense continues where Jablonka and Bergsten ends. Tine Wedege is concerned with how in mathematics education research we connect different theories, for what purposes and with what qualities. However, in order to be able to discuss such questions in a scholarly manner we need first to develop a terminology that allows us to distinguish between and characterise different ways of connecting theories in our field. Such terminology or meta-language is argued to be a necessary step from a bricolage approach to professionalism in connecting theories in mathematic education research. The paper offers a contribution to the development of such a terminology in form of a matrix combining six different strategies (or purposes) for connecting two or more theories with the three theoretical levels - Principles. Methodology, Questions – taken from the work of Luis Radford. Some of the cells in this matrix are illustrated using four research papers of which one case from the author's own work is analysed in detail. The paper is rounded off with a discussion of criteria for quality in research papers in relation to using and connecting different theories in mathematics education research.

The paper by Barbara Jaworski and Anne Berit Fuglestad Developing mathematics teaching through inquiry: a response to Skovsmose and Säljö is a commentary to a paper published in NOMAD 2008, no. 3, that analysed two research projects: Learning Communities in Mathematics (LCM) and Information and Communications Technology in Mathematics Learning (ICTML), both conducted under the KUL project at the University of Agder in 2004–2007. In the paper the authors focus on the concept of inquiry in these two projects and discuss the criticism of the two projects raised by Skovmose & Säljö (2008). Hereby the paper represents

an interesting example of a discussion about the quality and relevance of a concrete research project in mathematics education. This type of debate is quite rare in the research literature and at research conferences. However, it is in relation to concrete research projects that the criteria for quality and relevance should be tested and developed. Therefore, we find the paper very relevant for this thematic issue, and we would also like more generally to welcome papers for NOMAD debating concrete research projects. One of the points of critique that the authors are responding to in the paper has to do with the different meanings of the key concept inquiry. The authors argue that in the KUL project, where didacticians and teachers are working together developing and analysing teaching situations, the development of communities of inquiry is the central unit of analysis. Although, the ultimate aim in the project is to develop a teaching practice where students are learning mathematics through inquiry based activities, the project did not focus on analysing teacher-student or student-student interactions in mathematical inquiry situations, and this was exactly a main point of critique raised by Skovsmose and Säljö. It seems as if the meaning of the concept of inquiry may vary significantly with the context and that the meaning therefore need to be established explicitly in each context. This seems to be a general challenge in mathematics education research.

The editors