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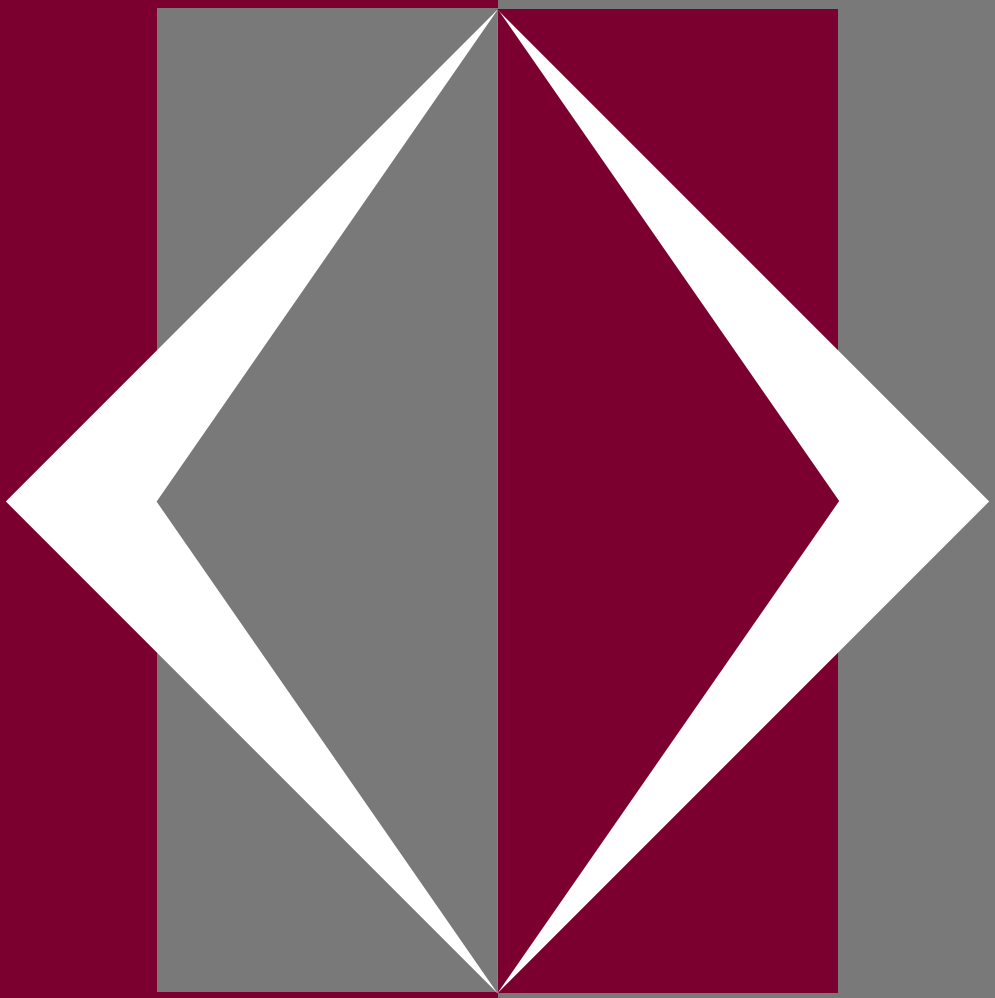

Adults and Mathematics

– a vital subject

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Lars Gustafsson & Lars Mouwitz

National Center for Mathematics Education, NCM

The Swedish government decided to establish a national resource center for mathematics education at Göteborg University with effect from January 1st, 1999. NCM will co-ordinate, support, develop and implement the contributions which promote Swedish mathematics education from pre-school to university college.
Further information on NCM is available from our web site
http://ncm.gu.se/
where there is also an English access point.

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Foreword

The National Center for Mathematics Education (NCM) was commissioned in October 2001 by the Swedish Government to carry out a survey and produce an analysis of the measures required to improve adult learning of mathematics. The results of this assignment were presented in June 2002 in the report *Vuxna och matematik – ett livsviktigt ämne* (NCM Report 2002:3).

Adult learning generally, and adult learning of mathematics in particular, are areas attracting ever increasing attention both nationally and internationally. International studies and comparisons e.g. the International Adult Literacy Survey (IALS), where the reading, writing and mathematics skills of adults have been studied, show that adult citizens from Sweden and the Nordic countries are very successful compared to countries which in cultural and social terms are similar. Among the explanations put forward for this are: the welfare system in the Nordic countries, high levels of participation in community and organisational life, the high proportion reading newspapers, as well as the high proportion of the population participating in some form of popular adult education. This shows the importance of both non-formal and informal learning. Different aspects concerning this are given prominence to and discussed in the report.

Given this background, we have decided to have the summary of the report translated into English[1] in the hope that the perspectives from which we view adult education and adults' learning of mathematics may be of interest to readers outside the Nordic countries.

Gothenburg in December 2003

Lars Gustafsson Lars Mouwitz

This English translation of the report is available in PDF format from the web site of NCM, http://ncm.gu.se/

[1] In its entirety the report covers 174 pages.

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The assignment

This report is the result of the Government decision (U2001/3808/V) to commission the National Center for Mathematics Education (NCM) at the University of Gothenburg to carry out a survey and produce an analysis of the measures required to improve mathematics learning among adults. This report, on which long-term and continuous development work will be based, identifies a number of areas where special measures should be introduced.

Background rationales

In addition to the assignment itself, this report also refers to current ideological and scientific perspectives and background rationales, which together provide a contemporary basis for the assignment. Among the variety of goals for education and learning, we see both individual educational and cultural goals as well as societal goals linked to the labour market and the business world. Other goals relate to citizenship, democracy and fundamental values.

A notable feature of adult education today is the large number and variety of providers of education. Forms of flexible teaching and distance teaching emerge with the assistance of ICT and a more widespread familiarity with computers. This opens the door to a considerable degree of individualisation and broad geographical coverage.

Today, teaching is commonly described along two dimensions: lifelong learning and life-wide learning, and learning is seen in relation to three different types of learning environments - formal, non-formal and informal. From national, European and global perspectives it should be noted that learning is a lifelong process that takes place in many environments and is not limited to the formal school environment. The way we look at learning and knowledge has changed; learning is not necessarily connected to the concept of ”teaching”, and knowledge is often referred to as ”perishable goods” that can quickly become old and outdated. Thus the question of how to recognize the knowledge adults actually possess has become something of a central question: what kinds of knowledge do adults gain from different learning environments, and how can we place a value on these different kinds of knowledge from the perspective of lifelong learning?

Nevertheless there is good reason to maintain a critical distance from the background rationales presented here. Although they bring both energy and substance to the debate, they are not free of problems. Broadening the concept of learning to encompass virtually all kinds of experience may serve to dilute its importance. Describing learning as lifelong may be perceived as prolonging the period of youth; we must try to remain "young" until we die. Furthermore, the society of tomorrow is often presented as if it existed today, which is in conflict with the democratic idea that man creates his future through his own activities. Lifelong learning is also described as an individual life project, and one may ask if learning as a collective project does not have at least as much value. Knowledge is often described in general terms as "a perishable item" that ages quickly, but is there not a certain type of knowledge, in mathematics for example, that is both vigorous and durable?

The school environment, life and learning

If teaching is to become a part of lifelong learning, it is essential that the adult perceives learning as meaningful; it must appear understandable and relevant and be of practical use in the adult's living world. At the same time, teaching should be seen from a broader cultural perspective. Learning then becomes an opportunity to transcend the limits of one's personal experience and confront the unknown. Seen from the perspective of lifelong learning, acquiring an all-round education is an unending process of personal development in its broadest sense. An individual equipped with greater mathematical proficiency is better able to understand and deal with the world.

Teaching, and social interaction, are factors that also have a very strong impact on the possibilities for learning, as does a reasonable pace of study that is adapted to the adult's situation. Its is well documented that in some forms of adult education a high percentage of participants fail to complete the course. Mathematics courses, with their condensed content and tight schedules, are often cited as an illustration of this.

A central issue for adult education is the question of how to draw attention to, and give a value to the often extensive learning the adult has gained from non-formal and informal environments, such as the workplace, family life, activities in voluntary organisations, and recreational life. The value given to the adult's mathematical proficiency is frequently expressed only in terms of the school syllabuses in mathematics, which places the adult in the humiliating position of having to go back to "the classroom". Taking a more flexible approach to the focus on the informal knowledge of adults and its evaluation will build self-confidence and motivation, and may perhaps also eliminate any residual anxiety and learning blockages about mathematics.

Our knowledge of the kind of mathematics and applied mathematical competence adults need in different contexts is limited by the small number of studies carried out in this area. There are, however, research findings showing that the "school mathematics" of today does not necessarily provide a sound and unassailable foundation. Rather, school mathematics may sometimes interfere with the adult's informal knowledge, and the adult will then produce lower test results after completing the course than before.

It is interesting to note here that the informal knowledge the adult acquires in various life environments may be utilised, and may even contribute new dimensions and content to the teaching materials used in school mathematics. In this context, the requirements expressed by different receivers should be problematised; as a rule they are only expressed in relation to the courses taught in the regular school system.

Who, in fact then, is "the adult"? There is no simple answer to this question, it is more a matter of the distinctive heterogeneity of the student body in adult education in terms of, for example, previous knowledge, the goals and purposes of studies, age, occupational background, family profile, socio-economic circumstances, ethnicity, and motivation and attitudes to studies.

In educational contexts, adults are often described from an "age psychology" perspective. The phase of interest here - adulthood - has two parts. The first is a period dominated by forming a family and pursuing a career, while the second is characterised more by cultural and social activities of an experiential nature. In the first part of the adulthood phase studies tend to be narrow and instrumental, while studies in the second part are often driven by curiosity, pleasure and the desire for personal improvement.

For a number of reasons, some groups of adults fall outside the educational system. These include persons who have completed only a brief basic education, people with reading and writing difficulties, older adults, unemployed people and some people with immigrant backgrounds. An important question is how these groups can be motivated and brought into adult education and encouraged to complete a programme of study. Here, negative experiences from mathematics as taught in the school often has a strongly inhibiting effect on studies.

The adult is used to taking responsibility for his own life and making his own decisions; what an adult perceives as meaningful, he will want to learn. If this is not reflected in the teaching, the adult may well develop a more or less vigorous resistance, or decide to leave the course. If an adult does not view the educational situation as a choice voluntarily made, it is unlikely that he or she will learn anything; in such situations, for example, as when there is at risk of unemployment or exposure to the pressure of moralising comments such as "education is a lifelong task for everyone". Recognition of prior lear-

ning (RPL)[2] and guidance are key issues here, as are possible conflicts between societal and individual motives and purposes. The knowledge an adult acquires from informal learning environments is an integral part of personal identity and self-regard, and must not be "rejected" by the demands for development that our society imposes.

One way of defining "adult students" is to examine their participation in different educational systems, as is done in this report. An adult student is a person who attends classes at the compulsory or upper secondary school level in adult education programmes arranged by the state or the municipality, or studies at a Folk High School or an adult educational association. As mentioned earlier, the perspective of lifelong learning identifies many other learning environments. The problems associated with the question of whether or not adult pedagogy is a separate discipline are complex. In certain respects, there are major differences between the framework and motives of adult education and those of the compulsory school system. They include the adult's need for flexibility in relation to his life situation, experience from and connections with other learning environments such as the workplace, and the risk of conflicts related to the adults' motives for learning, and their knowledge.

There has been little research on the process of learning mathematics among adults in Sweden, and the development of special research environments only began very recently. Both at the international level and in some Swedish surveys, research has often drawn attention to the gap between "school mathematics" and the mathematics that adults actually use or need in a range of life situations. The adult uses completely different strategies and methods, and is often unaware of their mathematical nature. A related problem is the question of the extent to which mathematical proficiency can be transferred from one situation to another, i.e. to what extent the learning has a transfer effect. Other areas that have been the subject of research or development work are the affective aspects of learning mathematics, questions about critical citizenship and democracy, and design of syllabuses based on the student's everyday life, perception of reality and life experiences.

Notwithstanding the lack of research and development work, adult education in Sweden is undergoing a phase of dynamic development, as is reflected, for example, in the goals and strategies contained in the Government Bill on Adult learning and the Development of Adult Education (Bill 2000/01:72).

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[2] The Swedish term *validering* has different translations in different contexts and countries. In Great Britain the term accreditation of prior (experimental) learning (APL/APEL) seems to be the most frequently used. In Australia and South Africa recognition of prior learning (RPL) is used. In USA and Canada the terms prior learning assessment (PLA) respectively prior learning assessment and recognition (PLAR) is in use. In this report we consistently use the term recognition of prior learning (RPL).

Here, it is of great importance that these development plans are co-ordinated with appropriate research projects and pedagogic/didactic[3] development work, not least in the field of mathematics.

Why mathematics?

Mathematics is a multi-faceted subject, and a unique construction of human thought. Despite its high degree of abstraction, the subject has many deep and living connections with our daily world, in both simple everyday events and advanced scientific matters. Mathematics is both a basic scientific discipline with "a life of its own" and a powerful tool that is applied in numerous other disciplines. Increasingly mathematical models are being applied in dealing with both economic and social conditions. Mathematical models are embedded in technical and social artefacts and are thus generally invisible to ordinary people.

In "school mathematics" an encounter takes place between the subject of mathematics and people's attitudes, experiences, feelings and thoughts, which creates special complexes of problems, not least in adult education. Finally, mathematics is also a domain for a particular kind of aesthetic experience, it provides moments of clarity and beautiful patterns that can create highly euphoric feelings of unexpected insight and overall understanding. Regrettably, many people's experiences of mathematics are quite the reverse: they associate mathematics with feelings of failure, anxiety, humiliation, suspicion and disassociation. The experience of school mathematics thus becomes a life-inhibiting stigma.

Mathematics is to be found everywhere, but to the individual it appears to be almost nowhere, a situation usually referred to as the relevance paradox of mathematics. An adult who feels anxiety and suffers learning blockages when faced with this subject is therefore likely to conclude that the subject is meaningless; it neither improves understanding of the environment nor adds to practical knowledge.

Since many education courses are mathematics-intensive, adult students need a high level of, or at least formal certification of competence in the subject. Here, school mathematics has a specific role to play as a "critical filter", a sorting instrument for admissions to many programmes in higher education. The state commission "Opening up higher education" discusses how higher education can be transformed from categorising and eliminating students into positively recruiting and supporting them. The role of the subject should be viewed from this perspective. In the long-term this could mean that the sub-

[3] In the report we use the term *didactics* (of mathematics). The term is used in Germany, France and the Nordic Countries as equivalent to the english term *mathematics education*.

ject of mathematics increasingly becomes an instrument for promoting broad all-round education and personal development.

Neither is it self-evident that school mathematics as traditionally taught is the most appropriate way of promoting lifelong learning. International research on numeracy, particularly adult numeracy, shows that the mathematics taught in schools today is seldom used in the individual's real world. The question, *why mathematics?* thus leads on to the more specific questions of *why school mathematics?* and *what kind of mathematics should be taught in our schools?*.

School mathematics – an ideological construction?

In this report, the term "school mathematics" refers to the mathematics taught in the formal educational system from pre-school up to and including the upper secondary school. By tradition, the subject has a high status. It is considered difficult to learn and yet, often without any detailed justification, it has a high value. Few adults are indifferent to the subject - they either found it easy and have a good appreciation of its content, or they have feelings of anxiety, and possibly disassociation, which may be attributed to the failures and learning blockages from their school years. For this reason, the adult's attitude to mathematics has a strong self-censuring effect on future studies and the life goals they regard as possible.

The goals and content of school mathematics change over time, influenced by the general objectives of educational policy, the outlook on mankind, ideologies, and popular perceptions of learning and knowledge. In the school system the subject is expected to provide knowledge and skills for future studies, future work and citizenship, and what is learnt is expected to have a strong transfer effect. Whether this assumption is correct in general terms, and in particular its validity for adult education, is an important question. Should mathematics studies for adults perhaps have completely different contents and points of departure?

From a socio-cultural perspective, "school mathematics" is not simply a set of syllabuses, but an entire educational and teaching culture that is supported by teachers and pupils, as well as parents, school heads and others. Teaching materials, expectations, traditions, attitudes and teaching styles permeate the school's daily activities, and it is difficult for new syllabuses, curricula and didactic advances in research to gain acceptance in the classroom.

From the perspective of lifelong learning and the rapid changes in the information society, abilities such as critical thinking, the assumption of responsibility, co-operation, seeking knowledge, argumentation and presentation all become important. How can this affect the content of school mathematics in general, and adult education in particular? School mathematics does not transcend our ideologies - to some extent both its rules and content are deter-

mined by political and economic interests. A general question here is whether the subject should reflect an adjustment to a future scenario defined by others, or if it should help develop individuals who shape their own destinies.

Furthermore, there is good reason to avoid the mirror perspective that used to be a prominent characteristic of adult education, in which the adult's studies were forced to comply with forms of governance, programme texts and descriptions of goals that are actually designed for children and young people. In this context an examination of the development of school mathematics from an historical perspective may help clarify the motives and purposes. This in its turn facilitates a problematisation of the current syllabuses, both in the regular youth school and in adult education. How can different learning environments enrich one another? Can the adult's knowledge of mathematics inspire advances in the subject of mathematics in the regular school system?

Mathematical proficiency for everyone

Even though the adult himself may have a limited instrumental purpose in studying mathematics, the curriculum as such must have a broader aim. The study of mathematics should thus be seen as part of a lifelong project, a process of study that enriches the personality, opens the door to new areas of knowledge and points the way to broader and unexplored opportunities.

The purposes and goals of school mathematics are described in some detail in our policy documents. To give an example, problem-solving involves training individuals in structuring their thinking and presenting both verbal and written arguments to support their ideas and proposed solutions. Such kinds of cognitive and communicative skills have more general applicability than mathematical proficiency per se.

Learning mathematics should also cover the school system's task of fostering democratic skills in its pupils. Hitherto, in Sweden it has often been argued that the role of the subject of mathematics in this context has been to facilitate the interpretation of statistical diagrams and economic calculations in political contexts. However, particular attention has been given in recent years to deliberative discussion - a discussion based on objectivity, respect for the arguments and viewpoints of others, and a desire to arrive at collective consensus. Here, the work forms of the subject of mathematics should be problematised, as the subject is often presented as a long succession of facts to be memorised and reproduced. An approach of this type may also be presented in an authoritarian and one-sided way by the teacher by virtue of the fact that only one textbook is used. The subject will then derive its legitimacy solely from a belief in authority and obedience, which would be a remarkable and counter-productive teaching culture that leaves no room for objectivity, argumentation and discussion.

An identification of a basic body of knowledge in mathematics must also be related to lifelong and life-wide learning. Seen from the adult perspective, it is vital that subject-specific knowledge is accompanied by a motivation to learn and to develop self-confidence and the ability to adopt a flexible approach to change. Here, the formal education system has the important task of providing a basic knowledge that can be developed in real-life situations and is an inspiration for future studies.

The contents should include a presentation of the "great ideas" of mathematics. Without a knowledge of these ideas, the subject will be incomprehensible and unusable. A knowledge both of mathematics and about mathematics is essential here. Another important aspect is that all students must learn about certain "growth points" that are the didactic foundation for further teaching and learning, perhaps in adult life as well.

In recent years, projects have been carried out both in Sweden and in other countries on describing mathematical knowledge in terms of a broader range of competencies as opposed to the more purely traditional, specialised view of "knowledge and skills". The advantage of using the concept of competence is that it allows different aspects of knowledge to be co-ordinated into an operational whole that encompasses both knowledge, skills and judgement, as well as affective elements such as self-confidence and motivation. The term "mathematical proficiency" has also been revived to convey the sense of a general ability to understand and handle a larger whole.

Basic mathematical proficiency can be illustrated through the following three aspects: as a life project, as competence in the subject, and as knowledge of the content of the subject.

One component of mathematics as a life project is the democratic aspect; the right and duty of individuals to take an active part in maintaining and transforming a society more and more permeated with mathematical concepts and models. Every citizen must thus learn the essentials of mathematics in order to understand, discuss and criticize societal issues in a democratic process. But democratic competence not only involves knowing some relevant mathematics; it also involves training in the classroom to develop a democratic approach to reasoning and discussion, both as regards mathematics and more generally.

A second component is the perspective of life-long and life-wide learning. Mathematics and mathematical reasoning is learned through an on-going process in many different situations covering school, working places, science centres, libraries and in every-day life. Constructive attitudes to mathematics are also of vital importance for e.g. attending adult education and applying mathematics in adult every-day life.

A third component of the life project is related to the shaping of a person in the broadest sense in accordance with the idea of Bildung[4]. This idea focuses more on who you are in contrast to traditional "education", which focuses on what you know. The post-modern society is complex and changing fast, and the future seems unpredictable and insecure. Under such circumstances it is more relevant for the individual to acquire general competencies such as how to learn and manage unforeseen situations, as opposed to acquiring a set of specific "future-adjusted" skills. Ideas similar to Bildung are highlighted in some reports and bills from the government. This idea of self-development has a bearing not only on developing cognitive abilities, but also includes preparedness to handle life and the future in both moral and affective dimensions.

The second aspect of mathematical proficiency is, as mentioned above, to develop competence in the subject. As a synthesis of different development projects, we propose the following eight general subject competencies: a productive approach (i.e. viewing mathematics as understandable, useful, and worthwhile, and connected to self-confidence), skills in exercising judgement, understanding concepts, mastering procedures, communicating , solving problems, presenting arguments and using aids. These competencies have a duality; both as regards applying mathematical knowledge, as well as as the art of developing new mathematical knowledge when needed.

The third aspect is knowledge of the content of the subject which can be illustrated as a progression based on a number of "strands" that run through the subject from pre-school to the upper secondary school. Describing the subject in this way has many advantages; it gives prominence to the central ideas of the subject and shows that there is progression through the entire educational system. A possible division might be as follows: numbers and operations, geometry and visualisation, representations of relationships and a familiarity with symbols, measurements and units, and statistics and probability.

A fourth aspect of proficiency in mathematics could be to draw attention to the subject and give value to its importance as a cultural and societal phenomenon. If we want to nurture democratic and critical awareness in indi-

[4] The meaning of the Scandinavian term *bildning*, which occurs throughout this report, is difficult to capture because it expresses so many different perspectives and meanings. As there is no equivalent in English, it defies precise translation. The word comes from the German *Bildung*, which has its roots in the Greek *paideia*. The literature in English contains the terms *liberal education*, *general education* and *popular education*, but none of these terms corresponds to the Scandinavian word. In this translation, the terms *all-round education*, *popular (adult) education* and *general education* are used interchangeably. (See Johansen, L. (2002) for a more detailed discussion of the concept *Bildung*.)

viduals, greater value must be given to this aspect of proficiency, not least in the subject of mathematics, which is sometimes seen as nothing more than a set of manipulations and operations that are carried out by means of mathematical symbols.

The teachers are, of course, key people in prioritising basic proficiency in mathematics for everyone. Teachers must not abdicate from their role as leaders and mentors. A passive teacher role leads to students working on their own using purely "mechanical " approaches in which a traditional and outdated view of knowledge becomes dominant. A long-term initiative to improve the basic proficiency of both young people and adults requires active and competent teachers who have both a command of the subject and effective methods of teaching it. All our experience indicates that all the players connected to the school must be involved if this initiative is to succeed. This includes teacher trainers, authors of teaching materials, researchers, parents, school heads, the mass media, trade and industry and others.

Learning blockages and resistance

As mentioned earlier, many adults have blockages about mathematics, and may also develop an active resistance to the subject. In light of this, it may seem paradoxical that adults nevertheless start studying again and are prepared to make considerable personal sacrifices to take up the subject. One explanation may be the need to overcome earlier feelings of inadequacy because of failures in youth. Many people also see success in mathematics as a sign of intelligence and an ability to learn, and mathematical reasoning is generally assumed to "sharpen" the intellect.

Many mathematics teachers in adult education maintain that blockages and resistance must be dealt with before people can even begin to start learning mathematics. Adults cite experiences from their youth to explain their negative attitudes to the subject, for example, the teacher could not give explanations or did not care about the pupils, the pace was too hurried or the content of school mathematics seemed boring and incomprehensible. They also bring up the lack of challenges and variety in the teaching. Many adults give concrete and emotional descriptions of the occasions when they were unable to keep up or felt humiliated in the classroom. This may easily cause lifelong bitterness and a lack of faith in their own abilities. Psychologically, active resistance is a more constructive reaction, but may involve a complete rejection of all forms of recruitment or guidance, and not just to courses in mathematics.

More research is needed on the complex relationships between the affective and the cognitive factors that influence the learning of mathematics. However, it is evident that a holistic perspective is essential; the foundations of the desire

and the will to learn mathematics must be laid and maintained at all levels of the school system as a part of lifelong and life-wide learning.

Mathematics in the national system of governance of adult education

Education to enable adults to acquire competence is provided in the form of municipal adult education (komvux), adult education for people with learning disabilities (särvux), and Swedish language courses for immigrants (sfi), and these programmes make up the public adult education system. Municipal adult education consists of basic adult education, upper secondary level adult education and supplementary programmes of education. The education is governed by the Education Act, curricula and syllabuses with their attendant grading criteria, as well as special ordinances governing different school forms, such as the ordinance on municipal adult education and the ordinance on the Adult Education Initiative (Kunskapslyftet). Adult education is also provided in the form of popular adult education through Folk High Schools and study associations which are governed by separate ordinances. Among other things, Folk High Schools and study associations are to work to eliminate educational gaps, raise the level of education in the community, and target socially and culturally disadvantaged people. The general courses providing formal qualifications will, in practice, also be governed by the relevant policy documents for the regular school system.

The current description of programmes in basic adult education corresponds to the nine-year compulsory school. Upper-secondary level adult education has the same curriculum as the upper secondary school for youth. This mirror perspective complicates the process of creating a flexible and individually-tailored system of adult education in which mathematics studies could be linked to other areas of learning and knowledge. Ill-considered mirror perspectives should also be avoided when it comes to recognition of prior learning of adults' proficiency before, during and after the education programme.

Work on syllabuses for adult education should take into account both tried-and-tested experience as well as current research, and relate to the vision of lifelong and life-wide learning.

Five areas of critical importance

The report identifies five critical areas in which research and development work are a matter of particular urgency. The five areas are:

Affective factors and adults' mathematics learning: The strongly negative feelings that many adults have about the subject of mathematics may have serious effects on the individual's entire life situation. The difficulty in recruiting young people and adults for studies involving mathematics may be explained by learning blockages and failures at school, which result in serious losses for both the individual and the community. How can anxiety and suspicion be replaced with a self-confident approach to mathematics and a desire to learn throughout life?

Adults' informal mathematics learning : Research has demonstrated that there is a great difference between the mathematical proficiency that is used in everyday life and at work, and the methods used in school mathematics. Formal proficiency in school mathematics has proved to be a poor indicator of how individuals manage in environments outside school and education. What lessons can we learn from this informal proficiency in mathematics, and to what extent should it influence school mathematics?

Recognition of prior learning of adults' mathematical proficiency: The question of how adults' actual proficiency in mathematics is to be evaluated is of very great interest to both the individual and the community. Today, evaluation is often only in relation to the syllabuses in the regular school system, which may be misleading, and also humiliating for the adult. How can adults' actual knowledge be recognized in a better way for recruiting people to studies and teaching, and in relation to receiver requirements?

Mathematics as a subject for Bildung: A school based on an instrumental view of knowledge appears to be increasingly obsolete. Both young people and adults need broader proficiency in order to keep pace with a rapidly changing and unpredictable future. The all-round educated person who has transformed proficiency and made it into an integral part of his make-up can apply greater imagination and creativity when navigating the future and in helping to create and shape it. What role can knowledge of and about mathematics play for children, young people and adults in such a journey?

Syllabus development in mathematics: It is of great importance that the work of drawing up syllabuses is both long-term and continuous. Permanent and long-term research, as well as commitment from the teaching profession can lay the foundations for dynamic development work on syllabuses and commentaries. The results of research and development concerning the critical areas mentioned above must also be a self-evident source of support in the design of syllabuses.

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Adults and Mathematics
– a vital subject

Lars Gustafsson & Lars Mouwitz

The National Center for Mathematics Education (NCM) was commissioned in October 2001 by the Swedish Government to carry out a survey and produce an analysis of the measures required to improve adult learning of mathematics. The results of this assignment were presented in June 2002 in the report *Vuxna och matematik – ett livsviktigt ämne* (NCM Report 2002:3).

The whole report *Vuxna och matematik – ett livsviktigt ämne* is available (only in Swedish) from the web site of NCM. Go to "Publikationer" and then to "NCM:s rapporter för nedladdning". The address to NCM:s webbsite is: www.ncm.gu.se

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