Engagement in Education:
Identity Narratives and Agency in the Contexts of Mathematics Education

Annica Andersson

Till Erica, Marica och Johan.

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Foreword

As we all know, a thesis is not a single individual's work. You are so many lovely persons around me that in different ways have shared your thoughts, reflections, critiques, ideas, challenges and – your warm hugs - with me during these three years of study. I am so grateful to all of you!

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During these years I have had the opportunity to be part of two research groups. At Aalborg University I have been part of SMERG. I want to especially thank Alexandre Pais, Cristina Carulla, Jette Schmidt, Martin Krabbe Sillasen, Ole Ravn and Peer Daubjerg who have read and commented on my work during these years. Gracias Cristina, por su amistad y por mostrarme su Colombia.

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Summary

In this PhD study I aimed fill some of the research gaps around the relationships between individual students' engagement in mathematics, and different contexts in and outside the classrooms, that impact on what occurs in the mathematics education practice. The discursive concept of identity narrative, defined as the stories the students tell about themselves, provided a way to understand the complexity of individual students' decision making about engaging in classroom activities at certain points in time.

This is a study in two layers. The bottom layer concerns an introduction of different teaching discourses in a specific Mathematics A course in a Swedish upper secondary school. This "disturbance" of the teaching needed to be realized, for me to be able to study students' narratives in a different mathematics education than they expected or imagined. The mathematics teaching became an intervention not to be evaluated as such, but to make possible the flourishing of other types of identity narratives, than the ones I had listened to as a teacher in traditional teaching environments. The second layer, the main study layer, concerns the students and their identities, experiences and emerging relationships with mathematics during this particular mathematics course where we had changed the teaching. This theme is the focus of the thesis.

A collaborative process was established with Elin Johansson (pseudonym), the mathematics teacher at Ericaskolan, in order to introduce elements of a critical pedagogical discourse. Within the mandated mathematical topics of the curriculum, the new pedagogy introduced project blocks that changed some key elements in the activities and the relationships between participants. With this pedagogical approach, Elin and I intended to bridge the gap between students' experiences in society and the mathematics classroom. To change a social practice such as mathematics education, and thus move between discourses was a multifaceted process and required support from different parts of the mathematics education network at different points in time. Relationships were re-established in order to proceed with changing the teaching organisation. Locating the experiences in the socio-cultural context of the school gave an understanding of the complex situations and processes. Elin's identity narratives and learning suggests that researchers need to reconsider the use of terms such as sustained change and success, and if sustained development is an actual possibility. On the other hand, Elin's continuous learning could be considered a success in itself, even if this was not the learning which we originally had anticipated from this collaboration.

The students who are the participants in this study chose the social science study program for their three years of upper secondary schooling. The students' narrated identities during their first compulsory mathematics course, Mathematics A, provided a way to understand their shifts in participation at particular times. These students' reasons for (dis)engagement with learning were brought to life in the stories they told and expressions they used in their relationships with Elin, the other students and myself in the mathematics classrooms. These students' experiences, of disliking mathematics or finding it boring, are not unique, but are representative of the large

number of young people whose well-being might diminish when they are asked to engage in mathematics education.

The results presented in previous research suggest that students' dislike of mathematics is a permanent feature of some types of students, and as a consequence their (dis)engagement in mathematics education is also seen as a constant characteristic. However, I argue that contextual changes to the way that mathematics is presented can alter the way students talk about their relationship with mathematics and mathematics education. To illustrate these changes, I analysed the stories that students told about themselves and their relationship with mathematics. Task contexts, situation contexts, school contexts and societal contexts intermesh as referents and groundings for the discursive practices in classrooms, through which students construct identity narratives. The students' narratives showed that different levels of contexts affected students' decision-making on whether to engage in mathematics learning activities at specific points in time. The analysis showed that students' identity narratives, such as that of being a "math hater" or having "math anxiety", also are intertwined with the learning opportunities that they are offered.

The findings point at problematic issues when research outcomes generalise students' learning of mathematics and conclude that specific groups of students act or behave in certain ways, or that certain pedagogies are to be preferred. The explored connections between identity narratives and contexts resulted in a re-evaluation of the usefulness of how students are categorised, or labelled in mathematics education research, and hence the impact that these reified labels has on individual students' agency and decisions to engage and learn mathematics. The ways in which I was able to connect students' identity narratives to contexts was a methodological research outcome from this study. The choices we researchers make about methodology, and the ways we interpret and communicate our findings, can, and often do, reinforce certain characteristics on students as being the only parts of them to which attention should be paid. Labels allow researchers to generalise principles, but it becomes problematic when the principles are applied to specific cases. The principle as such is not wrong, but when they are applied in this way then the labels can impede learning rather than support it. I argue that labels might reinforce the identification of a student by particular characteristics. These reflections reinforce questions such as: What stories do we want students to tell about their relationship with mathematics and their experiences in mathematics education? What is to be learnt in mathematics classrooms?

Resumé

I denne ph.d.-afhandling, Engagement i uddannelsen – Identitetsnarrativer og agency i matematikundervisnings kontekster, har det været min hensigt at begynde at udfylde tomrummet i udforskningen af relationerne mellem de enkelte studerendes engagement i matematik og forskellige kontekster i og uden for klasseværelset, som har indflydelse på, hvad der sker i matematikundervisningen i praksis. Ved hjælp af det diskursive begreb, identitetsnarrativ, der defineres som de fortællinger, de studerende beretter om sig selv, blev det muligt for mig at forstå kompleksiteten i de enkelte studerendes beslutningsproces omkring deres deltagelse i klasseaktiviteter på bestemte tidspunkter.

Der er tale om en undersøgelse i to niveauer. Det sekundære niveau omhandler en introduktion af forskellige undervisningsdiskurser i et specifikt matematik A forløb på et svensk gymnasium. Denne "forstyrrelse" af undervisningen var nødvendig for, at jeg kunne studere elevernes fortællinger i en anden form for matematikundervisning, end den de havde oplevet før, forventet eller forestillet sig. Men mit mål var ikke at evaluere den nye form for undervisning som sådan. Snarere det modsatte. Matematikundervisningen blev en intervention, der ikke skulle vurderes i sig selv, men som skulle muliggøre udfoldelsen af andre typer identitetsnarrativer end dem, jeg var vant til at lytte til som lærer i traditionelle undervisningsmiljøer. Det andet niveau, det primære undersøgelsesniveau, drejer sig om de studerende og deres identitet, oplevelser og emergente relationer med matematik i løbet af dette særlige forløb, hvor vi ændrede undervisningen. Det er dette tema, som afhandlingen har til formål at fokusere på.

Med henblik på at indføre elementer af en kritisk pædagogisk diskurs blev et samarbejde etableret med Elin Johansson (pseudonym), som er matematiklærer på Ericaskolan (pseudonym). Inden for rammerne af det obligatoriske matematikpensum introducerede den nye pædagogik projektblokke, der ændrede nogle centrale elementer i aktiviteterne og relationerne mellem deltagerne. Med denne pædagogiske strategi havde Elin og jeg til hensigt at bygge bro mellem elevernes oplevelser i samfundet og matematikundervisningen i klassen. At ændre den sociale praksis, som matematikundervisningen er, og bevæge sig mellem diskurser blev oplevet som en opgave med mange facetter og krævede undertiden støtte fra forskellige dele af matematikundervisningens netværk. Forskellige relationer blev etableret for fortsat at kunne ændre undervisningens organisering. Lokalisering af oplevelser i skolens socio-kulturelle kontekst gav en forståelse af de komplekse situationer og processer, der krævede yderligere belysning. Elins identitetsnarrativer og undervisning indikerer, at vi forskere måske bliver nødt til at revurdere, hvad vi mener med begreber som varig forandring og succes, og hvorvidt varig forandring overhovedet er mulig. Som jeg ser det, kan den undervisning, som Elin fortsat er

involveret i, siges at være en succes i sig selv, skønt det ikke var den læring, vi oprindeligt forventede som et resultat af samarbejdet.

De studerende, der har været i fokus i denne undersøgelse, havde valgt den samfundsvidenskabelige gren i deres treårige gymnasiale uddannelse. De studerendes identitetsnarrativer i første del af deres obligatoriske matematikundervisning, matematik A, blev et udgangspunkt for at forstå de skift, der skete i deres deltagelse på bestemte tidspunkter. Årsagen til disse studerendes engagement i undervisningen (eller mangel på samme) kom frem i de fortællinger, de berettede, og i de udtryk, de anvendte i klassen over for Elin, de andre studerende og mig. Som jeg ser det, er disse studerendes erfaringer ikke enestående, men repræsentativ for det store antal unge, der enten bare ikke kan lide matematik og/eller oplever matematik som et kedeligt emne, og som måske får deres velbefindende forringet, når de bliver bedt om at deltage i matematikundervisning.

Tidligere forskning indikerer, at studerendes modvilje mod matematik er et permanent træk ved visse typer af studerende. Som en konsekvens heraf anses deres engagement (eller mangel på samme) i matematikundervisningen også som et uforanderligt karaktertræk. I modsætning til disse forudgående forskningskonklusioner, argumenterer jeg for, at kontekstuelle ændringer i den måde, hvorpå matematik præsenteres, kan ændre den måde, hvorpå de studerende taler om deres forhold til matematik og matematikundervisning. For at illustrere disse ændringer, analyserede jeg de fortællinger, de studerende berettede om sig selv og deres forhold til matematik, og det viste sig, at forskellige kontekster påvirkede deres beslutningsproces angående deres engagement på bestemte tidspunkter. Fire studerendes identitetsnarrativer vedrørende deres matematik A undervisning præsenteres for at illustrere, hvordan disse gymnasieelever blev bevidstgjorte om sig selv og opdagede, hvorledes de besluttede sig for, om de skulle engagere sig i matematikundervisningen eller ej.

I nærværende undersøgelse har jeg dokumenteret, hvorledes opgavekontekster, situationskontekster, skolekontekster og sociale kontekster udgør en blanding af referencer og grundlag for de diskursive praksisser i klassen, herunder de studerendes konstruktion af identitetsnarrativer. Analysen viste, at de studerendes identitetsnarrativer, såsom det at være "matematikhader", er flettet ind i de tilbudte læringsmuligheder. De studerende beskrev handlemuligheder som en vigtig bestanddel i deres fortællinger. Relationen mellem de studerendes skiftende identitetsnarrativer og de kontekster, hvori de blev dannet, tilbyder en indledende forklaring på, hvorfor de studerende handler på bestemte måder på bestemte tidspunkter. Disse forhold giver et billede af, hvordan de tog aktiv del i deres matematiklæring i visse situationer, men ikke i andre.

Resultaterne tyder på, at identiteter ikke er konstante størrelser, således som tidligere forskningslitteratur ofte foreslår. De forbindelser, der blev fundet mellem identitetsnarrativer og kontekster fik mig til at genoverveje, hvordan studerende bliver kategoriserede eller 'sat i bås' i matematikundervisningen, og hvilken indflydelse, betegnelserne har på elevernes handlefrihed og indlæring af matematik. De studerendes identitetsnarrativer fik mig til at sætte spørgsmålstegn ved den måde, hvorpå forskning ofte afsluttes med objektiverende eller

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kategoriserende etiketter, og den indvirkning, dette har på, hvorledes visse grupper af elever, lærere eller forældre forventes at handle eller opføre sig på bestemte måder.

"Trigonometry and me"

by Genevieve Ryan

I am X.
I don't know my own value.
I'm waiting for someone else to work me out.
There are no clues
I've never been able to understand the logic of mathematics
I don't have the ability to know what I'm worth
I'm lost in a vicious triangle.
How can I simplify myself?

Genevieve wrote this poem at the age of 15 years. The poem originates from the book "…regards, some girl with words. Genevieve's Journey" by Elisabeth Ryan.
I am so grateful to Elisabeth for giving me permission to print Genevieve's poems, and for inviting my family to take care of their lovely house in Melbourne during the Christmas holidays 2009.

Setting the Scene

Jag har aldrig haft det särskilt svårt med matten, men ofta har jag upplevt den som seg och långtråkig p.g.a. att man ofta har jobbat med samma saker under en lång tid.

Jag har tyckt att matten känns väldigt meningslös när man lär sig sådana saker som det inte är uppenbart att man har nytta av i framtiden. (Erik, survey, 08-2009)

Jag har aldrig någonsin gillat matte och i högstadiet hade jag dessutom en lärare som inte gjorde det hela bättre. Jag är inte särskilt duktig. […]. Inte mycket av det man läser nu är nödvändigt, särskilt inte de uppgifter som handlar om exempelvis potenser. Det känns inte som att jag kommer använda det särskilt mycket i framtiden (Marie, survey, 08-2009)

Jag förstår ju att matte är viktigt och att det är till en stor hjälp senare i livet och jag försöker verkligen att bli bättre, problemet är nog bara att jag tycker att det är tråkigt... (Kim, survey, 08-2009)

I have never had big problems with maths; I have rather experienced it as tedious and boring because one has often worked with the same things during long periods. I have been thinking that maths is very meaningless when one learn those things, which are obvious that one will never have use of in the future. (Erik, survey, 08-2009)

I have never ever enjoyed maths and in lower secondary, in addition I had a teacher that did not make things better. I am not so good. […]. Not much of the stuff one learns is necessary, especially not when the exercise topic is for exponential calculations. It doesn't feel as if I will use that so much in my future. (Marie, survey, 08-2009)

Well, I do understand that maths is important and that will be of big support later in life and I really try to be better, the problem is probably that I think it is so boring… (Kim, survey, 08-2009)

"My" Students' Stories

Each year, when a new school year started and I met the new-incoming 15-year old students for the first time at Ericaskolan, a Swedish upper secondary school, I asked them to write a personal letter to me. In the letter I wanted them to describe their prior experiences in mathematics education, their personal goals in the new mathematics course, and if there was something else they thought I should know as their mathematics teacher for their three coming school years. After reading a number of letters during my teaching years, I realised that there were similarities in the ways students described themselves and their relationships with mathematics. Some students clearly enjoyed mathematics. Other students expressed that mathematics was "ok", even if it was not their favourite topic: "if it had been one of my favourite subjects I would have chosen the natural sciences study program instead". However, an increasing number of the letters contained statements such as: "Annica, I don't like maths, it's boring", "I hate maths", "My former teacher said I was not good at maths", "I have always struggled with the problem solving exercises", "I have tummy aches before math tests", or "I hope you can help me to just pass the course". These students were not positive about mathematics education even if they all indeed had passed, sometimes with high grades, prior mathematics courses. As their new upper secondary teacher, I became troubled about the way in which these students expressed their feelings and their depressing experiences when over the course of many years of schooling, they had put time and effort into a subject that we, the important adults in their lives, told them was good for them to learn.

It is not uncommon for students to have these experiences of mathematics education. For example, Yvette Solomon's (2009) book on mathematical literacy focuses on English students telling similar stories about mathematics education at all school levels from primary school up to university level. Feelings of exclusion in mathematics were expressed by students of all ages and also by those who performed well in mathematics, in opposition to students' *inclusive identities* which Solomon (2009, p. 27) suggested was comprised of "particular beliefs about oneself as a learner and about the nature of mathematics, an identity of engagement in mathematics and a perception of oneself as a potential creator of, or participant in, mathematics". Another example is the Australian research conducted by Bishop (1999) and Clarkson, Bishop and Seah (e.g., 2010). They introduced the notion "mathematical well-being" to talk about students' diminishing well-being when asked to engage in mathematics education. The notion "mathematical well-being" is constructed as an analytical framework. It links the cognitive, the affective and the emotional aspects of students' mathematical educational experiences with a focus on students' emotional relationships and students' values in their relationships with mathematics and mathematics education.

Additional to students' diminishing well-being, this group of students, when talking about their relationship with mathematics, repeatedly referred to boredom. This is one of the most debilitating emotions used by students as a description for mathematics according to a number of large-scale quantitative reports (e.g., Goetz et al., 2006). Brown et al. (2008) showed that *bored* was the word used most often by 16-year old students to describe their attitude in a questionnaire on students' affective relationship to mathematics. The questionnaire was distributed to 1997 students who were the same age as students who enrol in the first mathematics course in Swedish

upper secondary schools, the participants in this study. One of the reported reasons for students experiencing boredom was the lack of creativity, which also is showed by Lange (2009). Bibby (2008), although working with students at the end of primary school, suggested that boredom indicated lack of stimulation and challenge. Bibby reported that students' lack of control over tasks and direction was connected to students' feelings of boredom. It is largely this convergence of different depressing stories, told by students and acknowledged in prior research that provides the foundation for this study.

"Mathematics education" is an elusive term that requires clarification with many different definitions. In this study mathematics education is understood in a wider sense, consisting of all school mathematics practices. Mathematics learning, mathematics teaching, mathematics curriculum, mathematics as a subject etc. are seen as parts of "mathematics education".

My Teaching Background

I started to study students' identities with a background while being a mathematics teacher in Swedish upper secondary schools (in Swedish: gymnasium) and recently as a mathematics teacher educator at Malmö University. At secondary school, I mainly taught mathematics in the social science program, which is a 3-year upper secondary study program chosen by students with interests in social science subjects or language subjects, most of them aiming to go on to university. A number of students considered the social science program a good option to the other main theoretical study programs - the technical and natural science study program - as they did not enjoy natural sciences, mathematics or technical subjects.

Two important issues challenged me during my teaching years in this program and they also were important to discuss with my university students. The first issue that worried me concerned the increasing number of students that told me all these disappointing stories about their feelings and their experiences from their previous years of mathematics education, as exemplified above. Students talking about themselves as being "math-haters", having "math anxiety" or being "bored by maths" were present in every new group of students I met. The impact of all these depressing stories was that I became curious about their reasons for these stories. What contexts and discourses made these students talk about anxiety, boredom, engagement or disengagement in mathematics education? I queried why students, as individuals, decided to participate and engage in learning activates at some moments in time but resisted at other times? It troubled me, and my planning of the teaching, when I did not always understand what made students decide to engage or disengage in mathematics learning, and why they sometimes told that mathematics was meaningful for them to learn, but most of the time not. With these reflections I indicate the focus of this study: If mathematics education discourses were different, if the contexts in which mathematical topics were expected to be learnt were different, what stories would the students then tell about their experiences of mathematics learning? What discourses and contexts would allow them to reflect on their reasons for engagement or resistance? What if?

Students' Experiences and Governmental Expectations

I started to reflect on the students' particular study interests, how they viewed and talked about their world, what topics they seemed to engage in and discuss, and so on. As these students had chosen a three-year social science study program, I thought that mathematics relating to the social sciences might be a way to get them engaged. From my teacher perspective, there was a disjunction that mathematics education counts in society; however, society does not obviously count in mathematics education. This was the case even in the mathematics courses on the social science oriented study program. In these students' compulsory mathematics courses I thought that mathematics relating to society would be a mathematics teaching approach that would colour and permeate the teaching and the students' learning of mathematics. This argument is supported by the Swedish national curriculum. The aim of mathematics on the social science oriented study program is formulated in the following way by the Ministry of Education (2000b, p.94, original English):

> Upper secondary education in mathematics builds further on knowledge corresponding to that attained by pupils in the compulsory school by broadening and deepening the subject. The subject aims at providing knowledge of mathematics for studies in the chosen study orientation and for further studies. The subject should provide the ability to communicate in the language and symbols of mathematics, which are similar throughout the world. The subject also aims at pupils being able to analyse, critically assess and solve problems in order to be able to independently determine their views on issues important both for themselves and society, covering areas such as ethics and the environment. The subject aims at pupils experiencing delight in developing their mathematical creativity, and the ability to solve problems, as well as experience something of the beauty and logic of mathematics.

Mathematics is often associated with a basic literacy (e.g., Solomon, 2009). The way the OECD (2004, p.37) defines mathematical literacy corresponds with the intentions expressed in the Swedish national curriculum:

> An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.

The writings in the curriculum statements emphasis that one important focus in mathematics education should be mathematics learning in a societal context in the mathematics courses, especially in a social science oriented study program. There are also governmental expectations on development of students' creative and critical thinking in mathematics and how mathematics is used as a tool in society.

However, in contrast to the governmental expectations of a creative and critical mathematics education, the most common way to reach curriculum goals in mathematics in students' earlier schools seemed to have been quiet, individual, textbook work. At least this was the case if the stories the students told me about their previous experiences of mathematics teaching accurately represented the situation. This way of working was one of the most mentioned reasons for experiencing boredom. Boring mathematics education was described as mainly being organised

around an introduction that included teacher instructions, followed by quiet textbook exercise work, with students raising their hands and asking for "help" when they got stuck with a problem. For example one student commented: "Group work in mathematics? Is that possible?" This sort of comment was very common.

These students' experiences are reinforced in the context of Swedish mathematics education according to Lindqvist, Emanuelsson, Lindström and Rönnberg (2003), who concluded that textbooks in Swedish mathematics education seemed to define the essence of school mathematics. This way of organizing mathematics education is believed to support teachers in managing non-homogeneous group of students so that each student could work according to his/her previous learning and needs, as well as following curriculum and reform concerns (Johansson, 2006). As a teacher, I questioned both "quiet work" and the "textbook work" as the only, or best, way of learning mathematics. This was especially was the case in the social science program where the mathematics textbooks problems and exercises did not obviously connect mathematics to societal contexts even if the textbook was written especially for this particular student group. This issue became my first critique in my Ph.D. study and was elaborated further with Ole Ravn for the chapter "A critical perspective on contextualisations in mathematics education", in the book "Critique and politics of mathematics education" edited by Brian Greer and Ole Skovsmose. This text is printed in this thesis, and I thank Brian and Ole for the permission to print it.

A newly published quality report from the Schools Inspectorate (2010), while this research was being conducted, reinforced my experiences as a teacher. Their report was produced in collaboration with researchers in mathematics education. The School Inspectorate documented that mathematics teaching in the Mathematics A course, the first compulsory course for all students in Swedish upper secondary schools, is not obviously connected to students' chosen study programs as the formulated intentions in the national curriculum. The School Inspectorate highlights issues such as the fact that students' individual work dominates mathematics lessons, thus resulting in mainly mechanical calculations with lesser teaching time for students' discussions, collaborations and problem solving. They presumed that the consequences of the present mathematics teaching might be that students do not develop central mathematical capacities as mathematical creativity, critical reflections or understanding mathematical relationships. The School Inspectorate concludes that the observed teaching seemed to result in under-stimulated students, who experience mathematics as a boring and sometimes even "stupidizing" (fördummande, p. 8) school subject – verifying the students' narratives about their experiences of mathematics education.

Imagining Something Different

My personal experience from working with social science students in their compulsory mathematics courses indicated that an increasing number of them disliked mathematics or did not feel well when asked to engage in mathematics education. The students described mathematics education as competitive and their feelings of anxiety and resent, sometimes even hatred. Consequently, I thought it would be challenging to develop a more inspiring and nourishing

mathematics education. I started to imagine different mathematics classroom contexts and discourses, that is, different ways of talking and behaving in mathematics classrooms. Discourses and contexts are elusive concepts, and often defined in many different ways. Here, I start with a short definition of these concepts. However, a further elaboration and argument for how they are defined is provided in the theory section and in the articles.

Discourses are in this study defined as suggested by Tsatsaroni, Evans and Morgan (2007, p. 85):

> A discourse is a system of signs that organises and regulates specific social and institutional practises and provides resources for participants to construct meanings (including meanings for their emotions) accounting for their actions and their identities.

The concept of context is complicated to grasp as a single concept, but also in its relationship with discourses. According to the Oxford Dictionary of English, the noun "context" refers to "the circumstances that form the setting for an event, statement, or idea, and in terms of which it can be fully understood" and "the parts of something written or spoken that immediately precede and follow a word or passage and clarify its meaning" (Stevenson, 2010). Hence, the word is a reference to circumstances, but in our language use it also refers to, and makes discursive spaces possible.

In this research I wanted to study the students' stories when they experienced a teaching that was different to the "normal" expected teaching for them. I wondered if other opportunities were offered to the students, how would the stories about themselves, their engagement in and their relationships with mathematics education emerge in those different contexts? On the whole in affect research, students' opinions of their relationships to mathematics seemed to be missing, especially when contexts and discourses changed. In mathematics education research, context tends to be restricted to the immediate context of a particular classroom or studied activity episode (Morgan, 2006). Researching affective responses within only one context, like problem solving, means that students are seen as retaining the same set of affective responses, which consequently become objectified (Sfard, 2008).

Introducing a Disturbance

My research objective was to grasp the stories students would tell about their experiences of mathematics education, if the teaching was organised in a different way than they expected. I adopted the research idea explained to me by Michael Roth in a personal conversation: "If you want to understand how a queue works, you have to disturb the line". Roth continued to explain that the "queue" metaphor captures a research principle to either "look for places where there is trouble in the normal order of things, because then the normally invisible work of producing social phenomena reveals itself". In my study this implied to change or move the teaching away from certain expected mathematics education discourses. The "queue" reflects the present discourses in which students told stories about themselves disliking, being bored, not feeling well, or not recognising meaning in mathematics education. The notion of meaning in mathematics education is in this understanding not only connected to the mathematical

conceptual meaning. It rather concerns the students' whole experience of and relationships with learning mathematics (Skovsmose, 2005b).

Noss and Hoyles (1996, p.9-10) formulated their vision of a similar research idea in the following way when they introduced different ways of computer based teaching and learning of mathematics:

> We might like to be endowed with some special mental apparatus, which would give us a representation of another's mental state, but such is not available. Neither can we hope to take a mental snapshot of what is 'known' at a point in time (although much 'testing' is undertaken in the vain hope that this is either possible or useful, or both). Instead we can set thinking in motion, and try to study what happens; we can set ideas in turbulence and investigate how changes occur; we can introduce new notions and try to understand how the thinker connects these with what he or she already knows. Within educational discourse, the study of thinking tends to presuppose that what is to be learned is fixed; the study of thinking-in-change demands that we devote at least equal attention to what is to be learned, as well as the meanings the learners draws from the educational experience.

Consequently, I decided to "disturb" the "queue", the expected mathematics teaching including the expected classroom discourses. I intended to set ideas in turbulence, and study the students' stories in a changed setting. The *expected mathematics teaching,* seamed according to the students' stories to be a teaching based on an introduction of a mathematics topic with teachers' instructions followed by students' individual (quiet) textbook exercise work. Students were expected to ask the teacher (in some cases also their peers) for "help" when perceiving problems with the mathematics tasks. This way of teaching is by Skovsmose (2001, p. 123) labelled mathematics education within an "exercise paradigm":

> Most often, the mathematical textbook represents a 'given' for the classroom practice. Exercises are formulated by an authority external to the classroom. This means that the justification of the relevance of the exercises is not part of the mathematics lesson itself. Furthermore, a central premise of the exercise paradigm is that one and only one answer is correct.

In contrast to organising teaching within the exercise paradigm, I imagined a mathematics teaching connected with topics focused in society and media, as I taught social science students. In this way I proposed to contextualise mathematics education to society and these students' study interests. I visualised classroom discourses with spaces for students to critique, discuss and question mathematics. I envisaged classroom discourses allowing for reflections on the smartest way for individual students to learn mathematics and how individuals' best accounted for their mathematical knowledge in relation to curriculum objectives and assessment qualities.

My imagination became the start of a classroom *pedagogical discourse*, whish in this study is defined as "the complex set of language formulations, together with the systems of reason that emerge when people engage in the social practice of mathematics education" (Andersson & Valero, in press). My imagination also made me take the first steps towards the empirical part of this research project, which I carried out in collaboration with the mathematics teacher Elin Johansson in her two students groups at the Swedish upper secondary school Ericaskolan. The

school was chosen for the research of convenience reasons. I knew the school well as I had taught there for several years before this research commenced. However, I had not worked together with, or been in the same teaching staff as Elin, the participating teacher before she replaced me when I started to work within teacher education at university. I had met Elin previously a couple of times and she showed interest in my work when I started the PhD studies. Hence evidently it was a well-grounded and good decision to collaborate with her in this research. Our collaboration process is further elaborated in Andersson (in press, a) and in the methodology section.

Maybe what drove my research, my attention, time and creativity was a secret wish of, not quietness, but a sense of peacefulness and openness for different views, critiques and discussions in mathematics classrooms – both regarding the teaching of mathematics and of mathematics per se but also on the use of mathematics in society. Subsequently, within deep-rooted school-cultural and political structural boundaries, I changed, together with Elin, the "normal" order of things in a Swedish upper secondary mathematics education with the hope that the normally invisible work of producing social phenomena would reveal itself. In this way I aimed to study students' identities, and uncover how their stories were told in relation to the contexts in which they were told.

The research process

The Disturbance

In this section I explain the research process and the theoretical moves that I have made during these three years of study. The research process has not been a linear process; the case is rather the opposite. There have been theoretical movements, movements that are visible in my published writings. During the research process I have been navigating with broad research questions and the decisions I have taken at specific points in time are closely related to the direction my learning process for becoming a researcher has taken me at those particular times.

My teacher experience during years of teaching gave me a sense that students engage differently, or rather with different qualities of engagement, at those times when the teaching was organised with projects, tasks or exercises that either connected the mathematical learning with society, or where the students achieved spaces to decide on task contexts by themselves. When the students were invited to create connections between the mathematical topics presented in school and current issues discussed in society, students seemed to talk, act and engage differently than in a teaching mainly organised with teacher instructions and textbook work and students' "help-asking". In my very first peer-reviewed paper, *Mathematics education giving meaning to social science students*, accepted for proceedings at an international conference in Medellin, Colombia, I elaborated on students' experiences of my previous teaching through data I had collected for my masters thesis. At this time I wanted to understand why students engaged in a different way when they were introduced to a critical inspired mathematics education. For this paper I grappled with the concepts of meaning, meaningfulness, and students' subjectification processes in mathematics education. I used the notions of students' foregrounds, backgrounds and intentionality, concepts introduced by Skovsmose (1994) and developed further by Skovsmose

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(2005a), Alrø et al. (2009), and by Valero and Stentoft (2010). I started to use these concepts as they fit well with a critical mathematics education discourse (Skovsmose, 2005a; Valero & Stentoft, 2010).

During this time, as mentioned above, I also articulated my critique towards the textbooks and specifically how the textbook used at Ericaskolan, a textbook written especially with social science students in focus, contextualised the mathematics problems the students were expected to solve. The critique emerged in a book chapter, *A critical perspective on contextualisations in mathematics education*, authored together with Ole Ravn. This text was written very early in my study. We decided to make a critique of the task contexts from a "Wittgensteinian" theoretical perspective. This theoretical perspective is dissimilar to the theories that ground the main research study. However I decided that it still contributed to the thesis as it addresses task contexts, which are one of the focused topics in the thesis. I kindly thank Brian Greer and Ole Skovsmose for their permission to print this chapter in my thesis.

A difficulty for me was to build a new identity narrative for myself as a researcher, and hence distance myself from my teacher identity. This issue emerged after the six months I spend at Ericaskolan, the school in which I engaged in collaboration with the teacher. Two papers supported me to distance myself from my school experiences and continue forward in my researcher process. The first of these papers, titled "*Examining a critical pedagogical discourse for agency and social empowerment*" is a book chapter written together with Paola Valero for *Critical Mathematics Education: Theory and Praxis*, edited by Paul Ernest and Bharat Sriraman. At this time I needed to find a terminology that explained the ways in which Elin and I aimed at changing the students' mathematics teaching. It required a term that covered the change of contexts in which mathematics was introduced to the students, the different ways of working as alternatives to students individual work with textbook exercises, and the ways we intended to change the classroom discourses understood as the ways in which we talked with each other's as actors in the classrooms. Hence I started to use the term *pedagogical discourse* as a way to talk about the implemented pedagogical changes Elin and I did in collaboration in the classrooms. This book chapter, authored together with Paola Valero, tells the story of how we define the term, the different pedagogical discourses and how the mathematics topics were planned and introduced to the students. This book chapter is also of a more reflective character, and I believe it is of importance for the transparency of the research. I kindly thank Paul Ernest and Bharat Sriraman for their permission to print this chapter in my thesis.

The second paper, *A "Curling Teacher" in Mathematics Education: Teacher Identities and Pedagogy Development* is a journal article accepted for publication in Mathematics Education Research Journal (MERJ). This article highlights the collaborating teacher, Elin Johansson's identity narratives when adopting the challenge to change an expected mathematics teaching in Sweden. For this article I comprised the concept of identity and inscribed myself in the socio-cultural landscape of mathematics education research. Through the analysis of the teacher's narratives through the course, I received a better understanding of how the concept of identity narratives could support me in the coming analysis of the students' narratives. Elin's identity narratives tell the story of support and hindrances, of struggle and flow, when changing the pedagogical discourses in her classrooms. This article, that addressed the teacher and her

narratives, allowed me to distance myself further from my teacher experience, and to become more reflexive on my own engagement and involvement. Thus, writing this article supported me to clarify the impact I had on the teaching organisation and the collaboration process during the empirical data collecting time of my work. At the same time writing the paper was an important step in clarifying the methodology to study students' identity narratives in context and how to write my results.

Students' Identity Narratives

My main intention with the thesis was not to propose a specific mathematics pedagogy as a solution for particular students' disengagement or lack of motivation in mathematics education. My research interest was more accurately directed towards the students and their individual reasons for engagement and participation in mathematics learning. I became curious of the stories they told at different times, and how they ascribed meaning to the whole mathematics education situation. At this time I left the concepts of students' intentionality to participate in mathematics education, their foregrounds and their backgrounds behind. Instead I embraced the concept of identity as an opportunity for me to talk about students' experiences. I recognised the potentialities for my research through understanding identities in a sociological way and hence recognised learning as transformations of identities (Lerman, 2000; 2006). Students' talk about their experiences of meaningfulness, thinking, and reasoning are thus seen as products of social activities as suggested by Lerman, (2000, 2006). I positioned myself in the socio-cultural trend of mathematics education research. As a consequence, when considering the stories students' tell about their mathematics education experiences and their willingness to engage in mathematics learning, the notion of identity became particularly important for my study. I took on a discursive understanding of the concept of identity, and started to study the work of Anna Sfard (2008), and the framework she developed together with Anna Prusak (2005a,b). More inspiration and insight came from my readings of for example Black, Mendick and Solomon (2009); de Freitas and Nolan (2010), Solomon (2009) and Stentoft (2009). In this discursive understanding identity intersects with language use as discursive practises in context.

During the analysis stage, I came to the conclusion that the concept of agency needed to be incorporated in my study. I adopted an ecological understanding of agency that fitted well with the concept of agency. Two papers emerged out of these readings, both accepted for proceedings from CERME7, 2011. The theoretical paper, *Agency in mathematics education*, is written together with Eva Norén at Stockholm University. In this paper we elaborate theoretically on the concept of agency from Skovsmose and Biesta's different theoretical positions. The empirical paper, *Interplays between context and students' achievement of agency*, is the first of my writings addressing the students' narrated identities. In this paper I tried out the analysis of agency through the narratives told by Sandra, a girl who objectified herself as being "math-anxious". At this time I started to understand that there were relationships between how the students talked about and labelled themselves, and how they achieved agency in mathematics education. It seemed as the way the students objectified themselves had impact on their actions in the classroom, however in different ways in relation to different contexts.

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The meanings the students ascribed to their mathematics education experience where not only bounded to the classrooms contexts. The students referred to contexts both within and from the outside of the mathematics classrooms. This relationship is expounded more thoroughly in my last two submitted papers, authored together with Paola Valero and Tamsin Meaney. *I am [not always] a math-hater" Students' (dis)engagement in mathematics education* sheds light on Malin and Petra's stories in relationships with different contexts. The contexts their narratives referred to were situated in the classrooms, the school and in societal contexts. Hence, they emerged from different levels of the mathematics education network. Malin and Petra labelled themselves "maths-anxious" and "maths-hater" respectively, however, these objectifying labels were not always present in their stories. These results indicate problematic issues when students become objectified (Sfard, 2008). In Petra's case her way of (sometimes, but not always) expressing herself as a "math-hater" could easily be interpreted as she *had* an identity as a "math-hater", or as *always being* a "math-hater". However, this study showed that this not always is the case. Identities fluctuate and are less stable when connected to the available contexts.

The second paper co-authored with Paola Valero, *Identity narratives of [dis]engagement in mathematics education contexts,* focuses on Henrik's identity narratives in relationships with contexts, and in what contexts he achieved agency and engaged or disengaged in mathematics learning. The main points in this paper were threefold. First, Henrik's identity narratives in relation to the diversity of contexts highlighted the complexity and variability of his engagement in mathematics learning. Second, it was not the mathematical contents per se, but rather the array of contexts in which the activities for learning mathematics took place, that allowed him a space to act and tell stories about himself and his experience in qualitatively different ways. Third, Henrik's narrated identities indicate that identities are not always as consistent as they often appear to be portrayed in literature. These results indicate problematic issues when students become labelled as result of our research categorizations.

The way I allowed space for different students' voices in the different texts is a way for me to give evidence for the resemblances in the students' ways of talking about themselves. I studied individual students' identities through the course and could in this way show how individual students changed their way of narrating themselves.

Summing up, within the theoretical landscape of mathematics education research, I decided to position myself within the socio-cultural paradigm. I recognised mathematics classrooms as social and discursive practises where students talk about themselves emerged as discursive choices of identity. The ways in which the students individually talked about themselves, sometimes objectifying themselves, were linked to their actions, hence their achievement of agency. The students' identity narratives emerged in different ways, depending on what context the student addressed at that particular point in time when the narrative was told. In this way the different notions, which emerged during the research process, also linked to each other.

Now, when finalising my process I would have enjoyed putting more emphasis on the socio-political questions that arose from this research. However that is not possible for me to do within the defined time-space for this thesis. Instead I choose to talk about my political awareness, and my researcher attitude – not in my choice of theoretical or analytical framework but in relationship to my ethical considerations. With inspiration from Gee (2005) who defined

discourses with "capital D" and "little d", and from Valero (2004a) who argued for a socio-political trend in mathematics education, I would like to define the socio-political approach with "capital P" and "little p". Within a "capital P" research, power and relationships with a theoretically and linguistically consistency are emphasised, through the whole research process, in theory, methodology, analysis and conclusions. A researcher acknowledging a "little p" emphasises an awareness of political issues, is sensitive to power and relationships, and cares for research participants through a researcher "attitude". Paraphrasing Gee (2005, p. 26) I suggest that the distinction of socio-political research with a "capital P" and a "small p" can be formulated as: "A researcher with a "little p" adheres to the assumption that mathematics education is a social and political practice where power is exercised, and shows sensitivity for these questions through her researcher attitude. "Big P" researchers are in addition politically consistent in all "other stuff." In this study I would position myself as a "little p" researcher, recognising that power and relationships are important features, with sensitivity for participants and aiming for a reflexive researcher attitude.

What is unique with my research is the way I have aimed to take into account the students' whole educational experience, of which mathematics education is a part. Thus I broadened the research to comprise contexts emerging at different levels in society, in the school and in the classrooms. All students in the classes were invited to participate in the study, not only specific, labelled student groups. Their relationship with mathematics is voiced in this study in the way they themselves articulated their relationships with mathematics and mathematics education, in the available discourses and in relation to the contexts in which the narratives were told. My point is to show and talk about engaging in learning mathematics in terms of the students overall experience, not in terms of cognition or conceptual understanding of mathematics. Learning takes place in educational discourses, where students' identities emerge through the narratives they tell about themselves, and it is these narrated identities I intend to focus.

Distinguishing the Two Layers of the Thesis

I want to emphasise that this is a study in two layers, which at times have been complicated to identify as distinct dimensions, and at other times problematic for me to explain. The reason for this is that the two layers are intertwined and affect each other simultaneously. During the research process it has been a struggle to keep them apart and to not fall into the trap of evaluating the introduced pedagogical discourse per se. In this section I aim to clarify the two layers and in what ways I have strived to keep them apart.

The bottom layer concerns the specific "disturbance of the queue", which is the introduced teaching discourses in the specific mathematics course. This disturbance needed to be realized, for me to be able to study students' narratives in a different mathematics education than they expected or imagined. I believe that the imagination and arrangement of the introduced pedagogical discourse needed to be accounted for in detail as first, a part of the research process, second, to support a more fully understanding of the story I intend to tell, and third, for the reason of research transparency. I believed that the complexity of issues that occurred in relation

to different actors and stakeholders when Elin Johansson, the collaborating teacher, together with me changed her way of teaching needed to be explained in detail.

However, I underline that the pedagogical development was done with these particular students at this point in time, without any aims to evaluate if it was a "good" or "not so good" pedagogy and what (objectified) student group(s) would benefit from this particular teaching. If this study inspires teachers to develop their teaching organisation in for them more suitable ways I am happy. My aim was not to evaluate the teaching as such and hence give advice to other teachers on how to teach with inspiration in critical mathematics education, or teach "social-science students" mathematics, or engage "noisy" students. The case was rather the opposite. I intended to address the students' stories and experiences from their mathematics education. I was interested in understanding their stories and why they told these particular stories about themselves at those specific points in time. Hence, the mathematics teaching became an intervention not to be evaluated as such, but to make possible the flourishing of other types of identity narratives than the ones I was used to listen to as a teacher in traditional teaching environments.

The second layer, the main study layer, concerns the students and their identities, experiences and emerging relationships with mathematics during this particular mathematics course where we changed the teaching. This theme is what the main thesis is aimed to focus. I have strived to distinguish the hierarchical order of the layers in the articles, in a way that different articles, proceedings papers or book chapters address the different layers. Also in the wrapping I have aimed to keep them apart, calling them systematically the "disturbance" and "the study of identity narratives".

I need to acknowledge that there will be repetitions in the articles and in this wrapping as I, through the process of reading and writing, have developed further understandings and enunciated my ideas, hence engaged in learning.

Theoretical Considerations

Adopting a Socio-Cultural Perspective

The theoretical framework guiding the study is grounded mainly within a socio-cultural perspective. However also socio-political theories had a slight impact on my research. Mathematics education is thus viewed as a social and political practice (Valero, 2004a, Gutiérrez, 2010) and mathematics learning as a social activity (Lerman, 2000; Radford, 2008). In my understanding, a socio-cultural theoretical perspective implicates comprehending (mathematics) classrooms as social systems that are organised as shared practices. These social systems shape in different ways how individuals are expected to, allowed to and/or required to act and participate, yet they also form the meanings that subjects in these social systems make of their actions when participating in them (Gee, 2005; Holland, Lachicotte, Skinner & Cain, 2003; Roth, 2007).

Valero (2002a, 2004a,b) argued for a need to recognise students as whole social, historical beings with multiple motives for learning, and whose actions are embedded in contexts, which influence their intentions to participate and learn mathematics. Martin's (2000) illustrative painting of the interacting contexts that impacted on African-American youth's achievement, success or failure, in mathematics education is an example of research accounting for the complexity of students' engagement in mathematics education. Martin demonstrates the complexity of the interactions between socio-historical policies and practices, community contexts, school-level factors and the individuals' identities and socialisation processes into mathematics education. Radford (2008, p. 227) showed how students' developed a "communal self" in classrooms in which "the student elaborates a communal and active relation with his/her historical-cultural reality". And, the girl Kalila's narrated identities in Lange's (2009) case study of children in difficulties with learning mathematics indicate that students' identities in mathematics education are not separated from other life experiences. The case is rather the opposite: the identities Kalila told were closely linked to other aspects of Kalila's life. The different levels and the interplays between them that impacted on African-American youth's engagement in mathematics education explained by Martin, Radford's elaborations on students' "communal self" and Lange's examples of connections between young students identities in mathematics education, and other life features, clearly exemplify Valero's point about the complexity of students' reasons to engage in mathematics education, and I would ad - to disengage. These complexities are in this study illuminated through the narrated identities students voiced in Andersson (in press, b), Andersson, Meaney and Valero (under submission) and Andersson & Valero (under submission)

When I grappled with on the one hand the complexity of students' engagement in mathematics learning, and on the other hand the complexity of the interactions between different school practices, I needed to find a way to address these complexities in a structured way. Two metaphors for mathematics education supported my thinking at that point in time. These two metaphors are explained further in the next section.

Introducing two Metaphors for Mathematics Education

Martin's "Multilevel Framework Metaphor"

My study spans over different social practices and addressed different contexts that comprise mathematics education. First, to be able to investigate the processes of students' multifaceted identity work in a relational understanding, a multi-level framework developed by Danny Martin (2000) inspired my thinking. Martin expanded his initial classroom focus to comprise a socio-historical level, a community level, a school level and an "agency and mathematics success" level (p. 30). This empirically grounded expanded focus was required in Martin's research when he realised that observing teacher and student practises could not alone explain mathematics achievement and persistence among Africa-American youth. Martin's metaphor of mathematics education as a multilevel framework indicated in my understanding that there was a hierarchy between the different levels, from the top socio-historical level down to the individual students' level. The level metaphor also indicated to me that there were power relations between the different levels so that one level overpowered the next. However, Martin stressed that the relationships between the different themes and contexts within the framework were more complex than this hierarchical table indicated. The relationships between the different levels were multifaceted and participants within the different levels referred to other levels in the framework in an un-linear fashion.

Martin's writings made me reflect that (some of) these levels of contexts probably were of importance to all youth, not only for African-American youth, however in slightly different ways. Consequently, I reflected on what levels needed to be accounted for in a Swedish context when researching how students' identity narratives were told in relationships with mathematics education? With inspiration from Martin (2000), I decided the relevant levels for this particular study to be the school level, comprising governmental steering and the school structural boundaries as the national curriculum, national tests, timetables etc. This level referred to the boundaries the teacher needed to negotiate and impacted on her choices when planning and teaching. A classroom level was obviously required. I also took into account a family level. The parents would know their children's experiences and would be able to tell some of the identity narratives about their children's relationship with mathematics, as a large number of parents are deep involved in their children's education (Winbourne, 2009). Initially, I also intended to focus a students' peers' level and address the relevance of peers' and siblings' stories within and outside the mathematics classroom. However, from an ethical point of view I regarded it problematic to gather information from students' peers and friends. Even the data I received spontaneously from peers became problematic to analyse within the chosen analytical framework

and fit together with the students' narrated identities. So I decided to not systematically research students' peers' stories within the frames for this research.

Valero's "Network of Social Practises" Metaphor

A different way of conceptualising the field, however concerned with similar issues as Martin's framework, was offered by Paola Valero. I followed Valero's work closely during the time she developed her view on mathematics education as a "network of social practises" (Valero, 2010, p. LXX). The ways Valero's mathematics education metaphor illustrates the network, and takes into account not only classroom practises, but also practices on the school arena and national arena, allowed for a more structured expanded research focus in this study. Also the relationships between the different social practises were recognised. I obviously had a research focus on what occurred within the mathematics classrooms, but to be able to capture the students' narrated identities related to contexts emerging from outside the classrooms, Valero's model supported my thinking further. Different relationships emerged at particular times and influenced our work in the classrooms. Subsequently, when analysing the students' stories in this wider socio-cultural context, Valero's (2010) model of understanding mathematics education as a network of social practises became useful in this study.

For answering the research questions where the centre of attention was on the organisation of a critical inspired pedagogical discourse and on the teacher's identity narratives I span over a wider research area within mathematics education. From the teacher's perspective, social practices as for example the school leadership, the national policy making and governmental steering through the curriculum and examination policies, youth culture, and the way textbook authors constructed mathematics tasks impacted on her opportunities to develop the teaching in the way she wanted. Other issues such as relationships with colleagues impacted when she wanted to collaborate over subject boundaries and teach mathematics in a more interdisciplinary way. Sometimes these practises worked in our favour; sometimes they hindered the pedagogical development. Valero's (2010) representation of the social practises that impacted on planning and organisation of mathematics education supported my thinking and my structural way of addressing different slices of the network.

Valero's metaphor of mathematics education is not directional; hence there are no clear power hierarchies. The case in this study was rather that the relationships that emerged between the different social practises influenced and connected the different parts of the research in different ways at different times. In this way of understanding the two different frameworks by Martin and Valero, I believe that they were theoretically possible to coordinate. Wedege (2010a, p.109) explained the term coordinating as

> used when a conceptual framework is built by well fitting elements from different theories:
> elements, e.g. from the basic principles P, are chosen and put together in a more or less
> harmonious way to investigate a certain research problem. This can only be done by theories
> with compatible cores, which include the theories' accepted ground rules and norms.

The coordination of Martin and Valero's representations respectively of the mathematical education enterprise supported my structural approach in this research. Martin's levels supported

my thinking mainly in relation to the research questions relating to the students' identity narrations, Valero's supported me both in researching the students identity narratives and explaining the development and implementation of the pedagogical discourse (see Andersson & Valero, in press).
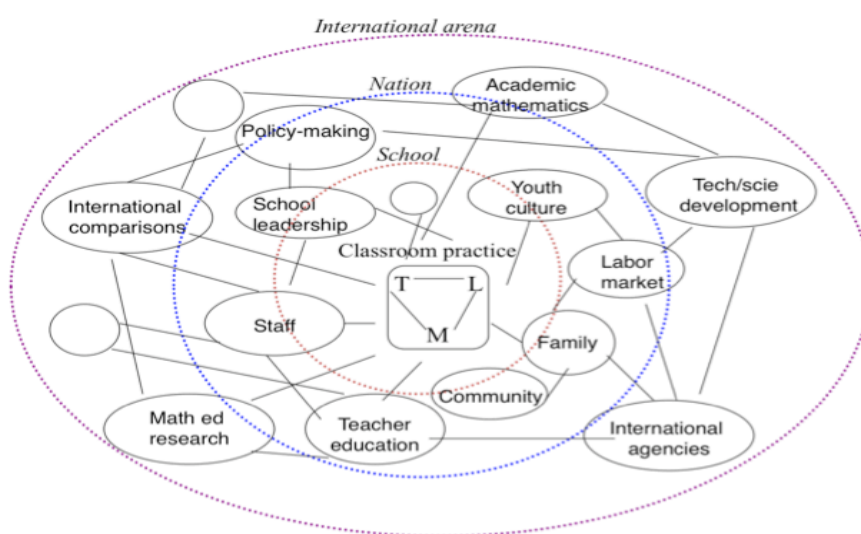
Fig 1. Valero's representation of the "Network of the social practices of mathematics education" (2010, p. LXX)

In this research I fore-grounded the classroom practice. Moreover, the introduction of the critical inspired pedagogy opened up spaces within the mathematics classroom for discourses from outside the classroom, and societal contexts became present in the mathematics teaching. Therefore I expanded the focus to 'outside the classroom practice' when students' talk referred to those other practises. These practises became foregrounded while recognising the important influence of families, school leadership and other staff as well as the structural organisation within the school. In the background of the research played national social practises such as governmental school steering, societal requirements of youth learning mathematics, possible future labour markets and further education. The back-grounded practises played an important structural role that we had to adjust to, as they were not conceivable for us to change. However these practises impacted on the identity narratives told by the students.

Navigating in the Socio-Cultural Theoretical Landscape

In the last ten years socio-cultural and socio-political theoretical perspectives have been increasingly adopted in the mathematics education research field (Gutiérrez, 2010). The

burgeoning interest has resulted in a growing number of edited books, research papers and journal articles (de Freitas & Nolan, 2008). When I positioned myself within the social trend in mathematics education research, several options on how to pursue my research emerged. I could choose to go in many different directions. For example I could understand mathematics classrooms as culturally constructed *figured worlds* (Holland et al., 1998) as Boaler and Greeno (2000) did when looking at able students' identity development in mathematics classrooms. Or, mathematics learning could be situated in classrooms, as "communities of practise" (Lave & Wenger, 1991). It is not possible for me to here argue for theories I discharged during my research process. Instead I choose to adhere to the suggestions of socio-cultural theories in future mathematics education made by Lerman (2006). Lerman (2006, p. 179-180) suggested three different future trends within the "strong social turn", that is the realm of socio-cultural theories in mathematics education research. The three trends were "recognising learning as identity formation, the application of the third generation activity theory; and also a greater focus on ethnography". In what follows I will discuss these different trends and argue for the decisions I took for or against these theoretical proposals during the research process.

Activity Theory

Drawing on cultural historical activity theory (CHAT), identities emerge from participation and engagement in socio-culturally mediated and action-oriented activities (Holland et al., 1998; Roth & Tobin, 2007; Roth, Tobin, Zimmermann, Bryant & Davis, 2002). Drawing on Engestrom (1987), Roth et al. (2002, p. 255) grounded their research with CHAT theory understood as "what people (subjects) actually do, the objects that motivate their activity, the tools they use, the community in which they are a part, the rules that pattern their actions, and the distribution of labour they take in activity". They concluded "participation and learning are coextensive with our changing self-understanding and identities as subjects in the activity system" (p. 279). Understanding a mathematics classroom as an activity system where students (subjects) learn mathematics (objects) would have been a possible way for me to move this study forward. Indeed, CHAT is a compelling social framework for understanding students developing identities, however it was not possible for me to distinguish the classrooms where this particular research was conducted as full activity systems with distribution of labour and possible (good) outcomes for a whole community as for example elaborated by Roth et al. (2002). CHAT might have supported me to answer questions about how the activities shaped by the introduced pedagogical discourse emerged, how the developed projects progressed and what the outcome for a community would be when working and learning within this particular activity system. However it was not my intention to answer those questions even if they are both important and interesting. Understanding a mathematics education classroom as an activity system supported my thinking but was left behind as an analytical framework.

Ethnography

Lerman (2006, p. 183) concluded "the future work in mathematics classrooms will not be so much on learning as cognitive development but ethnographic work on learning as developing appropriate school-mathematical identities". It is not clear to me what "appropriate school-

mathematical identities" comprises for features, emotions, attitudes, behaviours or actions. The way I see identities as developing and changing in social discursive practices makes it problematic to pin down what "appropriate school-mathematical identities" are. However, I agree with Lerman (p. 183) that ethnographic work is important to pursue within mathematics education research for "theorising power relations, identifying regulatory processes and pedagogical identities", which actually is part of the scope for this thesis. In the methodology section for the students' identity narratives, I elaborate further how I conducted ethnographic research in this study, with the purpose to develop an understanding for what identity narratives the students told in this particular setting.

Theories of Identity

In recent years students' and teachers' identities, that is how students and teachers view, describe and tell about themselves, has been increasingly used in mathematics education research, especially from researchers taking either a discursive psychological perspective, or a more sociological theoretical view (Gutiérrez, 2010; Lerman, 2006; Solomon, 2009; Sfard, 2008). The earliest thesis I have found focusing on students' identities in mathematics education research is Danny Martin's thesis from 1998, (rewritten 2000) where he addressed mathematics socialization and identities among African-American youth. At this time also Boaler and Greeno (2000) used the concept of identity when showing that the figured worlds as cultural constructs (Holland et al., 1998) of many mathematics classrooms are narrow and ritualistic thus leading able students to reject the discipline at a sensitive stage of their identity development. Boaler and Greeno (2000) argued that capable students discard mathematics as the views of mathematics education run counter to the students developing identities as responsible, thinking agents. They suggested that many students found the narrowly defined roles they were required to play within mathematics education incompatible with their developing identities.

Lerman (2000, 2006) accounted for "the social turn" in mathematics education research and argued for research potentialities in understanding identities in a sociological way and hence recognizing learning as transformations of identities. In this understanding, the concept of identity becomes processes of participation and enculturation, labelled for example "identity development" (e.g., Cobb, Gresalfi & Hodges, 2009; Hodgen & Marks, 2009), "identity work" (e.g., Mendick, 2006) or "identity formations" (Lerman, 2006, 2009). Hence, instead of talking about who we are in an essentialist way, identities on the contrary becomes recognised as movements or destinations. Recently identity researchers have moved further towards discursive sociological perspectives (see e.g., Black, Mendick & Solomon (2010), Solomon (2009) and Stentoft and Valero (2009)). In this understanding identity intersects with language use as discourses and contexts. Thus individuals' identities are referred as positioning in discursive practises and viewed as multiple subjectivities (Stentoft & Valero, 2009). Two different trends are more noticeable within the discursive sociological perspective. First, poststructuralist writers as Bibby (2008, 2010), and Black, Mendick, Rodd, Solomon and Brown (2009) are examples of researchers who explore students' identities from psychoanalytic theoretical perspectives. Individuals are from this perspective positioned within practices that construct them as subjects. Second, researchers as e.g., de Freitas (2010), Sfard (2008), Sfard and Prusak (2005a, 2005b) and

Stentoft and Valero (2009) define identities in a stricter discursive way. Sfard and Prusak (2005a, p.21) propose identities to be collectively narrated reified, endorsed stories about a person, which are produced through engagement with and connected to available discourses:

> The key move was to equate identity-building with storytelling. The difference between identity as a "thing in the world" and as a discursive construct is subtle. The kind of data that the narrative-minded researcher analyses in her studies is the same as everybody else's: These are stories that people tell about themselves or about others to their friends, teachers, parents, children, and bosses, as well as to researchers. The only distinctive feature of the present narrative approach is that, rather than treat the stories as windows to another entity that stays unchanged when "the stories themselves" evolve, the adherent of the narrative perspective is interested in the stories as such, accepting them for what they appear to be: words that are taken seriously and that shape one's actions.

This proposition made by Sfard and Prusak resonated well with my intentions to research students' stories during their mathematics education. I recognised identities as students talk about themselves that emerged in available discourses; however identities also shape their actions. This recognition resonates with Lerman' (2006, p. 180) statement about subjectivity:

> Subjectivity focuses on how individuals are both the subject in the sense of the actor in a discourse but are also subjected to the possibilities and limitations, the affordances and constraints, of that discourse. Identity is therefore produced in discourses and the notion of subjectivity captures that regulation.

Therefore, the relationship between identity on the one hand, and discourses on the other, is that discourses allow the spaces, and the boundaries, for the ways in which we as individuals can talk about ourselves as subjects, hence narrate our identities.

As Lerman (2000) accounted for the "social turn" in mathematics education research, Gutiérrez (2010) now reports a "socio-political turn". A socio-political stance in mathematics education research points to the fact that identities are recognized as subjectivities. As Gutiérrez stated (2010, p.2): "A shift toward focusing on social issues has allowed us to uncover the importance of students and teachers needing to belong to something larger and for changes in one's identity to serve as evidence of learning." Students are in this study recognised with multiple motives for learning, thus learning is not seen just about objectification of knowledge. It is also about *subjectification*, which is becoming someone (Bauchspies, 2009; Davies, 2006; Radford, 2008). Subjection, the ambivalence of mastery and submission, which takes place simultaneously in educational settings (Butler, 1990) or subjectification as becoming (Radford, 2008) are processes in which we as individuals become who and what we are in relation to others. However, I decided to keep within the socio-cultural terminological framework, and intend to use the concept of identity, not subjectivity or subjectification, in the following writings. Instead I will introduce the term identity narratives.

Defining Identity Narratives

The concept of identity provided ways for me to understand the complexity of students' decision making at certain points in time. Understanding identities as emerging in discursive activities

implicated defining identities in ways in which subjects think and communicate about themselves (e.g., Lange, 2009; Sfard, 2008; Sfard & Prusak, 2005a, 2005b; Stentoft, 2009). In this study *students' identities* are defined with the operational definition offered by Sfard & Prusak (2005a, p. 14). They propose to "equate identities with stories about persons. No, no mistake here: We did not say that identities were finding their expression in stories—we said they were stories".

With the goal to emphasise the discursive perspective in this study, and underline the importance of the idea that identities are defined as narratives about a person, I decided to use the phrase "identity narratives" for the purpose of this study. *Identity narratives* are the reified and endorsed stories that subjects tell about themselves, or others tell about the identified individual (elaborated further in Andersson, in press, a).

Learning as Closing Identity Gaps

Sfard and Prusak (2005a, 2005b) split identity narratives into two subsets: actual identities and designated identities. The *actual identities* are identities told in present tense stating the actual state of affairs. The *designated identities* are those expected to become actual identities in either close or far future. They are usually told in future tense with words expressing for example wishes, commitments or necessities. We as individuals create designated identities from narratives that are "floating around" and told by others or us. Sfard and Prusak (2005b, p. 1-46) put it: "identities are products of discursive diffusion – of our tendency to recycle strips of things said by others even if we are unaware of these texts' origins." With the diversity of stories floating around about a person, it is clear that multiple identities exist. The issue is complex. Lange (2009, p. 144) formulated the complexity, as "what we say about others and ourselves is a melting pot of social stereotypes, categories, and narrative genres." Hence a number of the stories that are "floating around" also become part of our narrated identities.

From the students' perspectives, prior experiences of inclusion or exclusion in mathematics education are likely to bring in significant identity narratives. These identity narratives are likely to impact on the students' decisions on how to engage or disengage in their learning of mathematics. Lange (2009, p.145) put the connection between children's' prior experiences in mathematics education and designated identities in mathematics education in the following way:

> Children's experiences with learning mathematics and of being in difficulties with mathematics are part of their lived experience. In identifying, the discursive counterpart of experience is produced through the reification of identity narratives. The narratives are products of discursive diffusion, created from stories floating around in the socio-political-cultural context. Being part of this context, the social valorisation of mathematics is imprinted on the identity narratives. Hence, (some) identity narratives link children's lived experience with the discursive field pervading mathematics education.

Sfard and Prusak (2005a) suggested learning to occur when the gap between actual and designated identities close. I interpret their suggestion that learning occurs when the designated identities become the new actual identities, with new future designated identities as an everlasting fluid process. I interpret learning as there also are smaller identity gaps closed along the way to reach a more future designated identity. In other words, there are future identities that

are stronger or different designated identities to those that are closer in time. Designated identities change as stories evolve and some stories may lose their endorsement while new stories become endorsed. Some stories may also be more significant than other stories to take up as actual or designated identities.

Sfard and Prusak's (2005a, 2005b) operational framework was designed with the to further understand the generalised differences in mathematics learning between two identified cultural groups of students. My intention of using their framework was to analyse how individual students narrated themselves in relation to contexts within a different pedagogical discourses than students' were used to participate in. Subsequently, in contrast to Sfard and Prusak's intentions, the objective of this study was rather to reveal how individuals' talked about their identities in mathematics education in relation to contexts over time and in relation to their wider experiences of life. So my objective was not to conclude about social science students' mathematics learning as a defined cultural group or doing other kinds of categorisations.

During the research process it became obvious for me that the notion of agency needed to be taken seriously. First, the ways in which the students' identity narratives empirically emerged indicated that decisions for acting and being agentic were an important part of their story telling. For some students the agency component and the open spaces for individual decision making also allowed for acting and learning mathematics in ways that were different to their stories about their prior identities in mathematics education. Second, given the concerns raised by the School Inspectorate (2010) about students' individualised work in Swedish upper secondary schools and issues raised by Bibby (2008) about students' experiences of lack of choices when invited to learn mathematics, the concept of students' agency needed to be incorporated in this study in relation to the disturbance. Third, as argued theoretically earlier, as the case is that discourses allow the spaces, and the boundaries, for the ways in which we as individuals can talk about ourselves as subjects, and the actions we take are shaped by – and shape - our identities, the concept of agency requires to be incorporated in this study. In the next section I elaborate further on different aspects of agency in mathematics education research, and how the concept of agency emerged in this study.

Defining Agency

Agency is an elusive concept that, according to the Blackwell Encyclopaedia of Sociology (Fuchs, 2007) is hard to pin down:

> Agency is a fundamental and foundational category and puzzle in virtually all social sciences and humanities. Debates over agency have emerged together with these fields, and continue unabated into the present time, with no resolution or consensus in sight. While many agree that agency, action, and actor are basic in some sense, controversies persist over the definition, range, and explanatory status of these concepts. In addition, agency is contested because it connects to core questions in metaphysics, philosophy, and ethics, such as free will, moral responsibility, personhood, and subjective rights. Agency is tied to the legacy of liberal humanism that is part of the core of democratic citizenship.

In mathematics education research, a person's agency can be understood as initiating ideas, agreeing with others, to elaborate and critique, question or disagree with others (Gresalfi, Martin, Hand & Greeno, 2009, p. 53). A larger number of research articles on students' agency in mathematics education (e.g., Boaler & Greeno, 2000; Cobb, Gresalfi & Hodge, 2009; Wagner, 2007) draw on Pickering's (1995, p. 21) metaphor of the "dance of agency". This dance is a metaphor for the tension between individuals' initiatives and structures or conventions in relation to mathematics per se. Pickering differentiated between different forms of agency: conceptual agency addressed how to develop meanings and relations between mathematical concepts, theories and principles. For example, conceptual agency allowed space for choosing methods when solving set mathematical problems or textbook exercises. On the other hand, disciplinary agency referred to the use of established solution methods and thus turned the concept of agency to the discipline of mathematics itself rather than being seen as the sole responsibility of the person.

However, I argue that other aspects of agency are engaged in the 'agency dance' in mathematics classrooms. Students' agency might involve making decisions about tasks, time allocation and work distribution, assessment and accountability, who to work with and so on (see Andersson & Valero, in press). It might also concern what sorts of decision-making students' can make in relation to their individual learning of mathematics. Hence, the agentic dimension "lies in the ways in which we have control over the ways in which we respond to the situation" (Biesta & Tedder, 2006, p. 20-21) or in what ways students can act within classroom culture (Sullivan et al., 2006).

For this study I required a definition of agency that resonated with my socio-cultural theoretical positioning. In Andersson (in press, b) I developed my argument for adopting Biesta and Tedder's (2006, p.18-19) ecological definition of agency. They connect the concept of agency to contexts, and they implicate that agency should be seen as achieved and not as an individual's capacity:

> … agency should not be understood as a possession of the individual, but rather as something that is *achieved* in and through the engagement with a particular temporal-relational situation. The idea of *achieving agency* makes it possible to understand why individuals can be agentic in one situation but not in another. It moves the explanation away, in other words, from the individual and locates it firmly in the transaction (which also implies that the achievement in one situation does not mean that it will necessarily be achieved in other situations as well).

For a further theoretical elaboration, I refer to the paper I wrote together with Eva Norén, where we on Skovsmose's and Biesta's writings concerning the concept of agency (Andersson & Norén, in press).

Resistance

Resistance, being part of the agentic dimension also needed to be addressed in this study. Resistance can be defined as certain behaviours, for example not bringing pens to lessons or raising regularly questions such as "Why do we have to do this?" (Martin, 2000). Alternatively,

resistance can be defined in a relational way, as students who do not accept teachers' invitations to learning activities and/or who challenge school authorities (Alrø & Skovsmose, 2003). In adult education, resistance is often explained as responses to learning situations with students' "lack of motivation and the symptom as non-learning" (Wedege & Evans, 2006, p.35).

In this study, I required a definition of resistance that captured the individual's decision to act, and talk within the classroom discourses, in relationships with others as part of a social practice. I also wanted a definition that included a focus on learning. I pragmatically defined *resistance* as "ways of acting, talking, engaging or participating in mathematics learning activities that neither are proactive for the student's learning or corresponding with the present classroom discourse and teacher's expectations".

This development of the concept of agency in this study is explored further in the recent two papers focusing on students' narrated identities (Andersson, Meaney & Valero, under submission; Andersson & Valero, under submission). The first paper, "*I Am [Not Always] A Math-Hater": Students' [Dis]Engagement In Mathematics Education"* tells Petra and Malin's identity narratives where especially Petra accounts for agency as a reason for engagement at some points in time but not in others. Henrik, whose identity narratives are reported in "*Identity narratives of [dis]engagement in mathematics education contexts*" is a student who talks about experiencing meaningfulness in mathematics education at those times there are spaces for him to act agentic in the mathematics classroom.

Objectifying Processes as Identity-Agency Dialectics

The relationships between agency and identity narratives had to be elaborated further as there were some other aspects of importance to emphasize. One of these aspects had to do with objectifying processes. Sfard (2008, p. 44) relates these processes to a reification of ones actions, which is "the act of replacing sentences about processes and actions with propositions about states and objects". To give one example: "Sandra repeatedly worried, and did not feel well before taking mathematics tests". After reification, a teacher might state that "Sandra has test-anxiety", hence Sandra's acts and actions become objectified as Sandra having, in this case, "test-anxiety". The labels originate in our way of talking when moving words from one discourse to another, or it might influence a whole discourse according to Sfard. This is done in order to describe mental processes and activities through introducing nouns "that helped to squeeze a lengthy story of repetitive but transitory actions into a narrative on permanent, even if evolving, entities" (Sfard, 2008, p. 45).

A number of the students I met labelled themselves "math-haters", "math-anxious" or as having "math difficulties" even if they indeed had passed nine years of prior mathematics education. As a student, being objectified with a label from experiences in the past possibly has an impact on how the student would act and behave in the future, and hence an impact on the students' agency achievement (Biesta & Tedder, 2006). Sfard (2008) identified *metaphors of object* as a special figurative expression with "roots in our tendency for picturing the perceptually inaccessible world of human thinking in the image of material reality" (p. 42). These metaphors can be both useful and potentially harmful depending on whose actions are objectified and in what way they

become labelled in the mathematics education language discourses. A risk with objectifying a person's former actions is that it can be read as a statement of a person's future and might function as self-fulfilling prophecies (p. 56, original emphasis):

> Indeed, words that make reference to action-outlasting factors have the power to make one's future in the image of one's past. As agents of continuity and perpetuation, the reifying and alienating descriptions deprive a person of the sense of agency, restrict her sense of responsibility, and, in effect, exclude and disable just as much as they enable and create. In particular, when the effectiveness of learning is seen as determined by such personal givens as *potentials, gifts or disabilities* failure is likely to perpetuate failure and success is only too likely to beget success".

De Freitas (2010, p. 131, original emphasis) pointed to another aspect, or reason for labelling students in mathematics classrooms. This aspect referred to students' cultural (or lack of cultural) capital in the available school mathematics discourses. Referring to Fairclough (2003) she argues that

> An order of discourse both confines what is possible in any given interaction and also produces the space for such possibility. For instance, repeated teacher references to the *difficulty* of a particular mathematics text can function as regulative discursive moves that position students and teachers in relation to cultural norms regarding ability and achievement. Students are then labelled as having "math difficulties" when the meaning of such texts remains un-accessed.

I would argue that there is a dialectic relationship between identity narratives and agency in a socio-cultural context. How we talk about ourselves, label or objectify ourselves impacts on what possibilities we acknowledge for taking action, hence our achievement of agency in an ecological understanding (Biesta & Tedder, 2006). Yet, on the other hand, our actions influence how we talk about ourselves and hence our identities (Sfard & Prusak, 2005a, 2005b). In the sense of how I think of identities, labelling is not helpful as identities and subjectivities are not consistent. With this said, I regarded it not possible for me to analyse the specific, or psychological reasons behind the students' talk about themselves as objects. That issue was beyond the scope for this thesis. Hence, to analyse or speculate about students' reasons for telling certain stories at particular points in time did not make sense. It was the narratives as such, in relation to the contexts and actions where the narratives were told, which were of interest. As Sfard and Prusak (2005a, 2005b) remind us, identities *are* the stories we tell about ourselves. Accordingly, if a student labelled herself "I am a math-hater", that statement was accepted as her identity narrative at that particular point in time, without any further suppositions of the individual psychological reasons for telling that particular story.

Concluding so far, I adopted a discursive understanding of identities, where the key to the students' relationship with mathematics is their evolving process of becoming, understood as a discursive activity. I see identity as a construction, relational and in dependence with contexts where people act. I was aware of the risk of objectifying students in the way proposed by Sfard (2008), labelling them as "mat-nerd", "math-anxious" or the like.

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I aimed for the concept of identity to signal the relationships and connections between the individuals and the social, political, historical context in which human beings are and become. I followed Sfard and Prusak's (2005a) definition of identities as stories about persons. Through the adoption of the concept of identity narrative I received an entry into how these students talked about their relationship with mathematics and what contexts they participated and engaged or disengaged or resisted at some points in time but not in others. Thus this allowed for understanding the sense they made from the whole mathematics educational activity and why they decided to engage in that activity – or not. In the next section I will elaborate further on the different contexts that emerged as relational to the students' different identity narratives.

Contexts for Learning

The notion of context comprises the network of relationships and available recourses in the social practises in which we act, but at the same time contexts are forming the ways and spaces where we act. Accordingly, "context" is in this study understood as a reference to circumstances, but also with an understanding that contexts influence discourses.

In mathematics education research, context tends to be restricted to the immediate context of a particular classroom or studied activity episode (Morgan, 2006). Consequently, there is a need to explore individual students' relationships with mathematics in different contexts, which can be considered in a number of ways. First, we recognised task contexts including contexts expressed in textbooks exercises and through developed pedagogical projects (Wedege, 1999). Looking historically on mathematics education research, the notion of context was mainly associated to the task-contexts in constructivist research (Valero, 2002b). Second, there are situation contexts (Morgan, 2006; Wedege, 1999), which in this study encompasses "the current activities, the other participants, the tools available and other aspects of the immediate environment" (Morgan, 2006, p. 221). From an anthropological perspective, socio-cultural studies of learning recognised situation contexts as social settings where learning activities takes place (Valero, 2002b). The school context refers to contexts on school level, for example school structures such as timetables, school leadership and so on, as elaborated by Martin (2000) when addressing African-American youths' achievement or failure in mathematics education. I also recognised socio-political contexts, defined from a students' perspective by Valero (2002b, p. 9) in the following way:

> The characteristics of their *socio-political context* may not be discarded when considering what a particular mathematical learning experience means for them. The idea behind this postulate is that understanding the real social, historical, political and economic situatedness of students' intentions of participation in mathematical learning implies keeping a strong and clear connection between the students' micro context –the task, the interaction and the situation context– and their broad socio-political macro context as member of a particular society in a given historical time.

As this is not a socio-political study with "big P", I delimited the socio-political contexts to the issues that arose in the students' identity narratives in relation to contexts from outside the

classrooms. Consequently, as there are different layers within a socio-political context, I divided the socio-political context into two components when referring to contexts outside classrooms that influence what occurs within the classrooms. First, there is a *socio-political school context* (Valero, 2004a), which is operationalized through governmental policies on schools and the national curriculum, ideologies and school policies. Second, societal discourses as "specialness" when being "good at mathematics" (Mendick, Moreau & Epstein, 2009), or "discourses of belonging, being accepted, or rejected, and succeeding or failing" that often contribute to people giving up mathematics' studies (Black et al., 2009, p.29) are examples discourses regarded as *socio-political societal context* in this study. The different contexts are elaborated further and exemplified in Andersson (in press, b), Andersson, Meaney and Valero (under submission) and Andersson and Valero (under submission).

As the contexts the students related to in their telling about themselves became available through the present discourses in the mathematics classrooms, I will in the next section elaborate further on different opportunities that emerged for defining discourses in this study.

Defining Discourses

Discourses are in this study defined as suggested by Tsatsaroni et al. (2007, p. 85):

> A discourse is a system of signs that organises and regulates specific social and institutional
> practises and provides resources for participants to construct meanings (including meanings
> for their emotions) accounting for their actions and their identities.

However, discourses could in this research have been defined in line with Gee's (2005) distinction of discourses with a capital D and a small d: "Discourses, with a little "d" to mean language-in-use or stretches of language (like conversations or stories). "Big D" discourses are always language *plus* "other stuff." "(Gee, 2005, p.26). I want to emphasise that I acknowledged there were several discourses playing and interfering at the same time in the mathematics classrooms. To give some examples, there were the larger societal discourses, as the requirement of mathematics for becoming a university student. There was the societal talk about mathematics as being important, however also difficult to do and learn. These societal discourses are exemplified in the work of Mendick et al. (2009). They show how successful students construct identities of being "special and rare", hence refer to societal discourses of "specialness" (p.81) of those who are able to do mathematics. These discourses I would refer to as "Discourses". Then there were the expected discourses the students were used to, and enculturated into, as "doing quiet exercise work", or asking for "help", also they "Discourses". An allowing space within the classroom for using language in other ways than the expected, as inviting students to other possible ways of talking and acting than the usual, was an example of a "discourse" with small d that Elin and I aimed to create within the classrooms. If we 'succeeded' or not would depend on how the students decided to talk and act within the new offered discourse. If they recognised and took on the different way of talking and acting the discourse might, after a certain amount of time, become a "Discourse" (Gee, 2005). De Freitas (2010, p.130, original emphasis) explain what I want to express:

> Our discursive habits involve a contradictory mix of *submission* to the cultural norms inscribed in the discourse, and *empowerment* through this act of re-inscription. Pedagogy always involves this contradictory mix of submission and empowerment, since teachers and student voice and authority are constituted by the very discourse that confine and disciplines their actions. Teacher and student agency thus emerges through this contradictory mix; they are hailed as subjects within an institutional discourse.

I kept Gee's (2005) distinction between D-discourses and d-discourses in my mind. However, rather than distinguishing between discourses with capital D and small d in the texts, I decided to discuss in more specific ways what level or in what social practise the present discussed discourse was situated. The concept of discourse will be written consequently with a small d in the coming writings however with an specific awareness of that there are several levels of discourses playing in mathematics education. Our responsibility as educators to examine what conditions of possibilities the "taken-for-granted" discourses in education as a discursive practise create for teachers' and students' identities is discussed by Davies (2006, p. 436), who pointed out that:

> The social, psychic and intellectual work is, rather, emergent. Our responsibility lies inside social relations and inside a responsibility to and for *oneself in relation to the other*- not oneself as a known entity, but oneself in process, unfolding or folding up, being done or undone, in relation to the other, again and again […] We must take responsibility for examining the documents and discursive practices that are taken for granted in our schools and universities, and ask: what conditions of possibility are they creating and maintaining for us and for students? (original emphasis)

Elin and I strived for challenging and change the present discourse, so often taken for granted, within the classrooms and hence acknowledge different ways of communicating and acting within the mathematics classrooms. However, I want to underline that the main analysis aimed at connecting students' identity narratives to the different contexts that these narratives addressed. I would argue that the present discourses rather invited the individual students to tell the identity narratives they chose to tell, in relation to contexts, at specific points in time.

Research Purpose and Research Question

Research Purpose

The purpose of my study was to understand the identity narratives that students tell in the context of, for them, an unexpected mathematics teaching and to see what kind of relationship there was between students' identity narratives and contexts. During the research process it became clear that I also needed to incorporate the notions of meaningfulness and/or meaninglessness, agency and resistance. The students were in their first year of upper secondary school. Most of them aimed to go on to university studies, but did not intend to pursue the mathematics, technical or science subjects.

The use of a different teaching approach allowed students to reflect on their relationship to mathematics education. The new approach enabled students to not only comment on their relationships to mathematics while engaged in this approach, but to also reflect on how they had previously been taught mathematics. These reflections formed the students' identity narratives. Without experiencing a different kind of mathematics education, it may have been difficult for them to reflect in depth on their relationships to mathematics and their mathematics education.

The pedagogical discourses that were prominent when different pedagogical approaches were used were the impetus that contributed significantly to how students talked about themselves. Thus, this study was *not* an evaluation of the pedagogy or a pedagogical discourse per se. It could obviously have been interesting to further analyse possibilities and hindrances when imagining, arranging and reflecting on the implementation of a critical inspired pedagogy in a Swedish context. However, that would have been a different thesis.

In this study the teacher and I changed the teaching practice within the current school structural boundaries, as those were facts we neither could, nor intended to, change. A number of critical researchers place themselves in an outside position in relation to the social school system and critique the schooling system as such (e.g., Biesta, 2009; Lundin, 2008; Popkewitz, 2004, 2009). This study was not intended to be a critique of schooling, nor a critique of the Swedish governmental policies on education. There is a background critique of the traditional teaching practice of mathematics which arises from the stories the students told. However, my aim was to challenge things within the schooling boundaries.

Research Questions

My overarching research question is formulated in the following way:

How do identity narratives, of students and the teacher, illustrate the complex relationships between mathematics education contexts, agency and engagement?

I have chosen to address this overarching question with three sub-questions. The first sub-question originated in my work during the first year of the research. At that time I imagined, arranged and reflectively reasoned on possibilities to change the pedagogy and classroom discourses within the schooling system boundaries in order to study students' identity narratives in relation to different contexts. This sub-question is being dealt with in the thesis, although it is of secondary importance for the overall aim of the study. The sub-question is:

1. What are the supports and hindrances for changing teaching approaches in mathematics in a Swedish upper secondary school?

The second and third research sub-questions are the primary sub-questions that constitute the main focus of the research. These questions address the students' identity narratives and how these narratives are told in relationships with the different contexts, students' talk about agency, and their engagement. These concepts became clearer during the analysis stage.

2. What are the relationships between identity narratives, communicated by individual students, and the different contexts that they were experiencing at particular points in time?

3. What is the relationship between agency and students' engagement in learning of mathematics at particular points in time?

The research questions, along with the purpose of this study, have developed and changed during the course of the research process. The theoretical framework together with the emerging insights from the data analysis influenced this final version. During the research process I progressed from using only socio-cultural theories towards becoming a "little p" socio-cultural-political researcher. As well, the notion of identity narratives developed during the analysis stage into a stronger discursive understanding than had been originally envisaged. Nevertheless, my original research interest has remained, which was to develop a better understanding of the stories students tell about learning mathematics.

Contextualising the Research in Sweden

In this section I situate the study in the Swedish schooling context, with the purpose of giving a deeper background understanding for issues raised during the research process. I start off with describing the Swedish school system, the social science program, and the specific Mathematics A course, which we "disturbed". I successively narrow in towards Ericaskolan, the school where the empirical part of the research was conducted. The school is described briefly to prevent the risk of identifying the school. However, I do give an account for some general working conditions at the school that might have affected the research.

The Swedish School System

Children in Sweden start school at six years of age. The first year they attend the compulsory preschool class and then continue in the compulsory school, *grundskolan*, for nine further years. After grundskolan, at the age of 15, more than 98% of the students (Ministry of Education, 2009) carry on to the non-compulsory *gymnasium* (equivalent to upper secondary school or high school) where they choose a three-year study program and a school. The major natural and technical and social science study programs give the students a theoretical subject base and hence prepares for continuing university studies. Other programs are vocational in their character. From the 1st February 2011 there is a new school reform implemented in upper secondary schools that change some of the compulsory courses within the programs, subject contents, and the grading system (Ministry of Education, 2010b). The students participating in this study followed the curriculum from year 2000 (Ministry of Education, 2000a, 2000b).

Within the different study programs, students attend different courses. Compulsory courses for *all* students, independent of what study program they have chosen, were the basic courses in Swedish and English language, and basic mathematics courses. Then there were courses that characterised the study programs, for example social science courses and modern language courses on the social science study program. Finally, students chose individually from a variety of courses to personalise their study program. The courses were of two lengths, either 100 points courses, equalling to an average of 100 study hours, or 50 points courses, equalling 50 study hours. The students needed to acquire 2500 points during their three years in the gymnasium. Consequently they were expected to at least pass 30-35 different courses. The students received marks after *each* finalised course and all grades were equally important when students applied for further tertiary studies. This meant in reality that the students got graded all time every year, at different parts of the year for the different courses. This issue added a pressure on the students from the very beginning of their gymnasium studies to perform well at all times. Some students

experienced this very stressful. If they got sick or missed a test of other reasons might have immediate impact on their grades. Swedish medical youth care is warning for high stress symptoms in Swedish teenagers, however that is another, but still important story.

From a teacher's perspective, my experience is that these short courses put an extra pressure on teachers who had a relatively limited time space to support students through the courses. It was difficult to change the teaching organisation because it implied risk-taking if it did not work out in the classes. It was also difficult to motivate larger cross-subject projects that might require extra time. This added pressure on the research in the way that the developed projects had to be very well defined, and prepared in a way that the given time schedule could be kept, still with possibilities for students to reach all curriculum stated goals on all examination levels.

The Social Science Program

The social science program is a three-year national study program aiming to provide the students with knowledge based on the social sciences and the humanities, and preparing students for higher education. The epistemological and ontological view on learning and knowledge, and how this knowledge relates to society in this study programs structure, was stated in the national curriculum by the Ministry of Education (2000b, p.6, original English) in the following way:

> Developing a scientific approach is an important part of the programme. Characteristic of the humanities and the social sciences is that their fields of knowledge are being continuously reviewed and reinterpreted in the light of new experience. Different issues have given rise to different scientific traditions and research methods. The ability to apply a historical and international perspective is essential. Knowledge of history enables participation in cultural heritage and contributes to an understanding of the present and creates preparedness for the future. Familiarity with people's lives and thinking from different periods and cultures is increased through reading literature. For studies in science, economics and civics to be meaningful, knowledge of mathematics is particularly necessary, as is an interdisciplinary approach.

The ontological and epistemological implications of the national curriculum impacted on the way (mathematics) teaching was organised. The way the curriculum emphasise a scientific teaching approach for supporting students to be able to apply, become familiar with and experience meaningfulness, indicated that mathematics education was expected to contribute to these features. The curriculum accentuated an interdisciplinary approach for studies in mathematics on this program and hence supported our way of trying to work across subject boundaries. These issues are elaborated further in Andersson & Valero (in press) and Andersson (in press, a).

The Compulsory Mathematics A Course

The Mathematics A course was the first out of two compulsory mathematics courses on the social science program. The course covered mathematical contents such as arithmetic calculations, geometry, algebra, statistics and first grade functions (Ministry of Education, 2000a). The problem with this course was that the mathematics content was not obviously connected to other subjects on the students' chosen study path in social sciences as reported by

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the School Inspectorate (2010), even if the national curriculum stated otherwise about the subject mathematics:

> The subject […] aims at pupils being able to analyse, critically assess and solve problems in order to be able to independently determine their views on issues important both for themselves and society, covering areas such as ethics and the environment. The power of mathematics as a tool for understanding and modelling reality becomes evident when the subject is applied to areas that are familiar to pupils. Upper secondary school mathematics should thus be linked to the study orientation chosen in such a way that it enriches both the subject of mathematics and subjects specific to a course. (Ministry of Education, 2000b, p. 94, original English)
>
> Mathematics A is a core subject course and is included in all programs. Students with widely different study orientations study the course. The structure is modified and the problems chosen should be based on the pupils study orientation. The course provides general civic competence and constitutes an integral part of the chosen study orientation. (Ministry of Education, 2000b, p.96, original English)

These aims emphasized in the national curriculum supported our intentions to change to a critical inspired mathematics education as concerns raised in critical mathematics education fitted particular well with the intentions in the national curriculum. Critical mathematics education stresses that mathematics may serve as a tool for identifying and analysing critical features of society, which may be global as well as having to do with the local environment of students (Ernest, 2002; Skovsmose, 2005a). Ernest (2002, p.8) elaborated the classroom work further:

> The aims of critical mathematics require the use of a questioning and decision making learning style in the classroom. Teaching approaches should include discussions, permitted conflict of opinions and views but with justifications offered, the challenging of the teacher as an ultimate source of knowledge (not in their role as classroom authority), the questioning of content and the negotiation of shared goals.[…]. Also the learners should be given the chance to pose their own problems and initiate their own projects and investigations at least some of the time.

Wile acknowledging the boundaries such as the Mathematics A course curriculum, and the epistemological view inscribed in the curriculum and social science program structure, we aimed for connecting the mathematics teaching with societal issues and, if possible, to work with an interdisciplinary teaching approach. Inspiration for the pedagogical discourse came from concerns raised in critical mathematics education mainly by Ole Skovsmose (1994, 2001, 2005a, 2010) but also from other critical mathematics teachers and researchers such as Ernest (2002), Gutstein (2006), Gutstein and Pederson (2006) and Marilyn Frankenstein (e.g., 1998, 2010). However, the pedagogical discourse needed to be fitted to the particular contexts where the research took place. How this was done is elaborated in Andersson and Valero (in press). The teacher's experiences from the innovation semester are elaborated in Andersson (in press, a).

The Students' Working Conditions

Ericaskolan is situated in the heart of a middle ranged city in southern Sweden. The school specialises in mainly theoretical study programs. The students came to Ericaskolan from different geographically situated lower secondary schools, grundskolor. Some came from city schools, other from suburbs or more rural areas.

The students were divided into two teaching groups, labelled class 1a and 1b. They were scheduled for mathematics lessons two times a week, 90 minutes each during the entire Mathematics A course. 1b had their mathematics lessons the last session Tuesdays 15.15-16.45 and the first session Wednesday morning 8.30-10.00. The Tuesday afternoon session was difficult as the students' were tired after a full day of language subjects and philosophy classes. So the main reflection on this session was that the students found it hard to concentrate. Wednesday mornings we experienced as the opposite. The students were more focused and worked harder during these sessions. If they wanted to work at home with mathematics they either had to do it between the last session Tuesday afternoon and Wednesday morning or they had almost a week between their lessons. The situation for 1a was different. Their lessons were planned early Wednesday afternoons 1245-1215 and Fridays before lunch 1030-1200. The classrooms were situated in two different blocks of the school, a ten minutes' walk out doors in all (read Swedish) weather conditions throughout the year from the main school block were the rest rooms, lockers and cafe were located. For the teacher this meant carrying books and other equipment in a large shopping basket between geographically spread locations. The students had to walk a long way back if they had forgotten their working materials and wanted to pick them up during lessons. These structural issues that impacted on the teaching and the students work in different ways were recognised as problematic both by the students, their teacher and the teacher union representative (Andersson, in press, a)

It was in this particular school context Elin Johansson, the teacher, and I intended to create the disturbance. Referring to Valero's (2010) network of social practises that influence mathematics education, we intended to change a Swedish mathematics classroom practise influenced by school leadership practises (e.g., timetables and geographically different locations), staff (e.g., for interdisciplinary teaching approaches), youth culture (e.g., students expectations) and the parents. These practises are examples of practises that impacted from the school arena. The national arena impacted through governmental school steering, policy-making documents, and assessment regulations. Also mathematics education research should be recognised as influencing Elin's planning of her teaching at this point in time. These mentioned practises are examples of social practises that were part of the schooling and mathematics education context at Ericaskolan this particular autumn semester.

Methodological Considerations

This section on methodology is divided into two parts. The first section explores the methodological framework for the disturbance. The second section addresses the methodology for the students' identity narratives. In this way I aim for further clarity of the two levels of the research, and how I proceeded as a researcher in each one of them. Ethical considerations are also addressed in relation to the different sections, however there is obviously also an overlap.

Methodology for the Disturbance

Valero (2004a, p. 20) wrote that

> Adopting a socio-political approach is not only a matter of choosing a particular set of theories and methodologies. It is an 'attitude' that seeks for consistency between the former and our activity as researcher.

My researcher attitude implicated the importance of finding a research methodology that would allow consistency between the adopted socio-cultural theoretical perspectives and the arranged pedagogical discourse. In addition, my socio-political "little p" awareness also considered ethical issues, and the emerging relationships between participants as valuable to recognise as a researcher attitude. Obviously, the methodology was also expected to support responding in the most appropriate way to my research questions and allowing for discussions of my research interests on the different levels (Martin, 2000; Valero, 2004a) mentioned above. Three methodologies emerged as potential good options. Vithal (2004) suggested that ethnography and (participatory) action research could be identified as methodologies resonating with a socio-cultural-political approach and critical inspired mathematics teaching. These methodologies resonated well with epistemological concerns in the adopted socio-cultural and socio-political theoretical perspective of the study (Atweh, 2004; Bryman, 2008; Mertens, 2005; Skovsmose & Borba, 2004; Valero, 2004a; Vithal, 2003). Also a design or intervention methodology could be seen as a possibility (e.g., Gorard & Taylor, 2004). These three methodological approaches were promising approaches depending on what research levels or social practises I wanted to foreground or what research questions I wanted to answer.

Why Not a Design Study

If I had intended to evaluate a critical mathematics inspired pedagogical discourse in a Swedish context, and account for example for students' mathematical learning and performance in this setting, a design study or an experimental study (Cobb, Confrey, di Sessa, Lehrer & Schauble, 2003; Gorard, Roberts & Taylor, 2004; Gorard & Taylor, 2004) would have been the more appropriate methodology. Design studies represent an approach to combined methods research and are typically associated with curriculum improvement or teaching and learning methods

development (Gorard & Taylor, 2004). My research has certain similarities with design research as I used the design process as a vehicle by which I was able to look at classrooms where the pedagogy and pedagogical discourse was aimed at changing. However, I was not intending to evaluate the design of or the intervention per se, or the students' accountancy for their learning within this specific pedagogy.

With a different set of data there could rather be another, yet important, thesis to write about the design processes and evaluate the pedagogical implementation as such. Evaluation of a pedagogical change might "refer to the process of determining the merit, worth, or value of something, or the product of that process" as denoted by Scriven (1991 in Mertens, 2005, p. 47). Mertens (2005, p. 48) recognised the inclusivity of different stakeholders and suggested that:

> Inclusive evaluation involves a systematic investigation of the merit or worth of a program or system, for the purpose of reducing uncertainty in decision making, and to facilitate positive social change for the least advantaged. Thus inclusive evaluation is data-based, but the data are generated from an inclusive list of stakeholders, with special efforts to include those who have been traditionally under-represented. It does not exclude those who have been traditionally included in evaluations.

There were certainly possibilities in this study to explore students' narratives as an evaluative tool for the invented pedagogy, and even maybe conclude if it supported disadvantaged students in their learning of mathematics. However, in the case of this research the intervention was created as a background setting when establishing the space enabling students' talk about themselves and their relationship with mathematics education in a different mathematics education setting than usual. This pedagogical development was done with these particular students at this point in time, without any aims to evaluate if it was a "good" or "not so good" pedagogy and what (objectified) student group would benefit from this particular teaching. I rather wanted to address a research meta-level and high-light the complexity of issues that occurred in relation to different actors and stakeholders when Elin together with me intended to change her way of teaching. Hence, the mathematics teaching became an intervention not to be evaluated as such. My experience from our collaboration, and my learning about the complexity of issues that occurred when Elin intended to change her way of teaching, is elaborated further in Andersson (in press, a).

Inspiration from Participatory Action Research

A possible methodology suggested by Vithal (2004) was participatory action research (PAR). PAR is a practice-based collaborative methodology that is characterised of cycles of planning, action and reflection when for example developing pedagogies and evaluating teaching methods (e.g., Atweh, 2004; Bryman, 2004; Mertens, 2005; Robson, 2002; Skovsmose & Borba, 2004). As previously stated, evaluating the arranged pedagogy as such was not an objective of this study. However, some aspects from PAR, especially ethical considerations, became useful for my thinking and for the description of the research background and the emerged collaborations between the actors in the school. Taking the position of a "little p" researcher implicated an awareness of power, sensitivity for emerging relationships and care for research participants.

Participatory action researchers have attended to these issues and I found support and inspiration for these particular considerations within participatory action research writings. Participatory action research theories emphasised the collaborative relationship in the sense that the research was done together *with*, not *on* teacher or students and this became important for me to address. As Atweh (2004, p.194) stated:

> Participatory action research problematizes the process of research itself and critiques it in terms of power relationships between the participants, the representation of their voices and relative benefits each obtains from the research. As a social activity, action research is never neutral.

This study did not contribute to the "teacher as researcher" tradition of action research (e.g., Atweh, 2004; Bryman, 2004; Cohen et al., 2010; Mertens, 2005). In our case, Elin, the teacher and I had different individual goals with the research project. Elin had clearly not a research role; she had other personal objectives to engage in the research. Elin wanted to develop her teaching and with support try out new ways of working in the classroom. I was engaged in my research project with focus on students' identities. Elin provided me with access to the classrooms, and she always had control over the final teaching decisions, students' examinations and time distribution. She also decided on how to position me in the classrooms. I had the knowledge and prior experience of working with projects in mathematics education, and connecting mathematics to societal issues. I assisted in planning and sketching out projects, and during tougher times when Elin of different reasons was constrained I took on more planning and preparations. I also took responsibility to push the research project further, especially at the time when the constraints in the school became so tough that the research project almost came to an end. These examples of issues became apparent through the ebbing and flowing as a tide of occurrences and movements during our collaboration. Thus they became important to acknowledge as relationships in relation to power.

Consequently, ideas that previously have emerged in participation action research writings regarding ethical considerations (Atweh, 2004; Atweh & Brady, 2009; Skovsmose & Borba, 2004) supported me when discussing issues that arose during the research process regarding the disturbance. Atweh (2004) stressed that concerns raised in participatory action research were consistent with critical understandings of mathematics education, in particular and aimed at "empowering participants as a result of their involvement in their projects" (p. 191). This statement resonated well with the intentions of this particular research where the collaboration with the teacher offered us both opportunities for learning. It also resonated well with my researcher attitude while searching for appropriate ways to communicate my interpretations of the research activity. Ethical considerations were important and fore grounded when I established the collaboration between the teacher and me, and for the emerged relationships between the teacher, the students, and the staff at the school and myself. I intended to care for the relationships in a way that we felt personal trust, with openness for discussions, ideas and critique. Accordingly, I decided to adopt ideas developed in participation action research for describing the research process, collaborations and emerged relationships in relation to the disturbance.

The qualities of Skovsmose and Borba's (2004) participation action research model resonated well with the theoretical standpoints in this research. Even if I did not conduct action research, the model assisted me in describing the research process and to address issues about distribution of labour, power relations, and reflexively my impact on the teaching change process (Andersson, in press, b; Andersson & Valero, in press).

Issues as how my collaboration with the teacher emerged over time, our discussions and negotiations needed to be addressed. My attitude as a researcher prompted me to allow for spaces where the teacher and students' voices could come through. I reflected on what Vithal (2004, p. 231) wrote:

> What needs to be understood is how and what mediates the way in which a researcher
> understands the theory-practise relation and chooses to act in particular ways as a researcher
> – in constructing research relationships, in deciding who collects data and what is construed
> as data, and in how research participants are to be involved in the analysis and writing.

The data used for only research purposes were the interviews with the students, their parents and Elin. Students' logbooks, the classroom blogs and teacher's assessment sheets were all developed together with Elin for both research and teaching purposes. Data mainly for research purposes were analysed by me. Elin used the blog comments and students' logbook writings to support and reflect on her teaching. In this way we both received opportunities for learning. Elin's voice came forward in Andersson (in press, a), which is a journal article where the emergence of the collaboration process together with Elin is explained further. Elin reified and verified these writings as describing the state of affairs. We also plan to participate and present the teaching and the pedagogical discourse together in future Swedish teacher conferences. Regarding the students, I have aimed for their voices to come forward as far possible through transcript excerpts in the different journal articles, book chapters and conference papers. One of the students, Henrik, also came with me to a mathematics teacher conference and talked about his projects and his experiences from his Mathematics A course. Ideas for these articles and conference presentations aroused during the research process as possibilities for allowing spaces for actor's voices to come forward in my research writings and conference presentations.

The next section goes more into detail what considerations needed to be taken when planning and introducing a change of pedagogical discourses in mathematics education.

Developing Pedagogical Discourses

My teaching background obviously had an impact when I together with Elin, the teacher, decided to introduce a pedagogical discourse inspired by concerns raised in critical mathematics education, and relate the mathematics teaching to current societal issues. A central concern in my practice when I taught at the social science program, was how society and social issues could be related to school mathematics. Critical pedagogical discourses acknowledge existing practices and introduces through collaboration with teachers, different possible organisations of the classroom practice that aim to generate other possibilities for students' learning and talking about mathematics (Skovsmose, 2001). Concerns risen within critical mathematics education fitted well with the curriculum for this course in the way it connected the mathematics learning to

society (explained in Andersson & Valero, in press. Critical mathematics education hence became the inspiration for the "disturbance", the change, and thus the research background setting. Accordingly, within deep-rooted school-cultural and political structural school boundaries, and realising the complexity of the whole mathematics education practise, I aimed at creating a change of pedagogical discourse and thus establish a setting for something different than the normal to emerge.

For the empirical part of this research I initiated and established a collaborative partnership with Elin Johansson, a mathematics teacher at Ericaskolan, where I previously taught. Elin and I worked together during a school year with further imagining, arranging, practical organising and explorative reasoning of an imagined pedagogical discourse. When we designed the pedagogical discourse, special thought was attended to variables that could have different values in different pedagogical discourses. The variables related both to the students' learning of mathematics and to the different ways actors interacted within the classrooms. In this section I point out some of the aspects attended to.

A first variable was the type of tasks students were expected to explore or solve for learning the required mathematics. There were obviously tasks and exercises in the textbooks that could be used. Nevertheless, there were also other possibilities as open tasks, problem solving exercises, investigative assignments, mathematical modelling and so on – there were numbers of possibilities depending on, for example, what mathematical topic was in focus. How the work within the classrooms was arranged was a second variable. Individual work, small or larger group work, workshops and interdisciplinary collaborations emerged as opportunities depending on what was to be learnt at that specific time. A third variable was how assessment could be conducted. In our case the students needed to be prepared to pass a national test, and they received final course grades after completed the course. Assessment and grading could be done in a number of ways, for example orally or written, as tests or hand-ins, individually or in groups. The fourth variable addressed the physical interactions between the different actors in the classrooms. We examined different ways to furnish the classrooms, as in what ways it was most productive to place worktables for optimal communication and collaborations in relation to different tasks and ways of organising the teaching. Aspects as responsibility and decision taking were other variables that needed to be addressed; who was expected to be responsible for what? The last, but not least, variable related to how we used the language within the classrooms. How we as different actors talked with each other and how we positioned ourselves in relation to each other as teacher, researcher and students through language use became important for us to recognise.

Reflecting back, I would conclude that the pedagogy as such became the organised teaching in this particular course. However, the pedagogical discourse Elin and I aimed for and worked towards continuously within the classrooms would not emerged as a pedagogical discourse unless the students decided to accept and participate and regarded the pedagogical discourse as a possible way of talking and acting within the classrooms. Elin and I were aware of, and continuously reflected on, how our language use structured ways of acting in the classrooms and how problematic and complex it was to change things. To use a Swedish metaphor, some discourses were "stuck in the walls," indicating that the (school) walls implicated certain ways of

talking and acting learnt through prior socialisation processes. Together we all as actors within in the classrooms needed to create the relationships and spaces in which the different systems of reasons could emerge.

To conclude, there were a number of variables of importance that needed to be attended to when a different pedagogical discourse was introduced. Those different variables required to be attended to, and reflected on continuously during the research process and during the collaboration that emerged between Elin and me. The development of the pedagogical discourse is explained further and reflected on in Andersson and Valero (in press).

Ethical Considerations Concerning the Disturbance

As stated above, my attitude as researcher made me consider ethical matters reflexively during the research process. It is not possible for me to address them all here, however in addition to issues elaborated on in the different articles I would like to provide three examples that illuminate different questions that arose.

The first concerns the epistemological arguments that mathematics knowledge, or mathematics learning is constituted through activity in a socio-cultural context reflected that mathematical knowledge never is value free (see e.g., Atweh, 2004; Skovsmose, 2005b; Vithal, 2003). This topic became important to critically reflect on together with Elin as we introduced and discussed a critical inspired mathematics teaching in the mathematics classrooms. Hence care was taken to ethical issues, as the assumption that mathematics is not a value free subject permeated the mathematics teaching and students' discussions during the course. Spaces were created for students to discuss and question both their learning of mathematics, mathematics per se and the role of mathematics in society.

Second, for the transparency of the research, and when reflecting of qualities of validity and reliability the intervention and the collaboration between the teacher and me but also with other actors involved became important to focus. I have aimed for providing a thick contextual description in the articles, the book chapters and in these methodology sections. An underlying expectation from this is that it will open up for critique and scrutiny of the implemented pedagogy and the research itself. Atweh and Brady (2009) argue in the prevailing mathematics education discourse that "better mathematics" often refers to the academic discipline of mathematics. This discourse often opposes a mathematics education that focuses social world applications. Atweh and Brady stress the need to balance these concerns for a quality mathematics education.

Third, yet another important ethical issue to address were students' examinations at the end of the course. As Elin, the teacher, and I changed a traditional expected mathematics education great care was required for students' security regarding reaching objective stated in the national curriculum, examinations and grading. All the students were expected to perform well in a compulsory national test in the end of the course. This test was given to all Swedish students regardless of chosen study program and classroom pedagogy. The grades the students' received on examinations after this course would impact on their possibilities for further university studies. These boundaries obviously affected our agency for changing things.

To be able to manage these concerns Elin and I together purposefully created an open, humorous and critical relationship where we continuously reminded each other of how we talked and acted within the classrooms and in our relationship towards each other. In this way we became aware of and learnt how problematic it can be to change pedagogical discourses in mathematics education. One example is the discussions that arose when students wished for alternative ways of assessing their mathematical knowledge when they realised they performed and accounted for their knowledge in 'better' ways when they had opportunities to write log books or present knowledge orally. To put it in a pragmatic way, if we offered spaces for negotiations, we needed to take the discussions that arose seriously and carefully examine the students' wishes as possibilities during the course. Some of these issues are discussed further in Andersson (in press, a).

Methods for the Disturbance

Collecting the Mathematics Teacher's, Elin Johansson's Stories

Different types of information were collected before, during and after the teaching change, with the purpose to give transparency to the implementation of the discourse pedagogy and to examine the teaching change process per se. Elin wrote a personal letter in the beginning of the project were she narrated her personal teaching story and her reasons to engage in the research process. I then interviewed her twice, once in the middle of the semester and one interview in the end. The first interview focused on her experience of the process so far, and the relations, expectations and challenges between actors in the different levels of the network. The second focused on her stories about the students through the semester and her personal experience of the teaching change. We also discussed future mathematics education analysing possibilities and challenges in her forthcoming teaching. The written information Elin shared consisted of a large number of reflective e-mails and face book comments.

The School Principal and Teacher Union Representative Stories

I needed information from the school principal, Karl-Gustav Bertilsson, to further understand the wider school context in which this research took place, when expanding the research outside the classroom level and address the school level further (Martin, 2000). He was interviewed on two occasions where we talked about specific school policy issues and his opinions on changing mathematics education both in general and specifically at Ericaskolan. He explored his view on social science students and on their mathematics education and how he interpreted the national mathematics curriculum for this particular student group. Hence this interview comprised governmental steering and the school structural boundaries for mathematics teaching. He also clarified his acceptance level for some students' remedial behaviour in mathematics classrooms. This conversation clarified issues that arose on the school arena (Valero, 2010) and by the school leadership that impacted on the research. The second topic we discussed was teacher development at this particular school. The principal explored his view on teachers' agency, ownership and support strategies when changing (mathematics) teaching. There were also issues related to the teachers working conditions that needed clarification. The interviews with the school principal clarified topics related to the socio-cultural history of the school and thus

expanded the school level research and gave an understanding for why particular incidents occurred at particular times during the semester.

The teacher union representative, Sophie Larsson, was interviewed on one occasion, as there were issues related to the teachers working conditions that needed clarification. This interview focused Elin's working conditions and teaching load from the union representative's point of view, in particular at the time when Elin became constrained of a large number of duties. The interview clarified the socio-historical background for this being the case, and why it was problematic to change the teachers' working conditions at the school. This interview expanded the understanding of issues occurring at school level further.

These concerns, raised in the school leadership practice, were important as they had impact on the research process. They clarified matters that are partly explored in Andersson (in press, a).

Methodology for the Students' Identity Narratives

Deciding on Ethnography

The methodology developed for my research work, which focuses the students' identity narratives, is grounded with (critical) ethnography. The ways in which I acted in the classrooms and how the information was collected and analysed for understanding students' identity narratives in this setting was most clearly described through an ethnographic methodological approach. According to Bryman (2004), ethnography is the most common type of qualitative methodology used in educational research designed to describe and analyse practices. The ethnographic research work consists of data gathered from a range of resources including documentary evidence of various kinds, conversations and participant observations (Hammersley & Atkinson, 2007).

Mertens (2005) described the focus of an ethnographic study to understand practises from both an *emic* (insider) and *etic* (outsider) perspective. An ethnographic study defined as by Robson (2002, p.89) "seeks to capture, interpret and explain how a group, organization or community live, experience and make sense of their lives and their world". It "typically tries to answer questions about specific groups of people" (p. 89), in this case social science students, or asks "about specific aspects of the life of a particular group" (p. 89), in this case social science students taking a compulsory mathematics course either they want to or not.

Seeing ethnography "as a family of methods involving direct and sustained social contact with agents" (Willis & Trondman, 2000, p.5) became purposeful in this particular setting. Through spending the large amount of time with these students in their mathematical classroom, and outside their classrooms during breaks and lunches as I did, offered possibilities to experience what took place, the contexts and discourses that were available in the classrooms and what identities the students decided to narrate in relation to these contexts and discourses. With ethnographic methods and my participation in the particular setting it became possible to explore students' identities when trying to understand why they acted, with or against, their own interests. Willis and Trondman (2000, p. 11) pointed to the issues that there are several 'youth

projects' driven by adults or experts regarding young people's citizenship and societal participation or as in this case, their education. They wrote:

> Rarely are these initiatives conditioned by ethnographic evidence concerning actual ways of life among the youth, yet more rarely by statements from youth groups concerning their *own* aspirations, priorities and agendas

My study is obviously also adult driven. However, the created spaces allowing students for agency, decision-making and taking personal responsibility were acknowledged by the students as something different. I need to recognise the limitations to our wish for democratic and student empowering approaches. We were obviously still imposing an external power and evaluation framework and this clearly restricted the ways we wanted to push matters of agency and student empowerment further. It was a balancing act with negotiations undertaken almost every lesson in relation to the students' wishes, the students and their teacher's responsibilities and the curriculum and assessment boundaries. I also want to stress that *empowerment* in this research is understood as students' possibilities to achieve agency and take personal decisions regarding their learning of mathematics which resonates with Biesta and Tedder's (2006) ecological understanding of agency achievement and/or resistance. If the critical reflections done during mathematics lessons will impact on students' empowerment further on in life or in their future societal citizenship we do not know, and that is neither a concern in this research.

In this research I as a researcher had an ethnographic *overt* role. I disclosed the fact that I was a researcher (Bryman, 2004). I introduced myself as a researcher for the students and their parents. Elin, the teacher, introduced and positioned me as an "assisting teacher", implying I had an active role in the classroom however with no examination obligations or any other responsibilities concerning mathematics teaching. However, if the students asked questions or wanted to discuss mathematical issues I happily took those discussions. My identities as an ethnographer and assisting teacher in the classrooms were reconcilable and also became fruitful for this research. In this way I could smoothly interact with the students and note what was occurring in the classrooms.

Some students chose to approach me more often than others. A number of students discussed on several occasions issues with me not relating to the mathematical topic but of importance for their whole educational situation. In this way I came closer to the students and got a stronger relationship with some of them. I listened to stories told by Arah about having to work late every night in his brother's restaurant. Arah was always hungry in the mornings because he needed to sleep later. I invited him to share my morning coffee break fruits with me on Wednesday mornings, when the morning mathematics lesson was scheduled. Malin told me personal stories about her prior school life that impacted on important decisions she had to take during this first year at upper secondary school. Yet Anna's parents had decided to get divorced and this made her sad, quiet and unfocused in all subjects she told me. And so on. This meant I had other information available of importance for the students' whole educational situation. I did not use this information in the analysis, but the nature of my relationship with these students required clarification as it might affect the qualities of the data I received. The way the students approached me and talked to me might have impacted on the quality of the stories they shared

during interviews and spontaneous conversations. The fact they felt comfortable with me, joked with me and talked quite openly with me might have impacted on the quality and spoke more freely with me during research activities.

However, from some aspects it is also possible to label this study "critical ethnography" (Cohen et al., 2010; Foley & Valenzuela, 2005; Thomas, 1993). Thomas (1993, p. 4) wrote in his book on critical ethnography that "Conventional ethnography describes what is; critical ethnography asks what could be". In my study inspiration came from asking, "what could be". The key premises in critical ethnography, suggested by Carspecken (1996), concluded that socially and historically located power relations mediate research and thinking, where language is central to perception. Cohen et al. (2010, p. 187) separated critical ethnography from other forms of ethnography in the following way:

> The focus and process of research are thus political at heart, concerning issues of power, domination, voice and empowerment. In critical ethnography the cultures, groups and individuals being studied are located in contexts of power and interests. These contexts have to be exposed, this legitimacy interrogated, and the value base of the research itself exposed. Reflexivity is high in critical ethnography.

Even if I have a political awareness, and I do label myself as a socio-cultural-political researcher with "little p", and I do expose contexts – I do not question the contexts and their legitimacy per se in this study. That I rather consider being the next research step, based on the knowledge received in this research.

Critical ethnography can be implemented in a variety of ways. Thomas (1993) stress, as quoted above, that the teachers have possibilities to challenge students' critical reflections. Not with the purpose to impose a "correct" line of thinking, rather to support students when considering different conditions of society and their and others ways of life from their own perspective. This statement resonates well with concerns for a critical mathematics education raised by Skovsmose (1994, 2001, 2005a) and also with the intentions that Elin and I had when imagining and arranging the pedagogical discourse. We wanted to arrange the teaching based on issues informed by the society outside the mathematics classrooms, and open up spaces for the students to be critical and reflective on their personal learning of mathematics but also on mathematics per se. To continue with Thomas (1993, p. 4, my italics) he also suggests that

> … critical ethnography is more than just the study of obviously oppressed or socially marginal groups, because researchers judge that all cultural members experience *unnecessary* repression to some extent.

This is important for me to stress. The students in Ericaskolan live in the best of social worlds concerning education for young people. All schools in Sweden are free of charge thus no school fees are requested. All schools are expected to follow a national curriculum, aiming for equivalent schooling for all pupils and students in Sweden. Books are free for all students as they borrow school owned books. Students at Ericaskolan also got their own personal laptop on lease from the school, as this was introduced this particular year. Therefore, in addressing what Thomas states as "unnecessary repression" I want to summarize, these students were not oppressed in the sense they all came from a certain oppressed group in society – even if some of

them did. Swedish law states, regarding education, that all students in Sweden should have equal opportunity to education. However, I would say that several students were evidently disadvantaged due to their experiencing of little or no mathematical wellbeing in prior mathematics education (Clarkson et al., 2010).

Choosing Methods for the Students' Identity Narratives

With the research objective to collect individual students' identity narratives through their compulsory Mathematics A course, different methods were required to catch different aspects at different points in time. The information was collected through my participation in two social science programs mathematics classrooms during a teaching semester in their Mathematics A course. I also interacted with the students during a large number of breaks. A certain amount of my time was also spent in the teacher's collective office where I followed their daily conversations about what went on in the school.

The data collection was designed in a way with methods that enabled students to reflect on and talk about themselves in relation to all aspects of their mathematics education. The quality of data I required were stories where students talked about themselves in relationship with mathematics education. Hence I decided to collect both written and oral data to be able to analyse language use and student talk about themselves from different sources. I considered the ways in which youth talk about themselves and what sources they usually use for communication.

Consequently, Ethnographic methods as field notes (Bryman, 2004; Cohen et al., 2010; Robson, 2002; Wolcott, 2005) and audio-recorded interviews (following guidelines from Kvale & Brinkman (2009) and Soyini Madison (2005)) were complemented with written materials such as personal letters, a survey, blogs, e-mails and students' logbooks. My research diary and field notes also became valuable data resources. My field notes gave me opportunities to analyse the contexts and available discourses present at particular lessons further. Hence, they also allowed for time framing when later relating the students stories to what went on in school, classrooms and society at particular times.

All names, including the name of the school and all individuals' names except my name, Annica, are pseudonyms all through the writings. The name switch was done in relation to the point in time when the information was collected, and most individuals chose their pseudonym themselves.

In the coming section the different information sources are explained in detail separately.

Collecting Students' Stories

The Survey

With the purpose to map out students' mathematics education backgrounds, their reasons for participating and their learning objectives in this course the students were asked to write personal letters to me as a survey. Before the students came to school and met their teacher, their classmates and myself, they received an invitation letter from me where I welcomed them to the

school, and asking them to answer some questions with the purpose to help me understand what mathematics education meant for them personally. They were asked to answer questions about their prior experiences of mathematics education and how they viewed themselves as mathematics learners and their personal wishes for their far future. Some question related to their parents, siblings and/or friends experiences and opinions about mathematics education. I wanted them to clarify their personal goals in mathematics education and describe their feelings, experiences and expectations on mathematics education and the coming course. Their view of themselves as mathematics learners, with regard to their personal thinking of motivation and usefulness of mathematics became illuminated through this survey. It was important for me to receive their personal letters prior to the course start, as I wanted their personal views before they met the teacher, their classmates and me. What I thus received were their expectations before the semester commenced and they started in their new school.

In this letter I also asked if they wanted to participate in the research. Those three students who clearly said they did not want to participate were not interviewed, cited or included in any other way in this research.

The Interviews

To be able to answer the research questions about what identity narratives the students told during their participation in this particular pedagogical discourse, I found open-ended semi-structured interviews to be the best option as research method for the students to talk about themselves (Kvale & Brinkman, 2009). The student interviews were conducted during the first two weeks of the semester and during the last week before the Christmas break. Eighteen students were interviewed in the first set, fourteen in the second before the Christmas break. All interviews were audio recorded in a quiet group room without any disturbances and each interview took about 45-60 min. The students were interviewed during longer breaks or free periods, or in relation to the beginning or ending of a school day. On one occasion was an interview conducted during a mathematics lesson, otherwise the interviews did not interrupt neither mathematics nor any other subject lessons.

For the first set of interviews, I brought the students' letters to discuss topics they had raised. This was done with the purpose to get a deeper understanding of their writings and to meet the endorsed quality in the analysis (Sfard & Prusak, 2005a). The second set of interviews/conversations focused on their experiences from the autumn semester change. The structure of the interview was, again, in a way that met the endorsed quality in the analysis (Sfard & Prusak, 2005a). Kvale and Brinkman (2009) points to the fact that for studying people's interactions with the environment informal conversations and field notes of observations usually give more valid knowledge than asking subjects about their behaviours during interviews. My experience from the interviews was that I mainly agree with their statement. What I found was that the interviews gave possibilities to talk about experiences that were observed through my participation, the students' own blog comments, from the survey and from the logbooks. Through asking why-questions during the interviews I got the opportunity for clarifications and deeper reflections. The interviews thus allowed for deeper understandings and mutual learning about why the stories had been told at particular times and in specific contexts. This point was

illuminated for example in the stories about Petra and Malin, where the interviews clarified the reasons for Malin's engagement at some point in time was the critical mathematics content whereas Petra had other reasons to engage or not engage at particular times (Andersson, Meaney & Valero, under submission).

The Blog

The third data resource was an Internet blog. Throughout the teaching year we had access to two web based forum spaces on the school intranet, one for each class. Here the students, the mathematics teacher and I had possibilities to discuss and reflect on what occurred in the mathematics classrooms, the work that was done (or not done), collaboration issues etc. The students had opportunities to comment on "today's lesson" at the end of each session. Sometimes Elin or I placed an open reflective question for the students to elaborate on. At other times the students choose their own reflective topic.

The data I received from the blogs allowed for understanding relationships especially between task contexts and situation contexts and the students identity narratives. The blogs were open for the teacher, the students within the boundaries of the teaching group they were part of, and for me to write in. This meant that students could read and comment on each other's comments. There were advantages and disadvantages with having it public for all classmates to read and comment. However, we decided to keep it public within the different education groups. Elin and I believed that if we grounded the teaching in concerns raised in critical mathematics education, and related the teaching to goals in citizenship education, then we should also discuss in the classroom how to write and express personal opinions in a public way without offending anybody or writing with personal attacks. In society we have to be able to express opinions and discuss in public, and we believed this should be possible also in mathematics education.

Matteprojekt	Inlagd av: 1B Pia Gran (E)	2009-09-22 16:10
Idag lyckades vi få ihop det mesta med räntor och priser för vår planerade resa. Det känns som att vi jobbat bra och vi kommer antagligen att hålla tidsplanen. Vi kom på att vi, eftersom vi inte lånade ett alltför stort belopp pengar, kunde dra ner på antalet år att betala av på. Då blev räntan mindre och vi förlorar alltså inte lika mycket pengar. Antagligen måste vi gå igenom räntan vi räknat ut igen, för det blev något fel i våra uträkningar, men förhoppningsvis kan vi börja med planschen redan imorgon! Pia & Kim		
Today we managed to put together the most with interest rates and prices for our planned trip. It feels like we've worked well and we will probably stay on schedule. We realized that we, because we borrowed an excessive amount of money, could cut down on the number of years to pay off. Then the interest rate will be lesser and we will not lose as much money. Probably we have to go through the interest rate we figured out again, because there was something wrong in our calculations, but hopefully we can start with the poster tomorrow! Pia & Kim		

prov	Inlagd av: 1A Stina Stromgren (E)	2009-10-07 14:14

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Picture 1. Two examples of students' blog-entries. The first entry was written in the context of the first project work, a week before it should be completed and handed in. The second entry was formulated a week before their first mathematics test (stress has the same meaning in both Swedish and English language).

The students always had possibilities to comment on the blog (well, at least at those times the school intranet worked) both at school and from outside school between lessons. We sometimes, in a positive manner, encouraged them to write on the blog, but we never pushed them to comment. Nevertheless, an interesting reflection that Elin and I made was the higher number of entries during project work, even during the statistical project where the students reflected on their learning in the logbooks. This was not the situation during periods with textbook work, when the number of entries decreased distinctly. I can only speculate on what these difference in numbers may indicate, but it seems as the more active students, the larger space for taking personal decisions, the higher number of blog entries during mathematics lessons.

Logbooks

During the larger statistical project at the end of the semester the students used personal logbooks. In the logbooks they were expected to reflect on their individual work, their learning, their collaborations and the groups work. These logbooks expanded the students' level of the research and added students written identity narratives to the larger research data body. This information contributed to answering the subordinated research questions that focused students' identities in relation to contexts and how agency was achieved. The logbooks were expected to be updated regularly by the students, and handed in to the teacher at the end of the project in relation to the presentations of their work. The logbooks were used by the teacher to get a fuller picture of the students learning in relation to assessment and grading, but also for her more general reflections on the students' experiences and learning of mathematics in an interdisciplinary project.

The e-mails

After I left the school, I stayed in contact with the teacher and four of the students until the course ended with the national test examination. I continued my relationship through conversations via e-mails as personal letters. The nature of these letters can be viewed in Andersson and Valero (under submission), where Henrik's letters as e-mails are accounted for in almost full length.

The National Test Examination

From the teacher I received information, via her assessment sheets, on students' results on the national test that ended the Mathematics A course. However, I did not get access to the test it self, so I could neither view and analyse the tasks nor individual students results on particular test questions.

The Parents' Meeting

With the purpose of addressing the family level in the research (Martin, 2000), and address the families as a social practise within the educational network (Valero, 2010) I met the parents and gave a presentation of the project at the first organised school parents meeting. A large number of parents are concerned about their children's education and making the right choices regarding education matters (Englund, 2005). They are concerned about examinations, grading and reaching curriculum objectives as these issues impact on their children's future studies or employment opportunities. Subsequently, it was important for the research project that the parents were well informed and received possibilities to discuss the mathematics teaching and the pedagogical discourse we aimed at introducing. At the parents meeting I described the ideas that grounded the pedagogical discourse and strived for transparency when describing the curriculum goals and how we expected to meet them, and how the examinations were planned. Some parents asked some concrete questions about the projects, other just commented, "thankfully, at last somebody tries to do something about mathematics education" (field notes, 09-2009). No parent indicated resistance to let his or her child participate; the case was rather the opposite.

The Parents Web-based Forum

At the parents meeting, the parents got an invitation to use a "Parents forum" on the school intranet. The reason for the "parent's forum" was twofold. First, we were concerned about the parents' reactions to the project and thus needed a space were issues important to the parents could be discussed openly. The second reason was that we wanted to keep the parents informed about the mathematics education and the projects during the semester. The parents were invited to comment in the forum but none did this during the semester. However, Elin and I posted practical information regularly.

Interviews with Parents

The parents were at the parents meeting invited to be interviewed by me further on. The research objective with the parents' interviews was to get the parents point of view on their child's mathematics education and thus expand the students' level of the research further. The parent's stories about their children's mathematics education gave a richer understanding of the student's prior experiences, narrated identities and some of the reasons for their engagement or disengagement in their learning of mathematics. The parents' stories about their children were part of the stories "floating around" about persons that may or may not impact on the person's identity (Sfard & Prusak, 2005b). Four parents, three fathers and one mother, volunteered and were interviewed at the end of the teaching semester. The parents' interviews lasted between 50-90 minutes each and took place at the school in the evenings after work hours. I adjusted my time to the parents' requests and convenience.

Data Constructions of Parents Information

It is important for me to stress that the parent's stories accepted as data related to their own children only. The purpose was to clarify relationships between the parent's stories and their child's identity narratives. I did not analyse parents' stories as such for themes told by them as a

parent group. There were obviously stories, invoked by the idea of stories floating around within the culture. To give a pragmatic example: "It is important that [my child] gets the best education so that (s)he will get the job (s)he really wants and be happy" as all the parents told me, however in slightly different wordings. Other examples of stories floating around about schooling that affect parents' view on a general level might concern children's performance and ability grouping as elaborated by Winbourne (2009). In the case of Sweden, Englund (2005, p. 45-46) elaborated the concept of Swedish equivalent schooling in the following way "The concept of equivalence has now been linked with freedom of choice and education as a civil right (the rights of pupils/parents); in contrast to the tradition of uniformity, which has been more closely associated with the idea of education as a social right". Stories themed on parents and children's choices concerning schooling in general were a common topic during the interviews. They were often related to worries about making "the right" choice when deciding on school issues for their children. Those stories are by me understood as common public stories, told more widely in society and not only in the own home. Those particular societal or cultural stories were left behind as not being reified and endorsed narratives. The parents' stories that were accepted as data in this study connected to the individual child's narrated identities or clarified their stories.

Reflecting back, I would say there were no contradictions between what was told by the students or by their parents. The case was rather the opposite. The parents verified more strongly or with more intense the students narratives. It was also obvious that those parents to children who disliked mathematics were worried about their child's results in mathematics education, and their wellbeing during mathematics lessons.

As I received data from different subjects, I want to stress that I did not use data obtained from one subject when interviewing another subject. That is, I never told a parent what their child had told me, or vice versa. Neither did I discus particular students' stories with other students. I was very careful during the interviews and conversations with students not to reveal other sources. However, I often reflected my own experiences and asked if I had interpreted a situation or communication in a correct way.

Analysing the Empirical Material

When I prepared the data for the analysis stage some decisions on how to treat the data was taken. One aspect concerned the fact that the sources in which the identity narratives were told were very different. There were qualitatively differences in the kind of stories that were told in interviews and those told for example on the blog, in e-mails or in the logbooks. Identity narratives that emerged from the different sources as interviews, conversations, blogs and logbooks were all treated in the same way.

The blog was constructed in relation to the course and framed as a course activity. The blog provided data mainly in relation to task contexts, but also students' actions and behaviours were common topics on the blog. Hence reflections of agency (including resistance) emerged from the blog. The logbooks provided data in relation to the context of learning mathematics, accounting for mathematical learning, task context and situation contexts – as how the group work developed. The students received possibilities to make other kinds of reflections in the interviews. The interviews were clearly a research activity; hence the interviews provided more reflective data.

I treated the data in the same way and with the same value for the research even if it emerged from different sources. No source was seen as more important or relevant than another. It was the stories as such, and in relation to task, situation, school or wider societal context they were told that it was important for this research. I clearly wrote out where the stories originated from and the dates the stories were communicated. Then I related the data to the different contexts during the analysis process. Consequently, when I prepared the data for analysis and built up pictures of the individuals, all data was treated equally independent of data source. There were very few issues of inconsistency in the emerged data. In the cases of contradictions, the issue was rather that students' had commented on the blog in a sarcastic way. For example one student wrote, "Today I really liked maths, I was very focused and calculated loads of exercises in the book". This was clearly neither the teacher's nor my experience during that particular lesson. In this case I kindly asked her after class: "I didn't get the impression that you loved maths today?" And she answered with a laugh "Off course I didn't…" If there had been clear cases of inconsistency I would have reflected this further with the particular student later, and asked for clarification if it had felt comfortable to do it - otherwise not.

This issue could also be addressed as a theoretical issue. If I assumed that multiple identities exist, and students tell different identity narratives in different situations with different audiences, they were probably both true and relational to the contexts in which they were told. Hence the students could have chosen to present different kinds of meanings, hence different identity narratives to classmates on the blog, or to the teacher in their logbooks, than to me in conversations.

Analysing the Quality of Identity Narratives

A specific concept of identity gave an analytical base for understanding students' reasons and actions in this particular setting. Sfard and Prusak´s (2005a, 2005b) operational sense of identities were used to analyse the data. They proposed to "equate identities with stories about persons" (p. 14). An identifying story is endorsed, reified and significant for the identified individual. The signifiers for distinguishing between the storyteller, the recipient and the identified individual suggested by Sfard and Prusak (2005a, p.17) where used consequently through the analysis process:

> $_AA_C$ = an identifying story told by the identified person herself. This story we call A's *first-person* identity (1st P).
>
> $_BA_A$ = an identifying story told to the identified person. This story we call A's *second-person* identity (2nd P).
>
> $_BA_C$ = a story about A told by a third party to a third party. This story we call A's *third-person identity* (3rd P).

Even if Sfard and Prusak talk about 2nd and 3rd person identities, I interpret this framework in the way that the identifying stories need to be reified, endorsed and significant for the identified person in order to be considered as an identity narrative. It first becomes an identity narrative when the individual take it up as part of her identity stories[1].

The Reifying Quality

The stories told or written by the students' about themselves or stories told by others about the identified person were analysed by locating the suggested verbs and adverbs (stressing repetitiveness) for identifying the reified quality. Stories that were told/written with expressions such as "I am worried …", or "I have always…" indicated reification. As Kajsa commented on the blog after a lesson using the verb 'have':

> $_KK_{R,T,S}$:   I have shown very bad behaviour today, from my point of view. I have been quite un-concentrated, tired, and dull and probably annoyed some people in my surrounding. Reason = tired, hungry, a lot of impressions bumping into each other in my brain. I have though completed what I should. (Kajsa, blog, 10-2009)

Her teacher reflected in the following way after the same lesson, also using the verb 'have':

> $_TK_R$:   Kajsa and her group have been very loud. Kajsa gives the impression of being an ambitious person, so I had expected more from her" (Teacher, Interview 2, line 18-19)

In these examples the 3rd person identity told by Kajsa's teacher resonated with the student's identity narrative.

An issue regarding the reified quality needs to be discussed. Words of repetitiveness were common in students talk about their experiences from previous mathematics education. However, this was not the case for talk focusing the present teaching semester. As the

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[1] I thank Troels Lange and Tamsin Meaney for clarifying conversations about identities and identity stories.

pedagogical discourse was new, there was probably no opportunity for the students to identify with words as "usual" or "most often" – as their experience of this mathematics teaching was both different and novel to them.

The Endorsed Quality

To reach the endorsed quality, that is what a person endorsed as true about herself, I first distinguished, in line with Sfard and Prusak's (2005a, p .17), suggestions between the individuals identities authored by different people with names indicating the relation between the story teller, the identified person and the recipient. To further reach the endorsed quality in the analysis, three different options emerged. First, I asked the identified person if I had understood previously told narratives correct or I reposed a question in a slightly different way to see if the answer supported previous answers. At interviews I sometimes asked the identified person to comment on transcripts from the survey, a prior interview or conversation or the blog. A second way was for me to compare and recognise resonating narratives told by different actors as being endorsed. Third, I used my experiences from the classrooms and my field notes to reflect different circumstances with the identified person. For example I asked Malin on one occasion: "I remember you talked a lot and that you were very engaged in the discussions in the afternoon?" which gave her an opportunity to elaborate and explain further her interpretation of what occurred (see Andersson, Meaney & Valero, under submission).

The Significant Quality

As the narratives were told or written by the students, they were in this research understood as being significant for them. Why would they otherwise communicate them? I regarded it problematic to speculate about storytellers' feelings with regard to membership in a certain community in this study. The aim of this research was not to generalise the research outcome for certain cultural groups, as was the case in Sfard and Prusak's (2005a) research. I could ask what kind of significance would be appropriate in this research. In the case of Sfard and Prusak's research it made sense to relate the significant quality to the students' cultural communities. However, it was not apparent for me what community would make sense to relate to in this research. There were several possibilities, for example the school community, their learning groups, their family communities and so on. In the case of this study, I regarded the students' stories were rather to be significant for the students' relationships with mathematics education as such. Consequently the decision was taken that the stories were significant as they were told in relation to the contexts they addressed at that particular time. Subsequently Sfard and Prusak's operational framework became adjusted to suit this particular research in the way that if the students talked about themselves in relation to mathematics education in the widest sense, then that story had significance without further interpretation.

Analysing the Storylines

The second part of the analysis connected the identity narratives to the different contexts and levels of the research. This was done through a four-step process:

First, all the identity narratives, that is what I took as reified stories with all the suggested verbs and adverbs (Sfard & Prusak, 2005a), were arranged in chronological order on a timeline that became a 'storyline', one 'storyline' for each student. Henrik (Andersson & Valero, under submission) illustrates these sentences when he says "I became a fan" (p.191), "far from the position I am in today as an upper secondary student" (p.192) and "I have been concentrated so I am tired" (p. 193). The narratives were collected and arranged on 5-8 m long, stripes of paper, on which I pasted all the information I had on each single student. I used different coloured notes and pens for information emerging from different sources. The different colours became especially purposeful when analysing what types of data emerged from the different sources. It also supported when distinguishing the different storytellers in relation to the individual student.

Second, events and incidents at particular times at different school levels were added on at the bottom of each 'storyline'. Also comments from the teacher, my field notes, the teacher's assessments comments and students' examination results were added on the timeline. In four cases, also parents' comments were added. The purpose with this exercise was to connect the students' identities as narratives with occasions in other aspects of the contexts for these narratives.

Third, a second analysis of the texts was carried out. As suggested by Sfard and Prusak (2008) attention was paid to the characteristics of the used language. Expressions that indicated action, for example "today we went to…" or "I decided to…" or "today I talked a lot and probably disturbed others" where added on the timeline. These sentences supported me when elaborating further on students' decisions for engagement, disengagement and qualities of agency achievement. These sentences are illustrated in the paper about Sandra who told that "We distributed the time…" (Andersson, in press, p. 162), by Malin who said, "I will try to get the others to be better at that" (Andersson, Meaney & Valero, under submission, p.174) and Henrik with his comments "Today Pelle and I checked on the information about the baker education and checked the costs. We have not chosen what bank to use yet. We will do that tomorrow" (Andersson & Valero, under submission, p. 192)

Fourth, my field notes and the careful detailed documentation of what occurred within the classrooms supported when deciding on the different contexts referred to in the identity narratives and action comments.

As a result, the timelines consisted of narrated identities, decisions and actions in chronological order where patterns of the stories emerged. These patterns showed how different discourses, events and incidents in and outside the school impacted on students' identities through changes in their identity narratives. The ways that students narrated themselves changed in relation to contexts, constraints, relationships, and time.

The "Absent" Identity Narratives

When analysing data it is important to question what is absent, what is not the case, what stories are not told (Skovsmose & Borba, 2004). As the students changed their ways of narrating identities in relation to contexts, also their "absent" identities, that are identity narratives told in

other contexts, became important to register during the analysis stage. For example, it became apparent that some students' objectifying identity narratives, such as "I am a math-hater" was told in some contexts, but not in others. My data collecting techniques allowed me to do assumptions that within the domain of positioning in relation to mathematics education, also "absent" identity narratives were of importance. Consequently, at some points I registered not told identity narratives as what they were: absent identity narratives in relation to contexts at specific points in time. See Andersson, Meaney and Valero (under submission) where Petra's identity narrative of being a "math-hater" is not told in some contexts. Also some of Henrik's identity narratives in Andersson and Valero (under submission) illustrate examples of "absent" identities.

The Overall Empirical Material

From the total of 36 students, nine students provided me with very rich data and their time-story lines were analysed further. The other students' talks about themselves were used as reference timelines. As will become obvious in the coming writings, some students' experiences will be described and quoted more often, or in more detail than others. This is not because they were more important, it is rather they were more verbal and captured my attention in the streams of conversations we had in the classrooms. They had the words, the language, to express what they wanted to say. This is not unusual in research. The way these students expressed their experiences and identity narratives, in their individual ways of talking and writing, illuminated issues that could be brought forward and asked to students who were more silent or did not (yet) have the words for expressing what they wanted. Other students were more silent, and yet some of them shared their written communication with me, but did not want me to interview them.

Problematizing Language Matters

Language issues have been constantly present and reflected on through my years as a PhD student. I am Swedish, with Swedish as my mother tongue and English as my second language. I was accepted as a doctoral student in an international research group at Aalborg University in Denmark with English as shared language. Regarding my supervisors, Paola Valero is Colombian with Spanish as her first language, and Tine Wedege's first language is Danish. These circumstances together implicated that English would/should be my thesis writing language. This decision has been integral to the research process and in my writing progress. However, when working with two different languages there might be ethical implications and validity concerns (Robinson-Pant, 2005). There are issues around validity and translation: How can it be ensured that the way the thesis was written up in English accurately reflected the experiences from fieldwork and what the students, teachers and other participants articulated in Swedish? Am I representing in a proof full way what my informants communicated?

Writing in English as a second language could be seen as an extra burden with additional workload e.g. translating the Swedish transcriptions used in the writings into English. Looking up words and finding a writing rhythm in a second language also took additional time. On the

other hand there were clear benefits to the research to note when writing in a second language. In this research the beneficial considerations are due to the researchers becoming language awareness. When translating and writing up the results an awareness of *what actually was said* by the participants in the Swedish language grow forward. An awareness of contexts, and subtexts, so easily taken for granted, became obvious during the translation process. It implicated a more complex and reflective approach to the developing research data. An awareness of the importance of describing the actual contexts where the conversations took place and explain the cultural- and historical background for the research setting at distinct times during the processes made the analysis more complex but also richer. Discourses became obvious in a way that was not anticipated during the translation processes as well as during times when I discussed or presented my research to an English audience. It implied a more comprehensive analysis of the cultural and linguistic contexts, and a recognition of the dynamic relationships between languages actually supported the data analysis.

Deciding During Translation Process

The purpose of this subsection is to describe some of the challenges that emerged during the translation processes, and how these were solved. I see this description as important in the research of both ethical and validity reasons, thus making (some of) the experienced challenges during the translation process explicit and thus open for scrutiny and critique.

When I first started the translating process I wanted the recipients' voices to come through in the research writings in the way they were uttered by the recipient. So, initially I translated all expressions word-by-word. I thought this was important, to give the recipient "a voice" and not to lose "the Swedish meaning" of the sentences. However, English speaking readers made a critique – the translated utterances did not always make sense in the English language. When the original translation was not comprehensible in English, I focused the whole meaning of the utterances instead of translating word-for-word. By that I followed concerns elaborated in text linguistics (Martin & Rose, 2007). A second difficulty I experienced was the richness of metaphors, in Swedish language. How was it possible to translate metaphors, with their historical and/or cultural subtexts, into another language without losing information? In these cases I had three options (Robinson-Pant, 2005). First, if possible, I translated into an English equivalent metaphor. For example, "Moment 22", a metaphor used by one of the students with the idiomatic use; a no-win situation, obviously had an English equivalent in "Catch 22"[2]. If this was not possible I either clarified the cultural/historical conceptual meaning of the metaphor. For example, the metaphor "Curlingföräldrar", "Curling parents" had no equivalent in the English language. Not even the sport named curling made sense to English readers. In this case, I described the idea of the sport curling, and why and in what way it objectified (some) contemporary Swedish parents (see Andersson, in press, a). The remaining cases, neither direct translatable or possible to describe through a thorough explanation I followed the discourse context and translated the intended meaning into English (Martin & Rose, 2007). The optimal solution would obviously have been to provide the readers with the original narratives in

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[2] Originating from the book "Catch 22" by Joseph Heller, when describing the paradox in which the attempt to escape makes escape impossible.

Swedish side by side with the English translations, as was the case in Lange (2009). However this has not been a possibility for me due to limited text space in journal articles.

The purpose of the discussion above has been twofold. First, I intended to open a space for critique of the research in relation to issues as ethics and validity related to writing up a thesis in English as a second language. Second, I wanted to illuminate difficulties, but also benefits and possibilities for writing up and discussing the research in a second language. Reflecting back on the language issues, I believe that working in two languages in my case was to be seen as an advantage in most respects. Constantly comparing and refining concepts between the different languages and analysing the data in terms meanings within that particular discourse have enriched the data analysis. Translating Swedish language, so rich in metaphors, into English made me question what was actually said in ways that had probably not been done otherwise.

Ethical Issues in the Swedish Context

Even if my researcher attitude made me consider ethical issues and reflectively strive for transparency in my writings, I cared for addressing the ethical considerations required when conducting research within the research field of social sciences in Sweden. The Swedish ethical guidelines (Vetenskapsrådet, 1990) described four main areas of importance for ethical acceptable research: the demand for information, the demand for approval, the demand for confidentiality and the demand for appliance.

The demand for information stated that the researcher should inform the participants about the objectives of the study. This was done in the survey, but also orally and in writings the first time the students met me in class. Their parents got this information at a planned parents meeting in September. The participating students, teachers and school principal signed a hand-out statement telling that they had received and understood the information and that they gave their approval to participate in the study.

The demand for approval stated that the students themselves were to decide on their participation. The students got this information several times in class, and the information was written in the hand-outs to be signed by them. Their parents received this information at the parents' meeting and for the interviewed parents in the hand-outs that was signed by all participants. The students were free to leave the research study if and when they wanted without any negative consequences for them. This information was repeated on occasions when students asked or when the teacher discussed assessment and grading issues.

The demand for confidentiality stated that all gathered information concerning the participants should be handled in a confident way and be kept in a way that unauthorized persons not could get hold of the information. The issue of confidentiality is closely related to the issues off publicity and secrecy, both described by Swedish law. "All gathered information about identified persons should be recorded, saved and reported in such a way that persons not can be identified by outsiders." (Vetenskapsrådet, 1990:12, my translation). To meet this demand as best as possible, all names of persons and geographical names were changed to pseudonyms, from the data collecting process and onwards. Special care was taken when reporting the research findings

to reduce the risk to identify the participating students, teachers, parents and school organisation staff through my writings. The only person whose name is not anonymous in the research writings is mine.

The demand for application stated that the gathered information only was to be used for research and that is the case with the collected data for this study. The data will not be used for commercial or other non – scientific purposes. This demand also stated that it is good custom to show the participants what will be published and give participants opportunities to comment before publishing. The teacher was asked to comment on all writings to confirm the state of affairs from her point of view.

I have strived for meeting all the requirements stated in Vetenskapsrådet's (1990) guidelines in a correct and purposeful way during all stages in the research process and my hope is that this has been clear through all my writings.

"A Fish called Wonder"

by Genevieve Ryan

I fish through my stream of consciousness
I hook onto a thought
It isn't light.
I wind it up to get a closer look
It struggles and pulses,
It flaps itself around the deck of my mind
It tries to swim away
So I hold it down
I see the fear of capture in its eye,
as though it wishes not to be interpreted.
At a closer look
I see it is more complicated than I first thought
It shines
It's coloured
It has lines to read
It has feelings
a life of its own.
It begins to tear at the mouth
and the fear in its eye accelerates to terror
Then – it ceases flailing
It holds quite still…
It is hardly there now…
And I begin to wonder – I begin to
It's going…
I'm loosing it…
I can't think where it could have –
Although I am hungry
I pull the hook out as quickly as possible
and proceed to toss the thought back into the stream
It swims away,
Faster
as though the water is a source of strength

I watch it think away
And smile
Because it did not sink, or die, or float –
It was more alive now that it flowed with the stream
And I was not hungry
I was filled with a stream of consciousness and many fish of wonder.

Genevieve wrote this poem the day before her Year 12 exams. From the book "…regards, some girl with words. Genevieve's Journey" by Elisabeth Ryan.

The Articles in the Thesis

Articles About the Disturbance

The writings in these three articles concern the disturbance of the teaching and the introduction of the different pedagogical discourse. In these writings I have aimed to answer and discuss the first two sub-research questions:

1. Which are the possibilities and hindrances when changing mathematics teaching approaches in a Swedish upper secondary school?

2. How can a critical inspired pedagogical discourse be organized in an upper secondary school, on the social science study program's compulsory Mathematics A course in Sweden?

 The articles are presented in the following order:

1. Andersson, A., & Valero, P. (In press). Examining a critical pedagogical discourse for agency and social empowerment. In P. Ernest and B. Sriraman (Eds), *Critical Mathematics Education: Theory and Praxis*. USA: Information Age Publishing.

2. Andersson, A. (in press, a). A "Curling Teacher" in Mathematics Education: Teacher Identities and Pedagogy Development. *Mathematics Education Research Journal.*

3. Andersson, A., & Ravn, O. (in press). A critical perspective on contextualisations in mathematics education. In O. Skovsmose & B. Greer (Eds), *Critique and politics of mathematics education*. Rotterdam: Sense Publishers.

Articles About Identity Narratives and Agency

The writings in the following five articles aim to answer and discuss, from different perspectives, the sub-research questions that focused individual students' identity narratives:

1. How are the identity narratives, communicated by individual students, related to the different contexts that they were experiencing at particular points in time?

2. What is the relationship between agency and students' engagement in learning of mathematics at particular points in time?

These articles are presented in the following order:

4. Andersson, A., & Valero, P. (2009). Mathematics education giving meaning to social science students. Accepted for proceedings *at II Congreso Internacional de Investigación, Educación y Formación Docente.* Colombia: Medellin.

5. Andersson, A., & Norén, E. (in press). Agency in mathematics education. Accepted for presentation and proceedings at Working group 10; Cerme 7, 9th-13th February 2011, in Rzeszow, Poland

6. Andersson, A. (in press, b). Interplays between context and students' achievement of agency. Accepted for presentation and proceedings at Working group 10; Cerme 7, 9th-13th February 2011, in Rzeszow, Poland.

7. Andersson, A., Meaney, T. & Valero, P. (under submission)."I am [not always] a math-hater". Students' [dis]engagement in mathematics education. Educational Studies in Mathematics.

8. Andersson, A. & Valero, P. (under submission). Identity narratives of [dis]engagement in mathematics education contexts. P. Valero & O. Ravn (Eds.), The networks of mathematics and science education practices. Studies of philosophy, society, culture and politics. Intended publication with Springer at the Mathematics Education Library.

Mathematics Education Giving Meaning to Social Science Students. A Case From Sweden

Annica Andersson and Paola Valero

Abstract

Compulsory mathematics for social science students is problematic. We discuss the case of a group of students in Sweden who met a mathematics course inspired on the ideas of critical mathematics education and ethnomathematics. The evidence collected about students' experiences on this course indicate that opening a space for agency and linking mathematics to their foregrounds can be the basis for a more meaningful mathematical experience. Such as experience has the potential of contributing to the process of students' subjectification.

Keywords: Critical mathematics education, ethnomathematics, mathematics for social science students, meaning, agency, foreground, subjectification.

Introduction

That students have troubles with mathematics is not any news. Some countries and particular educational institutions believe on all the good reasons to teach all students mathematics, including those who, beyond basic compulsory education, have chosen study paths that do not in a direct way involve mathematics. That is the case of, for example, students who have chosen a humanistic or social science path in high school and who will probably join further studies where mathematics is not required. The political or administrative decision of offering advanced high school mathematics for all is far from being problematic since it forces mathematics teachers to instruct students who are highly unmotivated to engage in the learning of mathematics. From the part of many students, who either define themselves as "math haters", or just see it meaningless to spend time in studying a subject that they do not like, it is difficult to understand why they have to invest energy in a subject that they will never use for anything in their future lives. For both teachers and students the decision is problematic.

There are few studies and experiences in the international research literature addressing this problem. Still, there are many youngsters in the world who face a meaningless engagement with mathematics, dictated from higher national policy levels. In this paper we examine this situation for the case of Sweden, a Nordic country that has explicitly made the decision for compulsory mathematics in all the different paths of high school[1]. In particular we inquire why mathematics

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[1] Compared to the organization of high school in Colombia where all students have all the same compulsory subjects in upper secondary school, in many European countries there is a policy of study path which students can make. Normally students in a scientific path will have to deal with many mathematics courses preparing

education seems to be lacking meaning for the many students engaged in the social science line in Sweden. With inspiration in Critical Mathematics Education and the Ethnomathematics Program, we propose possible understandings of the issue of the meaningfulness of mathematics education in relation to the social science students in Sweden. We then analyze an experiment that Annica Andersson introduced in a mathematics class of social science students in Sweden. We conclude with a discussion of a central idea for this case —but arguably for many other cases of other students who struggle to make sense of their mathematical experience—: a mathematics education that allows students to construct their own project of subjectification has a chance of being meaningful in deep ways.

Mathematics For Social Science Students: Problems And Dilemmas

In 2008 in Sweden there were a total of 93.000 students taking the 3-year social science program in upper secondary schools. Of them 32.000 students were in their first year studying the compulsory Mathematics A-course. There are two closely connected study programmes: the social science media programme with 6.600 in their first year and the social science arts programme with 8.300 in their first year. This means that each year there are about 47.000 students, only in Sweden, taking the compulsory Mathematics A course on the social science programs at high school (Skolverket, 2008).

The Mathematics A course is a course covering elementary mathematical content such as arithmetic, geometry, algebra, statistics and basic-level functions. The course is on the borderline to more advanced mathematics studies. The problem with this course is that the mathematics content is not connected to other subjects on the students' chosen study path in social sciences. A large part of the course content is recognised by students as topics already studied in primary school; however, the new mathematical content introduced is a lot more advanced than what they have met before. Therefore, those who have achieved highly in mathematics earlier see it as being very boring; while those who have failed see it as an experience that confronts them with problems, contradictions and dilemmas. In any case, students struggle though the course. The question that emerges is: *How can mathematics education get to be meaningful for students on social science programs in upper secondary schools?*

The Notion of Meaningful Mathematics Education

The meaning that students ascribe to mathematics education as a whole experience is to be seen in relation to students' intentions to engage in the activity of learning, where their agency is fundamental. The intentionality for learning is also related to the students' foregrounds (Alrø et al., 2009). Thus, the notion of meaning in mathematics education is not only connected to the

them for scientific or engineering studies in higher education. Students choosing a humanity or social science line are not required to study mathematics. In Sweden, however, the choice has been made that all students have to complete different mathematics courses of different levels of difficulty and complexity.

mathematical conceptual meaning, as supposed by most of the literature in mathematics education (Skovsmose, 2005). Alrø et al. (2009) define a student' s foreground as referring "to a person's interpretation of his or her learning possibilities and 'life' opportunities, in relation to what the socio-political context seems to make acceptable for and available to the person" (p. 17).

Students in the social science program are a diverse group of students with different cultural foregrounds and backgrounds, different interests and with different experiences of prior mathematics education. However, they have in common their choice of study program and their interest in social science, and a possible foreground for them is to study or work in areas connected to the social sciences. The relation between the notions of agency, foreground and creation of meaning is explained by Alrø et al. (2009) in this way:

> A condition for a person making the decision of engaging in the act of learning is that the activity makes sense, that is, that the person finds and constructs a meaning. […] The notion of foreground highlights the fact that meaning is not only a function of what the student has already cognized, but also and especially of the student's dreams, illusions, aspirations and perceived realistic possibilities for his or her future life. (p. 18)

From the point of view of the individual, the sense that is given to the engagement in the learning of mathematics is connected to the students' possibilities of acting and to their foregrounds. For social science students, this idea can be a possibility for exploring how to make mathematics education more meaningful.

Furthermore, we have found inspiration in the philosophical framework proposed by Biesta (2005, 2009) where he argues that there are three main functions for education in our societies nowadays. The first reason is for students to get *qualification*; that is knowledge, skills, understandings and forms judgements that allow students to act and do. The second reason is the *socialisation* function where "students become members of and part of particular social, cultural and political orders" (2009). The socialisation reason plays an important role in the continuation of both desired and undesired reproduction of culture and traditions. The third reason relates to the students gaining *subjectification*. As Biesta points out, whether all education actually contributes to subjectifying students is debatable. But "any education worthy of its name should always contribute to processes of subjectification that allow those educated to become more autonomous and independent in their thinking and acting" (p. 8). This framework allows us thinking in the function of mathematics education, in general, and in its function to the particular group of social science students in particular. It also opens possibilities to highlight goals in education without constructing the social science students as a group with "specific problems" or "special needs" in the classroom. Instead Biesta's framework allows us to think what mathematics education has to offer these students as "mainstream" students who have to take compulsory mathematics courses, independently of whether they want it or not.

In this paper we start by considering critical mathematics education and ethnomathematics as fruitful ways to develop mathematics education for social science students and to make it meaningful both in relation to their possible foregrounds and to their chosen study program. We describe some of the theoretical foundations of these two trends in mathematics education. We then proceed to relate these ideas to active citizenship education and the aims of the Swedish mathematics curriculum. We then introduce the setting of a teaching sequence and describe its

design for creating meaning for social students. We finalize with an analysis of the sequence in terms of how it addresses possibilities for the students to engage in processes of subjectification.

Why Critical Mathematics Education?

Critical mathematics education, philosophically described by Skovsmose (1994), is a mathematics education raising issues as social justice, democracy, equity and political issues in mathematics classrooms. Different people have developed educational practices with these concerns (e.g., Frankenstein, 1999; Gutstein, 2006). They use mathematical projects, tasks and assignments to teach students to "read the world with mathematics" (Gutstein, 2008). Critical mathematics education is context-bound (Skovsmose, 2005). To give some examples Gutstein (2006) develops practice in inner-city Hispanic communities in Chicago and Knijnik (1999) in the camps of the Brazilian Landless Peoples Movement. For Skovsmose (in press) it is important that mathematics education addresses both the critical position of mathematics education and that of mathematics. Skovsmose states that problem-based and project-organised forms of mathematics education are ways of realising critical mathematics education: "the problems should emerge from real-life situations, while the project organisation should ensure that students become owners of their learning processes" (p. 1).

In our research work we are starting to see that critical mathematics education, if giving opportunity to students of experiencing meaning and agency, might be strongly connected to their process of subjectification (e.g., García et al., 2009). But if the mathematical activities just are a new set of tasks created by the teacher with real life situations not recognized by the students, we could risk loosing the opportunity to pursue the subjectification goal.

Why the Ethnomathematical Program?

Ethnomathematics can be seen as a concept, a research field and/or an educational discourse (Andersson, 2007). The concept was defined by D'Ambrosio (1985) as the mathematics you find in different identified culture groups. In 1994 D'Ambrosio developed the concept of ethnomathematics further by dividing it into *ethno-mathema-tics*. *Mathema* refers to understanding and coping with reality, *tics* refers to techniques and art and together with the prefix *ethno* it comes to mean the culturally embedded techniques of understanding. In this way the definition includes more than first understood as ethnomathematics (D'Ambrosio, 2006). The ethnomathematical research area defined by Gerdes (1996) as "the cultural anthropology of mathematics and mathematical education" describes the research field as a field "in its own right, a field that reflects an acceptance and a consciousness of the existence of many forms of mathematics, each particular in its own way to a certain (sub) culture" (p.915). D'Ambrosio's arguments for mathematics in identified groups and Gerdes ethnomathematical implications for research gives possibilities in research to address social science students as an group who could have its own mathematical practices and with its own special objectives in mathematics education.

The context in an ethnomathematical educational discourse often becomes socio political and by that stimulate students to reflect on societal issues (Gerdes, 1996). Ethnomathematical

educational discourses have also been described as ways to bring discussions about global fairness and justice issues into the mathematical classroom (Archer, 1998).

Both critical mathematics education and ethnomathematics discourses are ways of seeing mathematics education with a concern to address environmental, equity, global and local social issues within mathematics teaching. Since social science students might have possibilities to connect these issues to other social science subjects in school and might have foregrounds with interest relating to these issues, then critical mathematics education and ethnomathematics might be approaches that suit these students particularly well. But, we want to stress the fact that we believe *every student* could benefit from critical mathematics education or ethnomathematics education, especially as it relates to the intentions of active citizenship education, now addressed and stated important in several (Western) curricula.

Citizenship education and the Swedish mathematics curriculum

The Swedish upper secondary school is committed to offering an education that, with all its subjects, contributes strongly to the construction of citizenship. One of the aims in mathematics teaching is stated in the Swedish curriculum as "The subject aims at pupils being able to *analyse, critically assess and solve problems* in order to be able to *independently* determine their views on *issues important both for themselves and society, covering areas such as ethics and the environment."* (Utbildningsdepartementet, 2000, my italics). One main goal in the compulsory Mathematics A course states that "Pupils should be able to *formulate, analyse and solve mathematical problems of importance for everyday life* and their chosen study orientation" (Utbildningsdepartementet, 2000, my italics). Our interpretation of the role for mathematics teaching in upper secondary schools is that it has to give students mathematical knowledge and competence for taking well-grounded decisions in everyday life, to interpret the flow of information and, thereby, follow, understand and participate in political discussions in society. Since the goals in the Swedish mathematics education curriculum are stated with words such as "democracy", "critically analyse", "society", "ethics" and "the environment" we see obvious possibilities for making mathematics education an education for citizenship and democracy. We argue that mathematics education could have a strong potential for education for an active citizenship. Ross (2008) states that three major elements can be distinguished in any effective citizen education programme: values and dispositions, skills and competences, and knowledge and understanding (p. 495). Mathematics ought to be an obvious subject for students gaining knowledge, skills and competencies to become active citizens in society. Vithal (2004, p. 227) also points out that "hard evidence to support or counter theoretical propositions and associated practices seeking to realise notions of empowerment, emancipation, democracy, social justice, equity and so on through mathematics education is still rather thin" and thus gives the research presented in this paper both significance and relevance. We conclude that bringing citizenship education and mathematics education goals together ought to be an important continuation in the development of both mathematics education and active citizen education.

In the literature on critical mathematics education and ethnomathematics the importance of answering the questions "why" and "what is the meaning" of mathematics teaching and learning in a particular context are highlighted. A teacher engaging in critical mathematics education and/or ethnomathematics need pedagogic content knowledge as well as abilities to deal with the

development of students' reasoning skills. The focus of teaching concerns the mathematical contents, skills and competences, as well as the ethical and political dimensions in the tasks, in the teaching situation and in the students' learning activity. The activities have to be owned by the students and be authentic for them. In the following pages we present the characteristics of a teaching sequence designed and implemented by Annica Andersson. We together analyse this experience in relation to the issue of meaning and subjectification.

The Classroom Experience

In an upper secondary school in southern Sweden 30 social science students participated in a compulsory Mathematics A course during two school semesters. The curriculum for the Mathematics A course was followed in detail but how the different parts of the curriculum and different topics were addressed differed from "traditional" mathematics teaching. The parts of the that changed were designed with inspiration from Critical Mathematics Education and Ethnomathematics. They will be described in detail in the next section. The topics chosen related to issues raised in current debates in Sweden. The remaining topics in the course curriculum were taught in a more traditional way with introductions to the mathematical topic by the teacher followed by counting exercises on work sheets or in the mathematics textbook. The Swedish ethical guidelines (Vetenskapsrådet, 1990) were all followed and addressed during the research process. As the teacher, Annica, developed tasks for the social science students that connected to the Swedish context. Annica wanted to establish in the classroom a type of interaction where students experienced a high level of agency and of negotiation. We have chosen to describe the following five tasks in detail as they represent a width of what happened during the course.

Example 1

Drawing on Mukhopadhyay & Greer (2001), an activity intending to address the learning of proportional calculation was designed. Barbie dolls were an important artifact in the activity. The students worked in groups of three, using the height of one student in the group as reference. They answered questions like "If Barbie had been the same size as you, how big would her feet be? Her eyes? Her breasts? Fingers?". Annica pushed this exercise a bit further than Mukhopadhyay & Greer (2001) do. Firstly, as there were boys in the classroom they worked on the Ken doll. This raised typical questions among the students as what are gender specifies, why do they exaggerate e.g. Barbie's eyes and Kens shoulders? Why are Barbie's feet so small (like baby feet) while Ken's are big if they were in our size? Why do they both have (very) small waists? Secondly, pictures on front pages of Runners World magazines were analysed. Runners World is an international magazine with a reputation of promoting good health information with good and well-written articles on food, training and health issues. The six volumes we looked at were from 2007. They had running women on their front pages except one with a male runner. The analysis was pushed further discussing what sells, why do women look as they do on the front pages, and whether they looking like "real" women. And what about the man? Measures were maid and discussions on how we can use mathematics to see through e.g. adverts for (male and female) magazines, clothes and make-up were raised. Some students turned up with

examples from other magazines the week after and showed how eyes were bigger; finger nails longer in adverts for make-up. And this wasn't possible to see at a simple glance: the students had to measure to conclude whether the pictures had been exaggerated or not.

Example 2

Inspired in Frankenstein (2008), Annica introduced the students to mathematical argumentation. A pile of daily newspapers were brought into the mathematics classroom and based on different news articles we discussed how to make sense of the big numbers mentioned in different articles. Questions such as why do they show this number in percentage, but that as a fraction? when is it smart to use whole numbers? were raised. Then we decided to use an article on big numbers, concerning the yearly salary of the director of a Swedish insurance company at that time. How much money was it really about? What could that money be used for? The issue was not only to compare, but also to break down the numbers in a way that made sense to the students and hence made the big number comprehensible. The students were free to choose topics and came up with ideas such as the numer of schools for children in Africa that could be built; the number of people who could go on holiday to Thailand; the number of school books that could be bought in Swedish schools; the possible human aid issues like e.g. providing AIDS medicine in South Africa etc. The mathematical topics touched during these sessions were percentage, fractions, decimal numbers, large numbers and basic counting skills.

Example 3

The most popular task (according to the students' evaluation) was a statistical task, which covered mathematical goals to reach in statistics both in the Mathematics A course and the Mathematics B course. This group work was conducted at the end of the Mathematics A course and in collaboration with the teachers in religion education and computer education. The students had, as an assessment task, to do a survey and then write an essay on the chosen topic where they presented a data analysis in appropriate diagrams and tables. They reached statistical goals as e.g. counting means and deviations. The content of the survey was based on questions supervised by the religion education teacher. As counting and data diagrams were done in the data program Excel and presented in Power Point the students also had the opportunity to reach goals in the Computer A course. In the Mathematics course the boundaries were pushed a bit and the students were asked to take a stance in relation to their survey and argue for it in their diagrams and text. By that they were "aloud" to manipulate diagrams (but not fake them) —just as newspaper articles and advertisements do. With this exercise they learnt to see through, or at least become aware of, that diagrams and tables seen in the press are usually constructed with a hidden agenda behind.

Example 4

An art class inspired a geometry task. The students got a frame and papers of different colours to choose. The task was to make a pyramidal shaped lantern with a specific volume. The mathematical skills needed were geometric formulas, Pythagoras theorem, algebra calculations and basic arithmetic knowledge. The lanterns turned up in different colours and sizes as the

students used different pyramid bases in their calculations. They trained skills as cooperation, collaborative decision making and argumentation within the groups. This task was not planned as a critical mathematics task. However, the students' evaluation of the task motivate it being described here.

Example 5

An ongoing discussion over the whole course period was inspired by the ethnomathematics movement. During some lessons we discussed e.g. different Indigenous peoples counting systems and that mathematics different from our "Western" mathematics actually do exist in all communities. Relating discussions raised issues of the dominance of the "Western" mathematics. Possibilities to discuss issues related to topics e.g. in the social science classes were obvious.

Mathematics Education, Meaning And Subjectification

The evaluation of what had *given meaning to the students* in the compulsory Mathematics A course took place six months after the students completed the course and was answered by 20 students, using a mixed method research design. First the students evaluated the 15 different topics and education methods from the course in chronological order using a 4-point Lickert scale (Bryman, 2004). Second they were given the opportunity to make comments on each topic they evaluated and these comments we found much more informative than their grading on the scale. The students' own comments to the scale gave us an understanding of why they evaluated the way they did in some cases. The third part of the evaluation was an analysis of students' test results and grades during the course.

Evaluating what had *given meaning to the students* and *their experienced personal meaningfulness* two of the topics on critical mathematics education scored highest: the statistics task and the group work on percentage-counting. But also the geometry task making lanterns, scored high. We see that these three activities have in common the possibility for students to exercise a degree of personal agency, defined here as the opportunity to act deliberately according to one's own will (Lange, 2009). Contrary to many other mathematical tasks that do not allow agency, these activities allowed students making decisions on the processes of mathematical enquiry This was cofirmed by Peter, one of the students in the class: "one gets more engaged when one works with something of your own will".

Many students expressed their experiences of agency and meaning. Karin, commented on an awareness of her personal learning strategy: "I learn best when engaging in group work and less when counting in the book. For me it was meaningful because I learnt a lot. One could talk to each other in the group and help each other. That was fun and one understands better when it is fun." Vanessa also reflected on a similar issue: "This was very meaningful. I learnt statistics in a fun but also in a very worthwhile and instructive way and I know I am very clever reading diagrams in metro☺." Knowing to be critical is described by Kim as: "It was fun to learn because I didn't know one could cheat in these ways". Jacob's comment resonates with Kim's: "it was very good for me how to e.g. see through 'layout' art how one gets cheated by a diagram". So did Maria's: "Helpt (me) to become critical to diagrams one sees". Sandra, describing herself as a

"math-hater" said that "this (the statistical task) is the only time in my life I have found maths useful for me" but, sadly, she also said "and I will never find that again". Anna concluded with a more general comment: "I think maths becomes more meaningful if one puts it in a real/actual perspective".

We interpret the students' answer as an expression of having experiencied agency in the course. Many students express their opinion in a very personal way, while few produce more general, objectified sentences about their experience. Independently of the form, we interpret these assertions as an experience that is real/actual for the students themselves. Being able to become more autonomous in thinking and acting is an important part of the subjectification process.

Analysing subjectification and qualification together we conclude all students completed and passed the final national test in this course. Examining the individual students' results and grades a tendency for other students performing well in mathematics than the usual ones in ordinary mathematics test situations could be noticed. It seems as students not usually performing well in mathematics education achieved better results on the topics educated differently when examining the national tests. If this tendency can be verified in coming research it is a very important argument for a different mathematics teaching.

As expected, students who were going to take further mathematics courses pointed to the importance of counting in the mathematics textbook to be well prepared for their future studies. Tina's comment "Counting in the book is good for you" and Robin's "useful for further studies in mathematics" illustrates these opinions. These comments we see more related to these students' personal foregrounds, and as seeing future mathematics education in the discourse they have known mathematics education through primary school.

One of the purposes of our research is to elaborate the above-described findings further and suggest implications for mathematical education recognising the students as cultural and social participants. Or, as Pring (2000:18) puts it:

> Central to educational research […] is the attempt to make sense of the activities, policies and institutions which, through the organization of learning, help to transform the capacities of people to live a fuller and more distinctively human life. Such research needs to attend to what is distinctive of *being a person* – and of being one in a more developed sense.

We conclude by agreeing with Frankenstein (2008, p. 11): "integrating math with social studies is an effective way to bring math alive to students".

References

Alrø, H., Skovsmose, O. & Valero, P. (2009). Inter-viewing Foregrounds. In M. César & K. Kumpulainen (Eds.), Social Interactions in Multicultural Settings (pp. 13-37). Rotterdam: Sense Publishers.

Andersson, A. (2007). Ett kulturellt besök i matematikundervisning: En aktionsstudie med ett etnomatematiskt perspektiv. Ics , Malmö Högskola.

Ascher, Maria (1998).Ethnomatematics. A Multicultural View of Mathematical Ideas. Belmont: Wadsworth, Inc.

Biesta, G.J.J. (2005). What can critical pedagogy learn from postmodernism? Further reflections on the impossible future o critical pedagogy. In Ilan Gur Ze'eve (ed), Critical theory and critical pedagogy today. Toward a new critical language in education (pp.143-159). Haifa: Studies in Education (university of Haifa).

Biesta, G.J.J. (2009). Good education in an age of measurement: on the need to reconnect with the question of purpose in education. Educational Assessment, Evaluation and Accountability. Retrived 31032009 from http://www.springerlink.com/content/

Bryman, A. (2001). Social science methods. Oxford: Oxford university press. University, Illinois.

D'Ambrosio, U (1985). Socio-cultural bases for Mathematics Education. Brazil: Unicamp,

D'Ambrosio, U (2006). Ethnomathematics. Link between Traditions and Modernity. Rotterdam: Sence Publichers.

Frankenstein, M. (2008). Quantitative form in arguments. In Matos, J F; Valero, P and Yasukawa, K (Eds). Proceedings of the fifth international mathematics education and society conference. Lisbon: Universidad de Lisboa and Aalborg University.

Gerdes, P (1996). Ethnomathematics and Mathematics Education. I A.J. Bishop, International Handbook of Mathematics Education (pp 909-944). Dordrecht: Kluwer Academic Publichers

García, G., Valero, P., Peñaloza, G., Mancera, G., Romero, J., Camelo, F., et al. (2009). Reinventando el currículo y los escenarios de aprendizaje de las matemáticas. Un estudio desde la educación matemática critica. Bogotá: IDEP - UPN.

Gutstein, E (2006). Reading and Writing the World with Mathematics. Great Britain: Routledge.Lange, T. (in press). "Tell them that we like to decide for ourselves" Children's agency in mathematics education. Accepted at CERME in Lyon 2009.

Mukhopadhyay, S. & Greer, B. (2001). Modelling with Purpose: Mathematics as a Critical Tool. In English, L.D. (ed). Handbook of international research in mathematics education. Second edition. United States of America: Sheridan Books Inc..

Ross, A (2008). Organizing a curriculum for active citizenship education. In Arthur, J; Davies, I and Hahn, C (eds). The SAGE handbook of education for citizenship and democracy.pp 492-505. Ondon: SAGE Pubblications Ltd.

Pring, R. (2000). Philosophy of educational research. London: Continuum.

Skolverket (2008). Statistical information. Retrieved from www.skolverket.se 2009-01-04

Skovsmose, O (1994). Towards a philosophy of critical mathematics education. Dordrecht: Kluwer Academic Publishers.

Skovsmose, O (2005). Meaning in Mathematics Education. In Kilpatrick, J; Hoyles, C; Skovsmose, O and Valero, P Meaning in Mathematics Education. USA: Springer Science + Business Media, Inc.

Skovsmose, O. (In press). Critical mathematics education - in terms of concerns.

Utbildningsdepartementet (2000). Kursmål för Matematik A. Retrieved from www.skolverket.se

Valero, P. (1998c). Struggles of a 'constructivist' curricular innovation. In O. Bjorkqvist (Ed.). Constructivism in the classroom (pp. 43-60). Vasa: Department of Teacher Education, Abo Akademi.

Valero, P. (2004). Socio-political perspectiveson mathematics education. In Valero, P. & Zevenbergen, R. (eds). Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology. Dordrecht: Kluwer Academic Publishers.

Negotiating Critical Pedagogical Discourses: Stories of Contexts, Mathematics and Agency

Annica Andersson and Paola Valero

Introduction

Mathematics education counts in society. However, society does not obviously count in mathematics education. This disjunction challenged Annica during her years working as a mathematics teacher in Swedish upper secondary schools. How society and social issues could be related to school mathematics was a central concern to her practice in Ericaskolan2, a Swedish upper secondary school, offering a three-year social science program. As part of Annica's Ph.D. study, a collaborative process was established with Elin, a female, young mathematics teacher, in order to introduce elements of a critical pedagogical discourse. Keeping the compulsory National curricular framework, the new pedagogy introduced project blocks that, addressing the mandated mathematical topics of the curriculum, allowed changing some key elements in the activities and relationships among participants. In this chapter, we want to explore possibilities as well as limitations when imagining and implementing a new pedagogical discourse that is heavily inspired by the concerns of critical mathematics education, and that intends to bridge the gap between students' experiences in society and the mathematics classroom.

Moving pedagogical discourses

The term pedagogic discourse has been broadly discussed in Bassil Bernstein's work, and has been recontextualized to mathematics education by different authors to refer to the ways in which knowledge and values are inseparably transmitted in the classroom (Lerman & Zevenbergen, 2004). With inspiration in Bernstein' framework, this type of research has offered a discussion of how mathematics education and power are connected in society (e.g., Dowling, 1998; Jablonka & Gellert, 2010). We have chosen not to adhere completely to this tradition, but to use the term pedagogical discourse in a different sense. A pedagogical discourse in mathematics education is the complex set of language formulations, together with the systems of reason that emerge when people engage in the social practice of mathematics education. Drawing on Valero (2010), the pedagogical discourse of mathematics education operates not only in classrooms, but is also present and at the same time constituted in a large network of the social and political meanings that different participants in the large network of practices of mathematics education in a society construct at given times in history. The pedagogical discourse, then, can be seen in classrooms as

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well as in schools or even in educational policy. In the case of this paper, we foreground the elements of that discourse as they are brought to live in the relationships between Elin, Annica, and the students in the mathematics classrooms. We decide to keep in the background, the whole context for the discourse as it is played in other spheres of practice.

There are many possible pedagogical discourses disputing a place in classrooms. Different curricular proposals, varying from the established and dominant tradition (Lampert, 1990) in the teaching and learning of mathematics, to constructivist-inspired, ethnomathematics-inspired, modelling-inspired, etc (Jablonka & Gellert, 2010). We find inspiration in critical mathematics education (Skovsmose, 1994, 2010; Skovsmose & Nielsen, 1996) understood as a series of concerns on mathematics, its role in society and its potentialities in education. A critical pedagogical discourse as we understand in this paper, acknowledges existing practices and introduces, through collaboration with teachers, different possible organizations of the classroom practice, with the aim of generating other possibilities for students' learning.

In order to describe the new critical pedagogical discourse, we want to use Skovsmose and Borba's (2004) and Vithal's (2003) model to discuss researching critical mathematics education. Skovsmose and Borba suggest three situations, as an analytical tool for exploring what is, what is not but what could be in the mathematics classroom. The current situation (CS) describes the situation prior to the attempt to introduce an alteration in the pedagogical discourse, and explains the problematic features of such state of affairs. The current situation is imagined as different, with new alternatives, on the grounds of inspiration from theoretical proposals or from new experiences. This new vision is labelled the imagined situation (IS) and can obviously be related to the teacher's, Elin, experiences and wishes for her teaching, but also supported by Annica's experiences and readings in critical mathematics education and ethnomathematics (Andersson, 2007, 2010, 2011; Anderson & Valero, 2009). Out of this imagined situation developed an arranged situation (AS) as an alternative to the current situation, which is also different from the imagined situation. However, as Skovsmose and Borba (p. 214) underline, "in general, an arranged situation is a practical alternative that emerges from a negotiation involving the researchers and teachers, and possibly also students, parents and administrators". As we will show, the arranged situation was not only constrained by negotiations between the direct participants. It was also framed by structural and practical limitations such as students' timetables and the mandatory frames given by the National curriculum and the examination system.

In analysing the move towards a critical pedagogical discourse in this paper, we will describe the three situations as we see they unfolded. However, we also want to emphasise in this chapter the qualities of the process, what Skovsmose and Borba (p. 215) labelled pedagogical imagination, practical organisation and explorative reasoning (ER). First, the relationship between the current situation and the imagined situation, labelled pedagogical imagination (PI) has to do with "conceptually exploring educational alternatives" (p. 216), with sensitivity for the socio-cultural and socio-historical school situation, but without taking the current situation as given. The qualities of the pedagogical imagination are here discussed in terms of co-operation, discussions and negotiations. Second, the relationship between the current situation and the arranged situation, labelled practical organisation (PO), denotes the pragmatic or more realistic version of

the imagined situation. The qualities of this process are discussed in terms of co-operation and negotiations in a wider school context. Considerations have to be made in relation to the teacher and students however also to issues as e.g. curriculum, examinations, timetables, staff relationships etc. needs to be acknowledged. The third process designates the analytical strategy to reflect on the imagined situation based on observations from the practical organisation and arranged situation. This analytical process, labelled explorative reasoning (ER), is by Skovsmose and Borba understood as a strategy for analysing educational possibilities that have not been acted out. However, we also see those possibilities that have been acted out as important for our purpose to pin point possibilities as well as limitations when imagining and implementing a new pedagogical discourse.
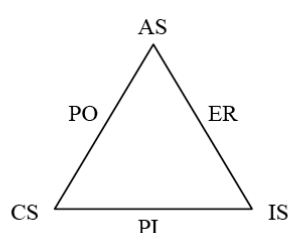
Figure 1: Model of critical research indicating what processes it might include

(Skovsmose & Borba, 2004, p.216)

The current situation at Ericaskolan and Elin's classes

Ericaskolan is a Swedish upper secondary school situated in a middle-sized city centre, and serves students from the city, its suburbs and from the countryside. The study programs at this school are all theoretical, thus preparing students for further university studies. The social science program is the largest program at the school, and within the program students has possibilities to orient their studies towards social science subjects, languages or journalistic writings. The students choosing the social science line usually decide to take this program because they need to enrol in a program that offers a good base to keep most doors open in relation to further studies, even if at the time of joining they do not have a clarity on which subject or profession they would like to follow. The students are generally interested in social sciences and humanities or attracted to language or media subjects. Besides, many of them have not enjoyed mathematics, natural sciences or the technical subjects, and thus see this particular program as an option to escape precisely these subjects.

Nevertheless, even if they have signed for a "social science" high school, the National Curriculum mandates that they have to pass two mathematics courses. The Mathematics A course, the first compulsory course for all high school students in Sweden, is thought to be:

> […] a core subject course […] The course builds further on mathematics from the compulsory school and provides broader and advanced knowledge in the areas of arithmetic, algebra, geometry, statistics and the theory of functions. The course is studied by pupils with widely different study orientations. The *structure is modified* and the *problems chosen based on the pupils' study orientation*. The course provides general *civic competence* and

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constitutes an *integral part of the chosen study orientation*. (Swedish Ministry of Education, 2000, our emphasis))

Such a declaration of intention opens the possibility for teachers and students to connect mathematics teaching and learning with civic competence and, thereby, with social issues of interest for social science students. The students in the two participating classes were in the age of 15-16 years old, and would soon reach the age of adulthood (18 years). They would then become recognized by society as competent citizens and expected to take on adult responsibilities. However, that was not what usually happens in these classes. A recent report of the Ministry of Education on precisely the Mathematics A course documents that teaching practices are very traditional, that is, the teacher dominates a type of instruction that is reported by the students as being meaningless and even "stupidizing" (Swedish Schools Inspectorate, 2010, p. 8). The teacher-centred approach gives students a reduced choice: to participate or not in mathematics education. If they decide to engage, they usually had to follow the instructions of the teacher and the textbook. The textbooks are full of mathematical exercises far from students' reality (Andersson & Ravn, in press). Time and planning responsibility lies with the teacher, and the authority in the classroom resides with the teacher or with the textbook. In other words, the Ministerial intentions in this course are far from being realized in the education of these future citizens.

This general situation reported to be the case in Swedish Mathematics A classes resonated with what usually happens at Ericaskolan, and with the way Elin was used to run her classes. It is also part of the expectation of the students who, coming from lower secondary school, have already experienced the "tradition". To give an example, when Annica asked the students with a "hands-up, how many of you have experienced team work and project work in prior mathematics education?" three students raised their hands. After completing the first 9-10 years of compulsory schooling in Sweden, only three out of 46 students, representing almost the same number of schools and classrooms as they came from different schools, had experienced mathematical group work. When asking these three students what kind of group work they had completed they all answered "solving problems or working through past national test questions in pairs".

The pedagogical imagination

Therefore, in addition to the aim of bringing societal issues into the mathematics classroom, it was important to establish an environment with possibilities for students to have a different experience with mathematics. In particular, Annica considered important to take seriously the Ministerial intention of making Mathematics A course to contribute to "civic competence". Therefore, achieving agency in/through/with mathematics were ideas that inspired the change in pedagogical discourse. We followed Biesta and Tedder's (2006) ecological understanding of agency, in which the possibility of acting and having control over one's actions "agency should not be understood as a capacity or possession of the individual, but as something that is achieved in particular (transactional) situations" (p. 27). This implies that agency is relational to the situations where becoming agentic is possible.

To change into this way of organised mathematics teaching required a different way of talking and behaving for all participating actors within the mathematics classrooms, and a new way of distributing power and responsibility between the researcher, the teacher and the students. In other words, a new pedagogical discourse was necessary. Consequently, the question that emerged was: how could an educational situation be arranged, imagined as informed by society and at the same time changing the classroom discourse into a discourse taking students intentions, experiences and reflections on their learning of mathematics seriously?

The mathematical content was not negotiable as the topics to cover were clearly stated in the national mathematics curriculum for the Mathematics A course. It was also of importance that all mathematical topics got well covered through the course as the students had to participate in a national test in the end of the course. However, the task-context (Wedege, 1999) in which the mathematics was learnt was clearly open to discussion, and actually approved by the same national curriculum. The curriculum for mathematics on the social science program stated that mathematics education should be e.g. "linked to" and "of importance for everyday life and their chosen study orientation" (Swedish Ministry of Education, 2000) and thus supported the plans to connect mathematics teaching to societal topics.

During the projects' design processes, concerns for mathematics education raised by Skovsmose (e.g. 1994, 2005) inspired Annica's thinking. First, concerns formulated as relating mathematics to citizenship and the preparation of students to become an active part of political life implicate understanding mathematics as a tool for identifying and analysing critical features in society and as seeing mathematics not only as a tool for critique but also an object of critique (Skovsmose, 2005). These concerns inspired Annica to rethink pedagogy in mathematics education that offered possibilities for project work, instead of working (only) with traditional textbook exercises. Mathematics textbook work was not excluded, that is not the point made here. Textbook calculations were rather used when this made sense to the students.

Project work became understood as a learning milieu (Skovsmose, 2001) where students, in pairs or groups, worked together on more complex tasks or activities over a period of time. Annica aimed for the projects to be designed with contexts relating to society, with possibilities for critical discussions and reflections and when possible with reference to the students' lives. Therefore students' were proposed spaces for themselves to decide on what were for them interesting contextual topics within the societal/mathematical framework. Critical researchers such as Frankenstein (2008), Gutstein (2006), Gutstein and Petersen (2006), and Skovsmose (2001, 2005) inspired Annica's thinking in how project work contextualising societal issues could be practically organised. Annica's plans included mathematical content development and teaching through critical projects and critical discussions, both of the learning of mathematics and of mathematics per se. The main teaching idea during the course was to vary the ordinary mathematics teaching with projects, if possible in collaborations with other school subjects.

Second, concerns about epistemology were taken seriously. Educational practices were understood in terms of acting persons and not as a transformation of a knowledge body. An awareness that classroom communication reflects power relations became important. Annica also wanted to take students reasons to engage in mathematics education (foreground/background/intentionality) (Skovsmose, e.g. 2005) seriously. This way of working

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resonated well with a critical mathematics classroom practise suggested by Ernest (2002, p. 8) in the following way:

> The aims of critical mathematics require the use of a questioning and decision making learning style in the classroom. Teaching approaches should include discussions, permitted conflict of opinions and views but with justifications offered, the challenging of the teacher as an ultimate source of knowledge (not in their role as classroom authority), the questioning of content and the negotiation of shared goals. […]. Also the learners should be given the chance pose their own problems and initiate their own projects and investigations at least some of the time.

These issues made Annica consider potentials, but also limitations, for a mathematics pedagogy discourse that takes students' empowerment, reasons for participating, etc., seriously. Annica started to imagine a different discourse in mathematics education. However, as pointed out by Ernest (2002, p. 8) "the approach must also honestly and openly address the instrumental and life goals of the learners themselves, both in terms of needed skills and passing exams." Examination issues became vital to address as the students course grades were of importance for potential further studies at University. It was also important that they passed the national tests at the end of course. Annica and Elin had to take care that the objectives stated in the national curriculum were possible to reach on all grading levels. Projects became designed to give students' opportunities to reach curriculum stated goals, on different grading levels. Elin created special examination sheets with "Objectives possible to reach within this project" and "What is required to show for different grade levels". The outcome of these sheets was threefold. First, it became clear for actors as head master, parents and students in the network that care was taken for examination issues as assessment and marking became transparent for all participating actors in the mathematics education network. Second, this way of working made it easier for students to achieve personal agency on assessment and grading issues. They got opportunities to decide individually on personal learning objectives to reach within each project. Third, the transparency supported Elin to assist all students to pass the course on their personal chosen examination level.

However, we need to stress that the National mathematics education curriculum puts limitations on project work. The tensions between students succeeding uniform national tests in the end of the course and the time constraints in relation to promoting effective project work mathematically and critically in different ways were very obvious during the teaching sequences. We also need to acknowledge the limitations to our wish for democratic and student empowering approaches – for the same reasons. We were obviously still imposing an external power and evaluation framework and this clearly restricted the ways we wanted to push matters of agency and empowerment further. These power relations are important to recognise when initiating a social responsible teaching (Atweh & Brady, 2009). It was a balancing act with negotiations undertaken almost every lesson in relation to the students' wishes, their and Elin's responsibilities and the curriculum and examination boundaries. Some of these discussions we took are exemplified in connection to the different teaching sequences below.

The pedagogy was developed and implemented in two classes' Mathematics A course in collaboration with Elin, the responsible mathematics teacher at Ericaskolan. At the start of this

research project Elin had identified why she wanted to change her teaching, she got support from the school organisation to work in the way she intended. Thus she had a substantial degree of ownership of the process. Annica's input was the theoretical perspectives that grounded the research as described above; Elin complemented these ideas with her school-situated knowledge and her personal teaching ideas, wishes and concerns. Annica and Elin discussed during a couple of months different options on how to build up the different teaching sequences and decided how to co-operate with(in) the new and different classroom discourse. Annica and Elin agreed on a coaching, mentoring role for the researcher (Kram & Ragins, 2007). Initially, the project and teaching planning was a joint activity, however when Elin asked for support or when her time was restricted Annica took on more of the planning responsibility. Besides Elin retained control of making the final decisions relating the content to the curriculum and examination qualities. Annica's role was encouraging and supporting but also pushing the process further, so that research goals not got lost during tougher times. The collaboration thus became very strong and Annica participated more in the implementation of the pedagogical discourse than was originally planned. In the classrooms Elin was the teacher with the classroom authority and she chose to position the Annica as a researcher and assisting teacher, although with no examination obligations. In other words, Annica participated in the classrooms, placed at a desk in the back of the classroom but also actively talking to students and answering their questions.

The arranged situation and the process of practical organization

The following section explores three carefully chosen teaching sequences where project work was conducted. The projects are chosen in a way that they illuminate three different ways of conducting project work. The three projects are situated in a larger school and societal context to explain reasons for the projects contextual development and the challenges that arose during the design process. In this way we aim for illumining different aspects of what occurred and the negotiations taken moving mathematics education discourses during a teaching semester.

The episodes are structured in the following way: first the projects are outlined as they were presented to the students. The students received instructions in Swedish; here we provide an English translation of the texts. Next, the societal background of the teaching sequence and the negotiations and reasons Annica and Elin had for creating the teaching sequences the way they did is described. Last an explorative reasoning paragraph, including voices from participants, casts light on some of the processes, possibilities and limitations they experienced together. The students' comments are quoted from a class blog, evaluation sheets, e-mails, interviews and spontaneous conversations, as well as from their personal logbook written during the last project. The students' voices are chosen in a way that they represent a group of students who, for different reasons, told stories about disliking mathematics or mathematics education or whose wellbeing diminished when they participated in prior mathematics education.

The teaching sequences are presented in chronological order to illustrate how Annica and Elin (referred to as 'we' in this section), together with the students, started off with small steps and then made the projects larger in content and time as the semester progressed. In this way all

participants learnt, together, how to work within a different pedagogical discourse when teaching and learning mathematics.

Project 1. "Making your dreams come true?"

The first project was presented to the students in writing as follows. In addition they also received examination sheets to all projects.

Box 1.

<div style="border:1px solid black;padding:1em;">

Making your dreams come true?

Reflect on something you would like to do, experience or buy, to yourself or others that cost so much that you need to borrow the money to cover the expenses. You need to find out how much money you need to borrow to finance the project and what interest the bank expects you to pay. We suggest the following: the interest and mortgage is paid to the bank once a year and you pay back the loan within five years. If this is not possible for you we will discuss that.

- How much will you be paying in interest costs per year? In total over the five years?

- How much do you need to <u>amortize</u> (pay to the bank) per year?

- What did the total cost add up to?

- Was it worth it? Why/why not?

We also suggest you discuss issues as what you personally might borrow money for. How do you find out the borderline for high or low interest costs? How do you find out if a loan offer is good or not?

Mathematical objectives possible to reach in this project in Mathematics A:

1. Be able to formulate, analyse and solve mathematical problems of importance for everyday life and their chosen study orientation

2. Have deepened and extended their understanding of numbers to cover real numbers written in different forms

3. With and without technical aids, be able to apply with judgment their knowledge of different forms of numerical calculations linked to everyday life and their study orientation.

The above stated goals imply you to be able to write fractions and decimals numbers as percentages (and the opposite), to be able to calculate percentages with factor of change (förändringsfaktor), be able to do calculate repeated percentage changes, have knowledge about the differences between percentage and percentage units and know how to calculate interest rates.

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Societal background and critical mathematical content

In Sweden an increasing number of young people (aged 18-25 years old) get in financial trouble after taking "quick-money loans". These loans are offered to young people often through mobile phone texts and are received within ten minutes. They typically come with high interest rates, and the companies usually do not check for credit record3 (Konsumentverket and the Swedish

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[3] According to Swedish statistical information, 25 % of those young persons (aged 18-25) who took these loans already had unpaid debts for debt recovery at the Swedish enforcement authority. Of these young

Enforcement Authority, 2007). If young individuals get into enforcement or police registers for not paying back debts, it usually decreases young persons' possibilities getting bank loans for further studies or a house later in life. These were the reasons why we regarded this topic as potentially critical for the students. We offered in this project possibilities for discussing issues with the students on how to act, negotiate and think when being in situations needing money.

The topic was introduced with a whole class discussion on how loan companies work with advertising to young people and about possibilities and risks with "quick-loans" as e.g. mobile text-loans. Even if these students only were 16 years old, it turned out that a couple of them had lent money from or through friends and parents and found it difficult to pay back. The critical discussions raised concerned e.g. when to/not to borrow money, different borrowing conditions and how to find different borrowing options. We talked about how different organisations in society work to pursue young people to borrow money, and how to look for information in the "small print" texts. As part of their project-work they had to either visit banks or search the Internet to find out the smartest option for their group and thus read conditions carefully.

Explorative reasoning

This was the first group work in these two classes and the students had known each other and their teacher for three weeks. During the planning process of this project Elin was concerned with the students and their beliefs about mathematics education, as this was their first year at Ericaskolan. Elin cared about both the students and herself to feel confidence about students reaching curriculum goals. So despite the research objectives and openness of the students' project, she argued that the students had to complete and hand in textbook exercises for assessment, in addition to the project presentations. This resulted in a mix of the old and new discourses, which at one point became problematic for the relationships between some of the students and Elin (Andersson, 2010). Reflecting back on this decision at the very end of the semester, Elin said:

> I would not do the percentage project again in the way that they had to hand in exercises
> from the book as well [as the project presentation]. But we had that discussion before we
> started and then I chose to bring in the book-part to feel sure about that they did something. It
> was a control point for me. (Elin, interview)

Consequently, when preparing the ensuing coming projects, Elin excluded prescribed textbook exercises. The textbooks became present in a different way than usual. Elin gave the students opportunities to decide for themselves (off course with supervision when asked for) what they needed to read or work with in the textbook to complete their projects. Alternative textbooks were offered on the front table for students to borrow if they required a mathematics book that explained mathematics in a way that suited a student particular well.

This way of organising the mathematics teaching, the new pedagogical discourse, was very new to the students. Through the students' blog comments during this project it became obvious that

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people 16% took new text loans to pay back prior loans. 64% regretted their loans and told that if they had needed to wait for the money 24 hours or more they would not have taken the loan in the first place (Konsumentverket and the Swedish Enforcement Authority, 2007).

this was a different experience of working in mathematics. They seemed to appreciate the new possibilities to decide for themselves. Rosie (who told she had not reached goals she wanted in prior mathematics education and thus experienced mathematics as a meaningless subject) wrote in her evaluation: "I think this was fun because one could decide topic and that means that one works with something one is interested in" (Rosie, written evaluation sheet, 10-2009).

When introduced to a different discourse in mathematics education, the students called for supervision both on the mathematical content and on how to work in teams to develop a mathematical project. Zizzi, a girl who described herself as "I have never had a real interest in mathematics, and I have never been encouraged enough to get the interest either" (interview, 10-2009), clarifies with her comment the importance of learning the skill to work project based:

> This was really meaningful and it was good to take personal responsibility for planning and for our own work labour. But this is new; we have to practise this way of working (Zizzi, blog comment)

Even if the students were enthusiastic about the activity and experienced possibilities for taking personal decisions and responsibilities, Elin had to pay attention to these issues when initiating investigation work. Skovsmose (2001, p.130) confirms our experience:

> Any landscape of investigation raises challenges to a teacher. A solution is not to rush back into the comfort zone of the exercises paradigm, but to be able to operate in the new environment. The task is to make it possible for the teacher and students to operate in co-operation within a risk zone, and to make this operation a productive activity and not a threatening experience.

Malin, a girl who told she had "mathematics anxiety" pointed to an issue needed to be recognised when conducting group work in mathematics. Early during the project she wrote: "This was ok. Something new and interesting and a good task because it was real, realistic. Can be good for me later on in life" (Malin, blog, 18-09-2009). Although, a week later she concludes after a bank visit:

> This was a good exercise because we had to find out stuff ourselves and thus become independent. I tried to calculate the interest rates but *realised we have to be better and more efficient to help each other with the mathematics tasks* in the group. I will try to get the others to be better at that, so we help each other ☺" (Malin, blog 23-09-2009, her smiley).

In her later comment we interpret her statements as worrying about reaching the mathematical goals in relation to the collaboration in the group. In this case Elin acted and supported Malin with extra mathematical discussions. In this way the blog became useful not only for research objectives and for students reflections on their learning; it also developed into an instrument for the teacher to become aware of what was going on in the groups.

Elin also made some critical reflections about the mathematical content. To make the project more authentic the interest calculations should not have been simplified to annual calculations in the way suggested. That decision was grounded in time constraints. As we wanted the students to work in an investigative way, and this way was new for them, we rather allowed time for e.g.

critical discussions, information seeking and planning for project presentation than conducting repeated interest calculations. However, the students did annual "interest-on-interest" calculations for the project presentation and thus showed they had the required mathematical knowledge to perform these calculations and Elin regarded this as acceptable.

Sandra was a girl who disliked mathematics. She did not want Annica to interview her because she did not want to spend more time connected to mathematics than was absolutely needed. However, Annica was very welcome to read her blog, evaluation sheets and logbook. In relation to her feelings for mathematics we regard her comment as interesting:

> I think mathematics has been a little more fun than usual. […] To plan time and content myself made me feel it was related to me. I feel the project has been meaningful and to look at mathematics from different angles (vända och vrida på matematiken) was positive. But I would have liked to have some more time for explanations from the teacher as mathematics is difficult for me (Sandra, evaluation sheet, 102009)

Summing up the first project, the students overall seemed to acknowledge the task-context of the project as related to their lives and realistic. They also seemed to enjoy being able to plan and take responsibility for their time and work distribution within the groups, as pointed out by Zizzi, Malin and Sandra above. No student indicated the opposite. At this particular time, possibilities for achievement of agency were appreciated by students who engaged and participated in a different way than their prior stories and experiences of mathematics education indicated.

Project 2. "The Newspaper Flyer Workshop" on critical mathematical argumentation

The second project was presented to the students in writing as follows.
Box 2.2

Newspaper Flyers/Headers with Mathematical Argumentation

The task for you today is, in small groups, to create a number of newspaper flyers that hits people, engages people, open up for curiosity, reflections and/or emotions – with a mathematical content! The goal is to acquire insight in how powerful numbers can be in advertisements- and news contexts.
There are 54 articles in "Convention on the Rights of the Child". See http://www.unicef.org/photoessays/50351.html We invite you to choose the one that interests you most and focus on that special one. Search and find information addressing the special children in your focus – information you consider important and want all people at the school to take part of. You might want to start a debate, it might be positive information, maybe information on the article is not followed – or something else. Reflect on how to present the numbers to get the message on your news flyer through in the best way.
We suggest you make at least three-four different flyers that address the convention you have chosen to focus on. The idea is to find out how the numbers can be exposed in the smartest way for the purpose of your flyer. Try different variations and show the ones you are most proud of to the class so we can take a critical discussion. Then we post them in school and see others reactions.
Mathematical objectives you have an opportunity to reach with this project:
•   be able to formulate, analyse and solve mathematical problems of importance for everyday life and their chosen study orientation

•   have deepened and extended their understanding of numbers to cover real numbers written in different forms

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> • with and without technical aids, be able to apply with judgment their knowledge of different forms of numerical calculations linked to everyday life and their study orientation
>
> • be familiar with how mathematics affects our culture in terms of, for example, architecture, design, music or the arts, as well as how mathematical models can describe processes and forms in nature.

Societal background and critical mathematical content

Inspired by Frankenstein's (2008) work on numerical information and quantitative argumentation we invited the students, in a cross-class setting, to a full day workshop in critical mathematical argumentation. As Frankenstein puts it when discussing the importance of requiring a "sense of quantities whose significance we cannot grasp" :

> Changing the form can help us make sense of quantities whose significance we cannot grasp. Changing the form through basic calculations can allow us to feel the impact of those quantities through better understandings. Further, knowing the most effective form in which to present those quantities in arguing for creating a just world is an important skill to teach in a critical mathematical literacy curriculum. I would go so far as to argue that knowing the most meaningful quantitative form in which to express information is necessary in order to understand what's going on. (p.262)

The framing contextual topic of the project became the United Nations 54 articles on the "Convention on the Rights of the Child". The reason for this choice was that during a two-week period the students had been engaged in a "Human Rights" cross subject project in school. Mathematics had not been invited to participate in this project and Elin experienced this as complicated. In the "Human Rights" project the students were required to conduct a survey, which from Elin's point of view was done with very low expectations on the statistical content. This issue became recognised as problematic by Elin as mathematics was not seen as important to acknowledge by other subject teachers. She argued that the subtext assumption to the students could be interpreted, as "mathematics is what you do in the mathematics classrooms and not related to other subjects or the outside world". Consequently, we decided to challenge the "Human Rights" larger school project in a subtle way. The students were invited to a mathematics argumentation workshop with the purpose of learning different ways of representing numbers and to consider why to choose a particular mathematics representation over other possibilities. They were also invited to discuss placements of mathematics in arguments. We wished to provide them with mathematical tools to present their arguments in the "Human Rights" project and thus give them possibilities to achieve better with support of mathematical knowledge.

The context for the mathematical argumentation day became the United Nation's Convention on The Right of the Child as it connected in a nice way to the "Human Rights" project's context. But there were also other reasons for deciding on this topic. First, Ericaskolan's basic values referred to these articles in the convention and thus were of importance for the students. Second, the context connected clearly to citizenship and democracy objectives stated in the curriculum. And last, it gave us opportunities to discuss mathematics as a critical tool while working with an international political important document.

Explorative reasoning

Due to administrative reasons this day was called a "Math-day" in the students' timetables, without further explanations. It soon became evident that calling a whole-day workshop in mathematics a "Math-day" was not a smart idea at all[4]. All students commented on this "stupid", "silly" or "meaningless" way of naming a day in school. Some students told us about feelings of anxiety the day before. The teacher noticed that there were a larger number of dentist and doctors' appointments this particular day than a usual school day. Petra, a student identifying herself as a "true math-hater" (interview, and repeatedly in class) told:

> First I thought, a whole day of mathematics, I can't do it; I just can't be there the whole day. But when I got there it was actually quite fun and now, afterwards, I read and look in the newspapers in a different way. So I actually learnt something and that was really unexpected of a math-day. (Petra, interview, 13/10/2009,)

Another interesting point is the number of students who did not recognise what we did as mathematics. Zizzi commented in this way:

> A math-day, how fun could that be, and why did you call it a math-day? We worked on posters, we sought information, we rewrote mathematical stuff for best effect, but that is not mathematics! It was a really good day, but definitely not maths… (Zizzi, interview, 14/10/2009)

Another interesting angle of this line of reasoning was that, when studying the non-mathematics subjects' curriculum objectives, the students also reached goals in computer science and there were clear possibilities to reach rhetorical and argumentation goals in Swedish language, as well as citizenship and democracy objectives. That is, these links could have been made if teachers at this school recognised mathematics as a subject to collaborate with.

During this project the students' comments indicated that it was the critical content rather than achievement of agency that made the day interesting.

Project 3. The statistical project "Students' Ecological Footprints on Earth"

The third task became a cross-subject collaboration between mathematics and environmental science on the contextual topic ecological footprints. An ecological footprint

> … accounts for the flows of energy and matter to and from any defined economy and converts these into the corresponding land/water area required from nature to support these flows. This technique is both analytical and educational. It not only assesses the sustainability of current human activities, but is also effective in building public awareness and assisting decision-making (Wackernagel & Rees, 1996 p.6).

The project ran intensively for three weeks during mathematics and science lessons, with a whole day of displaying results with power points, papers, posters, presentations, discussions and interactions in the fourth week. The students got a detailed introduction of ecological footprints

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[4] As the school oraganiation required a name of the day for the students' timetable, and the topic was not decided yet, we decided "just call it a math-day for time being".

at the beginning of the project, where Per-Erik, the environmental science teacher, Elin and Annica participated. The project was presented to the students in writing as follows.
Box 2.3

The average Ericaskolan-pupil's Ecological Footprint: ☺ or ☹?

The idea with this project is to commence a statistical investigation at the school. The goal is to find out how many earths we need to live in the way a student here lives today. We suggest you choose a topic you find interesting e.g. food, travelling, housing, energy, consumption or anything else you are interested in. The communal objective is, that we all together, in the end, compare our results and together find out the ecological footprint we, as students at this school in Sweden, do on earth (http://www.minplanet.se)

A suggested working process you can follow and some advice on the way:

1. Study course goals and assessment criteria carefully so you gain knowledge and by that decide personally how and on which assessment level you want to work.

2. Construct interest groups of three persons and formulate questions within a topic area that grasps your attention. If needed, you can find inspiration on www.minplanet.se. The questions have to be prepared and formulated in a way so they give you possibilities to reach the goals you wish. We teachers will be happy to supervise you in this work before you conduct your survey. The reason for this is that we want you all to get as good data as possible to be able to reach the objectives in both subjects (mathematics and environmental science).

3. The chosen population needs to be representative for the students at the Ericaskolan as we want to calculate the ecological footprint a student at our school make.

4. We invite you to account for your findings with a presentation in class and a written documentation as a Power Point presentation, posters, an article or as your personal choice.

5. The examination includes

a) One part where your results are shown and commented in a correct mathematical way

b) One part where you result is manipulated in a way that they make an impact in an intended way. You need to take a stance and write it up together with your manipulated diagrams.

c) A written test in environmental science.

6. Material you have for disposal: different mathematics and environmental science books, logbook, articles, a time planer and computer programs as Excel, PowerPoint and Star Office. If you need other things please let us now.

As teachers we will work as supervisors during the project. This indicates that it is your responsibility that we get your attention if you need whole class or individual information, explanations, feedback, materials etc. ☺

Mathematical objectives possible to reached within this project:

Mathematics A

- be able to formulate, analyse and solve mathematical problems of importance for everyday life and their chosen study orientation

- be able to interpret, critically examine and with discrimination illustrate statistical data, as well as be able to interpret and use common co-ordinates

- be accustomed when solving problems to use computers and graphic calculators to carry out calculations and use graphs and diagrams for illustrative purposes

Mathematics B

- use with judgment different types of status indicators for statistical material, and be able to explain the difference between them, as well as be familiar with and interpret some measures of dispersion

- be able to plan, carry out and report a statistical study, and in this context be able to discuss different types of errors, as well as evaluate the results

Societal background and critical mathematical content

A large number of the news headlines during this autumn semester where related to news reports from COP 15, the UN Conference on Climate Change in Copenhagen[5]. This meeting attracted speakers as the President of the United States, Barack Obama, and other state leaders. The school, while situated in southern Sweden, got affected by this conference in different ways and we decided to use the themes of the Copenhagen meeting: climate change and climate sustainability for a statistical project as it run at the same time as the conference. The environmental science teacher was about to start a project on sustainable development and ecological footprints, so we efficiently developed these projects together.

The project was designed and introduced as a "three-stage rocket". The students first decided on interest working groups and chose a topic within the ecological footprint area. They designed the survey in parallel with gathering information about the topic per se. The second part of the project was to conduct the survey, and report it in a descriptive way. However, we also invited the students to take a stance in relation to their chosen topic and argument for their view with support of their statistical information and thus consider different ways to represent their data. By that they learnt how to manipulate (not fake!) diagrams and statistical information to strengthen arguments and thus experienced how different ways of (re)presenting statistical information in newspapers and advertisements affect our personal judgements.

The third step was an afternoon session where all the different collected information was summarised into an ecological footprint website, My Planet, with the goal to find out how large footprints the students at Ericaskolan actually made on earth. (For those of you who wonder: if we all lived the way that these students did, we would need 4.6 Earths to survive. Energy and transport were our large consume areas, maybe not surprisingly as we were located in Sweden). During this session we discussed and compared data from different countries (e.g. a person in

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[5] Please see e.g. http://climatecongress.ku.dk/ for detailed information of the congress.

Bangladesh used least while a person from the USA needed the largest amount of planets for their way of living) and we examined critical issues in our different ways of living.

Explorative reasoning

At this stage we all, students, teachers and researcher, realised the rich possibilities to continue the ecological footprint project further either in a global direction or in a more local way. The global approach, immersing into issues as sustainability, fairness, responsibility, economy and so on – had no limitations. Locally the students saw rich possibilities for influencing the school to work for a sustainable development in the good for the school community. Concrete information from different groups showed e.g. that students did not know where the paper collection bin was located or whether the fruit sold in the café was ecological (organic) or not. The students proposed writing letters to influence school leaders for taking environmental and ecological sustainable decisions. These were the times when there was energy in the classrooms, the students wanted to initiate change and influence climate sustainability in their school. However, it did not fit into the school system timetables, curricula and school labour distributions and thus it was not possible for us to push the project boundaries further and create changes at school or to take a more global approach. Disappointingly, the project became just another "school project", however with rich possibilities for expansion. To realise these possibilities of the project in the future, projects need to be planned and decided in such a way beforehand, together with school leaders and teacher colleagues.

How did the students experience this project? An excerpt from Sandra's logbook highlights some different aspects from the student's perspective:

> During the project I have learnt about different diagrams. E.g. I did not know about histograms before the project. I think it has been really interesting with manipulated diagrams and results – now I will be more observant when reading newspapers etc! What surprised me most though was how important role mathematics plays when talking about environmental issues. With support of mathematics we can get people to react and stop. […]
> I am so interested in environmental questions and did actually not believe that maths could be important when presenting different standpoints. I have probably learnt more now than if I had only calculated tasks in the book. Now I can get use of the knowledge in the project and that made me motivated and happy! I show my knowledge best through oral presentations because there you can show all the facts and talk instead of just writing a test. To have a purpose with the calculations motivated me a lot. (Sandra, logbook, conclusions).

Sandra's voice indicated that she first, changed her attitudes towards mathematics during the project and, second, she improved her performance during the projects. In this case Sandra related mathematics to a context that made sense to her and achieved agency in her learning and accounts of her learning. Sandra performed, together with her friend, a very well prepared presentation and carried out a questioning on both mathematical and environmental issues. She reached the assessment criteria in mathematics for "passing with special distinction". As Sandra told us that she never had received more than "pass" on a written mathematics test in her life, the point she makes, that she shows her knowledge best through oral presentations, seemed to be well reinforced. This gets me to the third reason for choosing Sandra's evaluation – her

awareness and reflections on her personal learning. During those teaching sequences when the students worked in projects their reflections on their learning experiences changed in character. Initially they used adjectives as e.g. fun, interesting, difficult, different etc. However, a different quality of their evaluations emerged during the semester. This phenomenon occurred even as they did not receive any feedback on their blog comments or their evaluations from the teacher or me. These tendencies emerged in different phases and at different stages in different students, but a trend were indicated.

To conclude the ecological footprint project, we could notice that the students advanced in their reflections on how to conduct project work, on how to use mathematics and on their personal learning of mathematics. The qualities of the students' comments became different during this time. During this project it was both the critical context and the possibilities to take personal decisions that impacted on the students' engagement.

What occurred after the researcher had left the school?

The students were expected to sit the national tests at the end of the spring semester and this goal influenced the organisation of teaching. During this time Elin varied her teaching with textbook work, a larger geometrical project and smaller peer collaboration exercises. However, Elin stressed that

> … during all lessons focusing on textbook work, through the semester, I gave them opportunities to do smaller group work tasks or to collaborate around different activities and problems, even when we were preparing for the national tests (Elin, interview, 24-07-2010).

Henrik was one of the students who decided to conduct a geometry project He wrote about himself and his prior experiences of mathematics education before he started at Ericaskolan in the following way:

> I have in general always disliked mathematics, it has never felt meaningful. The problem was not that I did not understand mathematics; I usually picked it up very quickly. The problem was rather that I could not write down the mathematics, I became tired very quickly. It was also tiresome that I did not experience any meaningfulness; I could not relate the knowledge to something I would need in the future. Just sitting down, focus, do the same tasks again and again felt meaningless (Henrik, personal letter, 08-2009)

After the geometry project Henrik wrote:

> I finished the Tetra-Pak project some time ago but got response from my teacher yesterday. That was a fun project to do, and I got a good grade on it as well. It was interesting to do a report in mathematics, that with mathematics solve a problem. My question was formulated: Why is a milk packet shaped in the way it is and what calculations has Tetra Pak performed to innovate such a good product?
> I feel I got an answer to the question. The milk packet is simple but has complex consumer requirements. The packet has to be sustainable, be small, cheap to produce and consumer friendly. It was groovy to design my own small milk packet with the same shape as the larger 1-liter packet but containing only 1 dl. (Henrik, e-mail, 14-5-2010)

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However, a mail received after the national test illustrated the contrast between his experience of project work and the national test:

> I am very happy with the semester and feel I have achieved as best as I could. The most interesting and instructive parts were the projects and theme works. Then it felt really realistic and meaningful, because we not only worked with facts but actually used it to create something new and creative.
> I did the national test last week, and that is really not my favourite and I performed quite bad – and felt that I lost some of my interest and motivation for mathematics. (Henrik, e-mail, 05-06-2010)

In Sweden, even being compulsory, national tests are not recognised as exam tests, they are rather expressed as a support for the teacher when grading (Swedish Ministry of Education, 2000). As Henrik performed very well during the other parts of the course he received a higher grade than his performance on the national test indicated. The transparency and foresight of teacher on assessment issues supported her when grading the students in the end of the course.

What Henrik's story indicated was that he both engaged and achieved well during the project sequences, thus resonating with Sandra's story above. However, Henrik's story adds knowledge about the vulnerability in the changing process. In his case, the national test confirmed his prior experiences of mathematics.

Further explorative reasoning and some concluding comments

The analytical framework offered by Skovsmose and Borba (2004) supported our thinking when we tried to understand the reasons for why the arranged situations unfolded as they did during the teaching semester. Through emphasising the different processes between the current situation and the arranged situation, and between the imagined and arranged situation. Foregrounding the elements of the new pedagogical discourse as they were brought to live in the relationships between Elin, Annica, and the students in the mathematics classrooms gave a further understanding of concerns required to be acknowledged when moving between discourses in mathematics education. The complex issues that arose during the processes of pedagogical imagination and practical organisation of the different teaching sequences were issues that in some cases supported our work, and in other cases hindered or put boundaries on the way we wanted to develop the new pedagogical discourse. Situating the projects in the socio-cultural background of the school and acknowledging what occurred at certain historical moments in the school and in society clarified why specific decisions were taken at particular times. Our underlying expectation is, through this way of reasoning, we open up for critique and scrutiny the projects and the implemented pedagogy itself.

We found inspiration for the discourse movement in critical mathematics education understood as a series of concerns on mathematics, its role in society and its potentialities in education (Skovsmose, 2010). A critical pedagogical discourse as imagined an arranged in the described context acknowledged existing practices at Ericaskolan and introduced, through collaboration with Elin and the students at Ericaskolan, different possible organizations of the classroom practice, with the aim of generating other possibilities for students' learning. The critical

pedagogy was in this piece of research realized as a pedagogical discourse connecting mathematical contexts to social/societal concerns while appreciating students' possibilities for achieving agency both in relation to their mathematics learning and in relation to task context. Thus the students became responsible, or rather in charge of, their personal learning of mathematics. This was a very different way of organizing mathematics education in Sweden (Swedish Schools Inspectorate, 2010). Initially students' required supervision on working and learning mathematics through projects and teamwork. However, students acknowledged the new way mathematics teaching was organized and engaged in the classroom work during the project sequences. Students who not usually participated or achieved well in mathematics accounted for differences both in their engagement and in their results. Their achievement of agency (Biesta & Tedder, 2006), differed in different situations and in different individuals. Concerns about relating mathematics to citizenship and as understanding mathematics as a tool for identifying and analysing critical features in society were addressed but could have been pushed further if opportunities had been given. The shared experience that we all, together, learnt how to work in this way needs to be recognized. Obviously it would have been challenging to explore and push the boundaries further with this group of students and their teacher over a longer period of time. To change the social practice of mathematics education and move between the discourses required support from different parts, or nodes, of the mathematics education network at different historical times. Valero (2010 p. LXXII-III) wrote:

> If mathematics education practices are seen as the network I proposed, the aim of the research field would be to provide insight into not only how each single node of the network operates constructing the meaning and significance of mathematics education, but also into how different nodes interconnect at particular historical times.

The different nodes Valero referred to are understood as different relationships we needed to establish in order to proceed with changing the teaching organisation this particular semester. E.g., the importance of an on-going dialogue with school leaders became apparent when timetables needed to be negotiated. This became an issue when investigation work together with other school subjects was to commence. At particular historical points, the negotiations worked in our favour, as with the ecological footprint project where both the mathematics teacher and the environmental teacher recognised opportunities with collaboration. This collaboration became fruitful for all participants and fitted well within the time allocated for the teachers, within the structures of the school and with a big event in society. At other times, nodes in the network hindered us, as when the relationships with the teachers responsible for the larger "Human Rights" project impacted on the way mathematics was recognised in the school. Our experiences indicated that change was possible, that a critical pedagogy could lead to students' achievement, engagement and reflections in mathematics education. Through locating the experiences in the socio-cultural context of the school gave us an understanding of the complex situations and processes that required to be addressed to realise the imagined situations.

This is a case study where Annica had the opportunity to collaborate with Elin, a teacher who, with support, wanted to change her way of organising her teaching. Therefore, in some significant ways, these researched situations in two different classrooms were different from regular classrooms, thus this article's account cannot be translated into any other mathematics

classroom. However, there are reasons to suggest that experiences from this setting are potentially transferable to other classrooms. Adjusting the situation context, thus addressing the immediate context of teaching and learning in the mathematics classrooms, in this case bringing in societal issues into the classrooms and opening up possibilities for students to achieve agency ought to be a transferable teaching idea when adjusted to other socio-cultural contexts.

References

Andersson, A. (2011). Interplays between context and students' achievement of agency. Accepted for presentation and proceedings at Cerme 2011, Working Group 10 in Poland.

Andersson, A. (2010). Making Sense of Critical Mathematics Teaching. In L. Sparrow., B. Kissane,. & C. Hurst (Eds), *MERGA33. Shaping the Future of Mathematics Education. Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia* (pp.51-58) . Fremantle: MERGA Inc.

Andersson, A. (2007). *Ett kulturellt besök i matematikundervisning: En aktionsstudie med ett etnomatematiskt perspektiv.* Ics, Malmö Högskola.

Andersson, A. & Valero, P. (2009). Mathematics education giving meaning to social science students. A case from Sweden. Proceedings at II Congreso Internacional de Investigación, Educación y Formación Docente in Medelin, Colombia 26-28 Aug 2009.

Andersson, A. & Ravn, O. (In press). A critical perspective on contextualisations in mathematics education. In O. Skovsmose & B. Greer (Eds), *Critique and politics of mathematics education*. Rotterdam: Sense Publishers.

Atweh, B. & Brady, K. (2009). Socially Response-able Mathematics Education: Implications of an Ethical Approach. *Eurasia Journal of Mathematics, Science & Technology Education. 5*(3), 267-276.

Bishop, A. (2008). Mathematics teaching and values education – an intersection in need of research. In Clarkson, P. & Presmeg, N. *Critical issues in mathematics education. Major contributions of Alan Bishop.*(pp. 231-238). New York: Springer.

Biesta, G. & Tedder, M. (2006). How is agency possible? Towards an ecological understanding of agency-as-achievement. Working paper 5. Exeter: The Learning Lives project.

Dowling, P. (1998). *The sociology of mathematics education: mathematical myths, pedagogic texts*. London ; Washington, D.C.: Falmer Press.

Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of mathematics education journal. 15* (1)1-16.

Frankenstein, M. (2008). Quantitative form in arguments. In J F. Matos, P. Valero. & K. Yasukawa (Eds). *Proceedings of the fifth international mathematics education and society conference.* Lisbon: Universidad de Lisboa – Department of Education, Learning and Philosophy, Aalborg University.

Jablonka, E., & Gellert, U. (2010). Ideological roots and uncontrolled flowering of alternative curriculum conceptions. In U. Gellert, E. Jablonka & C. Morgan (Eds.), *Proceedings of the*

Sixth International Mathematics and Society Conference (pp. 31-49). Berlin: Freie Universität Berlin.

Gutstein, E. (2006). *Reading and writing the world with mathematics. Toward a pedagogy for social justice.* Great Britain: Routledge.

Gutstein, E & Peterson, B. (2006). *Rethinking mathematics. Teaching social justice by the numbers.* Wisconsin: Rethinking schools, Ltd.

Konsumentverket and the Swedish Enforcement Authority. (2007). SMS-lån. En kartläggning av unga vuxnas erfarenheter. Rapport 2007:17. Retrieved 2010-07-12 at www.ungkonsument.se/

Kram, K.E & Ragins, B.R. (2007). The landscape of mentoring in the 21st century. In K.E Kram & B.R. Ragins (Eds) *The Handbook of Mentoring at Work: Theory, Research, and Practice* (pp 659-689). Thousand Oaks: Sage Publications Inc.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational research journal, 27*(1), 29-63.

Lerman, S. & Zevenbergen, R. (2004). The socio-political context of the meathemaics classroom. Using Bernstein's theoretical framework to understand classroom communications. In Valero, P. & Zevenbergen, R. (eds). *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology* (pp. 27-42)*.* Dordrecht: Kluwer Academic Publishers.

Swedish Schools Inspectorate (2010). *Skolinspektionens Kvalitetsgranskning Rapport 2010:13: Undervisningen i matematik i gymnasieskolan. Diarienummer 40-2009:1837.* Stockholm: Skolinspektionen.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer Academic Publishers.

Skovsmose, O. (2001). Landscapes of investigation. *ZDM. 33* (4), 123-132.

Skovsmose, O. (2005). *Travelling through education: Uncertainty, mathematics, responsibility*. Rotterdam: Sense Publishers.

Skosvmose, O. (2010). Critical mathematics education: in terms of concerns. In B. Sriraman, C. Bergsten, S. Goodchild, G. Palsdottir, B. D. Søndergaard & L. Haapasalo (Eds.), *The sourcebook on Nordic research in mathematics education* (pp. 671-682). Charlotte (USA): Information Age Publishing.

Skovsmose, O. & Borba, M. (2004). *Research Methodology and Critical Mathematics Education.* In Valero, P. & Zevenbergen, R. (eds). *Researching the socio-political dimensions of mathematics education* (pp 207-226). *Issues of power in theory and methodology.* Dordrecht: Kluwer Academic Publishers.

Skovsmose, O. & Nielsen, L. (1996). Critical Mathematics Education. In A. J. Bishop. *International Handbook of Mathematics Education* (pp 1257-1288)*.* Dordrecht: Kluwer Academic Publishers.

Swedish Ministry of Education, (2000). Upper secondary school: Goals to reach. Retrieved 2009-01-18 from www.skolverket.se

Valero, P. (2010). Mathematics education as network of social practises. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello. *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education. January 28th - February 1st 2009, Lyon (France)* (pp. LIV-LXXX). Lyon: Institut National De Recherche Pédagogique 2010.

Vithal, R. (2003). *In search of a pedagogy of conflict and dialogue for mathematics education*. Dordrecht ; Boston: Kluwer Academic Publishers.

Wackernagel, W. & Rees, E. (1996). *Our ecological footprint. Reducing human impact on the earth*. Gabriola Island: New Society Publishers.

Wedege, T. (1999). To know or not to know - mathematics, that is a question of context. *Educational Studies in Mathematics 39* .205-207.

Wedege, T. (2010). Sociomathematics: A subject field and a research field. In U.Gellert, E. Jablonka & C. Morgan (Eds). *Proceedings of the Sixth International Mathematics Education and Society Conference* (pp. 478-487). Berlin: Freie Universität Berlin.

A "Curling Teacher" in Mathematics Education: Teacher Identities and Pedagogy Development

Annica Andersson

Aalborg University, Denmark and Malmo University, Sweden.

Abstract

In this article, I outline processes that supported or hindered Elin, a mathematics teacher, to engage in pedagogy development. In a setting inspired by critical mathematics education, Elin was encouraged to bring societal themes into her upper secondary teaching so that mathematics was connected to social science subjects. A classroom environment was set up in which classroom discourses supported students' negotiations about their learning of mathematics. In this new pedagogical discourse, projects were introduced that while addressing the mandated mathematical topics of the curriculum, changed some key elements in how mathematics had been previously taught as well as the relationships between participants. Elin's narrated identities provided ways to understand shifts in Elin's ways of acting when gradually transforming her teaching. Elin's identities illuminated how she became aware of herself, her teaching organisation and her different ways of interacting with students. She identified and acted upon her perceptions of the new possibilities and different responsibilities that actors in mathematics classrooms have. Elin's fluctuating teacher identities reveal why she struggled at times and how she was constrained in becoming the teacher she wanted to become.
Keywords: teacher change, identity, agency, professional development, pedagogical discourse

Introduction

> It feels as if it always is there [where] I and the students end up. We start off with, for example, practical mathematics but then, suddenly, I stand there again, with a whole class explanation and think wait, what happened now, how did we get here? (Elin[6], interview, 18/09/2009)

Elin, a mathematics teacher at Ericaskolan, a Swedish upper secondary school, is probably not unique in finding that when she tried to change her mathematics teaching practices, she ended up

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[6] All names of school and persons in this article are pseudonyms.

instructing in her usual way. With the best of intentions and with motivation to try out a different way of organising her teaching, Elin experienced difficulties in succeeding and sustaining new ways of teaching. In this article, issues that supported Elin to change her teaching as well as those that hindered her are explored with an aim of understanding how success and sustainability in teacher development can be achieved.

The second aim of this article is to examine how a collaborative research partnership offered the teacher and me, as the researcher, learning opportunities. The gradual changes that could be seen in Elin's stories about her teacher identities suggest reasons for why some problematic issues occurred at particular times. As a participant in the research, I learnt, reacted and acted in response to the ebbing and flowing of events that impacted on Elin and hence on our collaboration. Together, we navigated through the different circumstances that affected our collaboration.

From this case study, which is part of a larger study, three vignettes are provided to illuminate how the teacher became aware of herself, her teaching practices, and her different ways of interacting with the students. The discussions allowed for an ebbing and flowing of ideas that contributed to further explorative reasoning.

Learning, discursive identity and agency

The theoretical framework used a socio-cultural-political perspective in which learning was viewed as a social activity, implying that learning processes "are constituted in the encounter between contextualised, historically grounded human beings and their activity in particular settings and spaces that are socially structured" (Valero, 2004, p.10). In this perspective, learning is not just about getting to know, learning is also about becoming someone (Radford, 2008). Consequently, the notion of identity became particularly important (e.g., Solomon, 2009; Valero & Stentoft, 2010) because it provided ways to understand how learning was enacted both by the teacher and the researcher. Kelly (2006, p. 513) wrote:

Teachers' identities are neither located entirely with the individual nor entirely a product of others and the social setting. They can be regarded as the ways in which practitioners see themselves in response to the actions of others towards them; that is they are the constantly changing outcomes of the iteration between how others construct practitioners, and how they construct themselves, in and away from social situations.

Kelly's understanding of identities as constantly changing within a socio-cultural landscape is a starting point, but such a theoretical view demands devising an operational tool to analyse identities in a relational way. Recently there has been a strong move towards discursive sociological perspectives within mathematics education research (e.g., Black, Mendick & Solomon, 2009; Sfard, 2008; Stentoft & Valero, 2010). For the purpose of the analysis of Elin's identities, Sfard and Prusak's (2005) understanding of identities as collectively narrated stories individuals and others tell about a person was adopted. They define identities as narratives, which are reifying, endorsable and significant to that person and produced through engagement with and connected to available discourses:

The key move was to equate identity-building with storytelling. The difference between identity as a "thing in the world" and as a discursive construct is subtle. The kind of data that the narrative-minded researcher analyzes in her studies is the same as everybody else's: These are stories that people tell about themselves or about others to their friends, teachers, parents, children, and bosses, as well as to researchers. The only distinctive feature of the present narrative approach is that, rather than treat the stories as windows to another entity that stays unchanged when "the stories themselves" evolve, the adherent of the narrative perspective is interested in the stories as such, accepting them for what they appear to be: words that are taken seriously and that shape one's actions. (Sfard & Prusak, 2005, p.21)

Thus a person's identities change with the stories that they accept about themselves. The stories become adapted as a result of the interactions between the person and the environment in which they operate.

Political and social contexts impact on teachers' identities and the ways that they can act as they change their teaching practices within a reform context (Lasky, 2005). Data collected for Lasky's research were part of a larger longitudinal, mixed-method study conducted in ten schools from two boards in Canada. The reform context, in which the teachers operated, constrained teachers' experiences of agency as it affected the choices that they could make. Lasky's findings indicate that identity stories that are linked to changes are most likely to include teachers' sense of agency. Biesta and Tedder (2006, p.27) provide an ecological understanding of agency, in which the possibility of acting and having control over one's actions are contextually situated: "Agency should not be understood as a capacity or possession of the individual, but as something that is achieved in particular (transactional) situations". So, in this article the concept of identity, defined by Sfard and Prusak (2005) as stories people tell about themselves and others, together with Biesta and Tedder's (2006) ecological understanding of the concept of agency will be used for further discussing issues that supported or hindered Elin to change her teaching.

Becoming agentic is tied to the specific circumstances within the educational situation. Clarke (2007) described the conditions that would support teachers to make changes to their teaching practices as "involving groups of teachers", "solicit teachers' commitment" and "allow time and opportunities for planning and reflection", and to share "the wisdom of practice" (p.28). Within situations where teacher change was occurring, teachers' feelings were seen as significant contributors to the process. Clarke (2007) acknowledged these by stating the need to "recognize that change is a gradual, difficult and often painful process" (p. 28). If teachers decide to engage in change processes, a stable change involves continuous learning by teachers including opportunities for reflection (Fullan, 1991). Although it has been suggested in the literature that change is required to be stable or sustainable over time (e.g., Fullan, 1991; Garet, Porter, Desimone, Binnan & Yoon, 2001), teachers' narratives of identity suggest that changes are not sustained over time. There might be a contradiction; if learning occurs continuously then change cannot be stable. So we have to query what learning actually results in. For example, will it result in changes that cannot be undone, or in continuing change processes? And what issues supports or hinders teachers learning during change processes? Learning is in this article understood as closing gaps between actual and designated identities. The actual identities are "stories about the actual state of affairs" (Sfard & Prusak, 2005, p. 18) and the designated identities consist of

narratives that are "expected to be the case, if not now than in the future" (ibid, p. 18). Regarding designated identities, Sfard and Prusak states that:

> … identities are crucial to learning. With their tendency to act as self-fulfilling prophesies, identities are likely to play a critical role in determining whether the process of learning will end with what counts as success or with what is regarded as failure (p.19).

Elin's identity narratives during change processes, and how she valued her learning are discussed further in the terms of learning as closing gaps between actual and designated identities.

Methodology

This article draws on data from a one-year, classroom-based study that explored students' identity narratives whilst learning of mathematics. As in participatory research, both Elin, the teacher, and I, the researcher, collaborated (Skovsmose & Borba, 2004). Although we had different goals for our collaboration, we developed the pedagogy in the classroom together. An underlying expectation from this type of research is that it will open up for critique and scrutiny of the implemented pedagogy and the research itself. As Atweh and Brady (2009) argue, in the prevailing mathematics education discourse, "better mathematics" often refers to the academic discipline of mathematics. This discourse often opposes a mathematics education that focuses on social world applications, e.g., a critical mathematics approach. Atweh and Brady stress the need to balance these concerns for a quality mathematics education. In our case we strived for integrating projects within the boundaries of the mandated curriculum mathematical, also acknowledging mathematics that were required for further academic studies through allowing textbook work. In Elin's and my collaboration I contributed with a theoretical base, including theories about critical mathematics education. Elin complemented these ideas with her school-situated knowledge, and concerns for curriculum objectives and examinations. Even though the stories in this article are Elin's narratives and experiences, the stories impacted on the larger research project and influenced how it developed. Through our collaboration, we learned together whilst we navigated through the course, over time and in relation to other participants inside and outside the mathematics classrooms.

Data collection

The data were collected during my participation in two social science students' mathematics classes during a five-month autumn semester. Elin taught both classes. After the semester, I continued to stay in contact with Elin and some students on a regular basis through the teaching year until the students sat the national Mathematics A test. The collected data consisted of three audio-recorded planned interviews and several shorter spontaneous conversations audio-recorded after lessons or during planning or coffee breaks. We discussed e.g. planning and teaching, or reflected on what occurred in the classrooms or in our collaboration. The interviews were complemented with written material such as personal letters, e-mails, Internet forum/blog comments and my field notes. In this way I received data from different sources, that was communicated both orally and in writings.

Working with two different languages, in this case Swedish and English has ethical implications (Robinson-Pant, 2005). For instance, there are issues around validity and translation to ensure that the English translations accurately reflect what was said or written in Swedish. For this article the audio-recorded interviews and conversations and the written material where all transcribed and analysed in Swedish, and those parts that are cited were translated into English by me.

Data analysis

The notion of identity gave an analytical base for understanding Elin's reasons and actions in this particular cultural setting. Sfard and Prusak´s (2005) operational sense of identities were used to analyse the data. They propose to "equate identities with stories about persons" (Sfard & Prusak, 2005, p. 14). An identifying story could be considered reifying if the words be, have, can, always, never, usually were used; endorsable if the identity-builder (the person the story is about) accepted the stories as being about themselves; and significant, that is if any change to the story was likely to affect the storyteller's feelings about the identified person, particularly with regard to membership of a community. The stories told or written by Elin about herself were analysed by locating the suggested verbs and adverbs (stressing repetitiveness) for identifying the reified quality. Stories that were told/written with expressions such as "I am worried …", or "For me it is always important that…" indicated reification. To reach the endorsed quality, that is, what a person endorses as true about herself, Elin read this article carefully and confirmed that the quoted narratives reflected the state of affairs. As Elin was the author of the stories, they were considered as significant for her.

In order to clarify why these particular narratives were told or written at particular times the data was arranged in chronological order on a timeline. Events and incidents at the school were added on to the timeline with the purpose of relating the stories to immediate classroom situations (Wedege, 1999) and to the wider school context. Our collaborations and reflections from the students' blog, e-mails and information from my field notes were attached to the timeline. The purpose with this exercise was to connect Elin's identities narratives with other aspects of the wider school context. Finally, stories that indicated action such as when Elin took decisions about her teaching where added to the timeline. In this way the timelines consisted of narrated identities, decisions and actions told in relation to different contexts.

The ways that Elin narrated herself changed in relation to contexts, constraints, relationships, and time. From the timeline, clusters emerged and these were labelled 'critical moments'. The 'critical moments' showed how events and incidents at the school impacted on Elin's teaching identities through changes in her narratives. Drawing on the idea of learning as the bridging of the gap between actual and designated identities (Sfard & Prusak, 2005), Elin is likely to engage in learning when she recognises a difference between the teacher that she wants to become and the teacher that she is currently being.

The contextual background

Situating the school context

Elin is a qualified teacher who had taught mathematics and psychology in upper secondary school for five years. We met at the time she replaced me at Ericaskolan where I taught for eight years before becoming a teacher educator. Locating Elin's experiences in the socio-cultural context of the school provides a better understanding of the complex situation in which Elin and I operated as we considered how the teaching could be re-organised. The school presented itself through policy documents as a school with creative and motivated teachers, where students participated in project work and had possibilities for personal growth and learnt to communicate knowledge in a convincing way (Ericaskolan's homepage, retrieved 10/1/2010). As part of their jobs, teachers also were expected to learn. In an interview the school principal emphasized that teacher professional development was important:

> I believe that in the long run you get better teachers [in this school] if they are allowed to test and experiment. These kinds of things [professional development and teaching in new ways] results in that you take away the conservatism in students and make them receptive for news and ways of working that will be good for them. It is challenging for the teacher colleagues as well (Principal, interview, 06-2009)

From the principal's response and the school's policies, we felt that teacher development, through collaboration was seen as being beneficial both for teachers and students. The principal stressed that he trusted the teachers' professionalism when trying out new ways of organising teaching and that testing new teaching methods was consistent with the school objectives. Since professional development and explorations of new teaching ideas was an expectation from the school leadership, support for a different way of teaching mathematics was given by the school. However, the students often were vocal about their dislike of mathematics. At Ericaskolan, there were a large number of students who chose a social science program for their three-year upper secondary schooling. Students commonly decided on this program because it provided a stable base for entry into university studies within the social sciences or language faculties. Another reason for taking the social science program was that the students did not enjoy mathematics and thus this program was a good option instead of the alternative natural science program. For instance, Petra, a 15-year old girl, decided to be present, "but do as little as possible so I just pass the Mathematics A course because I hate maths" (Petra, interview, 09-2009) or Malin who chose this program because she wanted a good base for further studies, however excluding mathematics: "What I am sure of is that I will not study anything containing mathematics. I would rather choose to study something societal as human rights or dance and music!" (Malin, survey, 08-2009).

A newly published quality report from the Swedish Schools Inspectorate (2010) documented reasons for why a number of students struggle with passing the Mathematics A course: Mathematics teaching is not obviously connected to students' chosen study program as stated in the national curriculum, and students' individual work dominates mathematics lessons resulting in mainly mechanical procedural calculations with lesser time for discussions, collaborations and problem-solving. As a consequence, students do not develop central competences such as

creativity, critical reflections and understanding of mathematical relationships. The report concludes that teaching seems to result in under-stimulated students, who experience mathematics as a boring and even "stupidizing" ["fördummande" in Swedish] (p. 8) subject. The school's requirement of teacher development and the conclusions in the Swedish Schools Inspectorate's report (2010) resonated with Elin's wish to develop her teaching further in this particular Mathematics A course.

Similar to Clarkson, Bishop and Seah's (2010) findings, Elin was concerned about her students' feelings that mathematics education was meaningless. In her view this meant they took no responsibility for their personal learning. In part this lead to Elin wanting to change her teaching:

> I have so many thoughts of how it could be [instead]. I have a feeling I only keep everything floating all the time. I want to change things because I am not satisfied with what is going on. But I don't really know how I can do it. Then I think this [research] project, the reason for why I want to do it, is that here might be an opportunity [for me] to try developmental work that feels more "safely anchored" (a Swedish metaphor that suggests the opposite to floating) and in a way makes it possible for me to see what happens. Because it's just this type of developmental work I want to do all the time and think what happens here, and how does it work, [both] in relation to myself and the students. (Elin, interview, 09-2009)

External reasons such as expectations from the principal and parents that a higher proportion of students should pass the course also affected her decision. Elin wanted a transformation. Yet she also understood she would need to allocate time for planning and evaluation if this was to occur. Further she wished for external support, particularly in thinking through how to change but still remain within the mathematics education curriculum and assessment boundaries. This became the starting point for our collaboration.

So, at the start of this research project it seemed as if all the necessary conditions were fulfilled for teachers to successfully change their teaching practices (Clarke, 2007; Fullan, 1991). Elin had identified why and in what ways she wanted to change her teaching, she got support from the school organisation to work in the way she intended and she had a substantial degree of ownership of the process. We were also aware that changing teaching practices could become a problematic process, so Elin and I had agreed that, as the researcher, I was to be a conversational and supporting partner like a coach or mentor (Kram & Ragins, 2007). At the beginning, the project and the planning of the teaching was a joint activity, but when Elin asked for more support or when her time was restricted I took on some more of the planning. Regardless of the circumstances, Elin always retained control of the final decisions relating the content to the curriculum and assessment. My role was one of encouraging and supporting, but also included pushing the process further, so that research goals for the larger research project were not lost during the tough times. In the classrooms Elin was the teacher with the classroom authority and she chose to position me as a researcher and assisting teacher, although with no assessing or grading obligations. I participated in the lessons, worked from a desk placed at the back of the classroom, but also actively talked to students and answered their questions.

Situating the research in (critical) mathematics education

The introduced pedagogy was designed with inspiration from the concerns for critical mathematics education. In particular this meant attending to three aims. Firstly, we wanted to design the pedagogy so that it contributed to "relating mathematics to citizenship", "understanding mathematics as a tool for identifying and analyzing critical features in society" and "seeing mathematics both as a tool for critique and an object of critique" (Skovsmose & Nielsen, 1996). These aims of critical mathematics education fitted well with the goals of the Swedish curriculum. The curriculum asserted that mathematics education for social science students should "provide general civic competence and constitute an integral part of the chosen study orientation" and be "of importance for everyday life and their chosen study orientation" (Swedish Ministry of Education, 2000). Secondly, the educational practice was considered to involve learning and becoming, rather than a simple transmission of a body of knowledge (Skovsmose, 2005). Our third aim was about ensuring that the power relations supported a classroom environment where students could be more agentic in relation to their mathematics learning (Biesta & Tedder, 2006). To accomplish this aim, a different way of talking and behaving between participants in the classroom was required. We wanted to structure the classrooms in ways that positioned the students so they had access to and contributed to the discourse. Ernest (2002, p. 8) had suggested:

> The aims of critical mathematics, require the use of a questioning and decision making learning style in the classroom. Teaching approaches should include discussions, permitted conflict of opinions and views but with justifications offered, the challenging of the teacher as an ultimate source of knowledge (not in their role as classroom authority), the questioning of content and the negotiation of shared goals.[…]. Also the learners should be given the chance to pose their own problems and initiate their own projects and investigations at least some of the time.

Elin wanted to bring societal issues into the mathematics classroom and establish an environment where students could make choices about their learning and thus be agentic in a way that stimulated their mathematical understandings. As well, the mathematics lessons would be connected to other school subjects and objectives in their chosen social science study program. In order to achieve these changes, Elin, the students and I had to take on different roles and responsibilities in how we related to each other and how we talked about the discipline of mathematics itself (Skovsmose, 2001; 2005). We intended to develop a discourse shift that supported students to take their own decisions for their learning of mathematics. For example, we developed projects aiming at encouraging students to make choices about their learning, task contexts and about mathematics per se (Andersson & Valero, 2011) within what Skovmose (2001) labelled an *investigative learning milieu*.

Nevertheless, successful implementation of such a discourse was a challenging task. The complexities involved in establishing a different pedagogical discourse are always considerable (see Franke, Kazemi & Battey, 2007). Together, we rethought participants' roles and responsibilities within the classroom discourse, and gradually implemented the different discourse. The role of the teacher became different and also difficult since there was little guidance about how to enact such a role in curriculum documents (Hunter, 2010). From a

teacher's perspective it might appear as moving out from a teaching "comfort zone" into a "risk zone" of uncertainty (Skovsmose, 2001, p.130). The experience of uncertainty became important to acknowledge in relation to our collaboration but also for the reasons Elin had to take particular decisions at different points in time.

Critical moments and change in identity

In what follows, three vignettes are provided to illustrate examples of changes in Elin's way of talking about her as she adapted her teaching. The vignettes illustrate how classroom incidents and school organisation occasions impacted on Elin's identities or identity narratives, at particular times. I want to underline that these vignettes are not to be seen as static critical moments, such as photographs, they are rather understood as pausing a movie, zooming in and analysing events at those three particular times.

The first critical moment: Telling an uncertain identity

A couple of weeks into the semester the first project, labelled "Making your dreams come true", commenced were the students got spaces for deciding on task contexts, time and work distribution. In addition to the mathematical content; interest rates, percentage and repeated percentage calculations, they were suggested to reflect on what they personally might borrow money for, how to find out boundaries for high or low interest costs or if loan offers were good or not. This was a critical topic as the news head lines at this time focused that an increasing number of young people (18-25 years old) got in financial trouble after taking "quick-money" loans received through e.g. mobile phones texts. If youngsters get into enforcement or police registers for not paying back debts, it decreases their possibilities to get bank loans for further studies or a house later in life.

During the planning stage Elin's focus was on the students, who were new to the school, to each other and to their teachers, and what she thought was their beliefs about mathematics education. Elin wanted the students to experience recognition, thus she understandably introduced the new pedagogy gradually. Elin was also worried about whether students would attain the curriculum objectives. As she said:

> Well, I don't know if it will work, as I have not taught in this way before. I have tried small-scale problems, but then I thought those problems became 'pseudo-problems', not real problems. I was uncertain about what the results would be if I did larger projects and what base I then would have for examinations and grading (Elin, interview, 09-2009).

Her actual identity at this time was told as e.g. "I am uncertain of…", "I am worried about…" or "I don't know what…" (Elin, interview and conversations, 09-2009) indicating uncertainty. Her designated teacher identity was characterised in three different ways. First, she wanted the students to learn mathematics within the frames given by curriculum and examination objectives. Second, she wanted the students to become agentic in their learning of mathematics: "For me it is important to clarify what mathematical goals we are working towards, and that the student can experience personal development and evaluate their learning" (Elin, interview, 09-2009). Third,

she also wanted the students to experience mathematics lessons as meaningful: "[The students usually] want to know "What are we going to use this for?" My experience is that when they have worked with maths that is useful in every-day life they experience mathematics as meaningful" (Elin, letter, 08-2009). She later concluded:

> My vision is that all students shall feel that they expand their knowledge base and their
> problem solving skills. I want them to get opportunities to work together and that they feel
> the mathematics they learn today might become useful for them (Elin, interview, 09-2009).

So, Elin decided that students would start a collaborative project work but also hand in individual textbook exercises for assessment. It was not easy to change the classroom discourse, and doing it gradually with mixing the old and new intended discourse did not simplify the process. There were two different discourses playing at the same time, one that appreciated students' achievement of agency and one that potentially restrained this achievement. This mix of discourses was problematic and some discipline problems occurred when some students did not understand how to navigate between the different discourses, or in both at the same time. Elin had a respectful and clear conversation with them about these issues. She wanted the students to reflect on possible outcomes from their different choices about how to behave, and what she expected from them if they wanted to pass the course and continue their studies. Some students chose to change their behaviour but some did not.

A critical moment was when Elin silently wrote on the whiteboard at the end of a busy lesson, at 17.00 on a Tuesday afternoon, "Those of you who don't attend, present your project and pass next week will have to do a written math test" (researcher, field notes). Zizzi, one of these students commented on the blog:

> This was really meaningful and it was good to take personal responsibility for planning and
> for our own labour distibution. *But this is new; we have to practice this way of working*
> (Zizzi, blog comment, my emphasis)

In this case Elin decided to take responsibility for the students' learning by giving them the option: participate, present and hand in or take a written mathematics test. She reflected at the end of the semester on this decision:

> I would not do the percentage project again in the way that they had to hand in exercises
> from the book as well [as the project presentation]. But we had that discussion before [we
> started] and then I chose to bring in the book-part to feel sure about that they did something.
> It was a control point for me. (Elin, interview, 12/2009)

The contextual openness and rich possibilities for the students to make decisions, especially during project work sessions made Elin concerned about students not participating. She was concerned that students' non-participation would result in a number of them not passing the course. Elin is not unique in having these concerns. A rich body of teacher change research has documented these issues (e.g., Fullan, 1991; Garet et al, 2001). At this point in time Elin started to reflect on herself as being a "curling[7] teacher". The metaphore of a curling teacher is

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[7] Curling refers to the winter ice sport where competitors sweep the ice in front of a stone to smoothly get it in the right, or best, position.

transferred from the idea of a "curling parent"; in Sweden a generalising objectification for parents who serve and 'sweep the way' for their children, solving possible problems and tensions beforehand and thus make their children's lives as smooth and easy as possible (Hougaard, 2004). Elin explained:

> I am fighting with the feeling that this in some way can contribute to them not doing the exercises and hence not reaching [curriculum] goals, and not passing the course. Maybe this is a "curling behaviour". Sometimes I feel like a "curling teacher". I bring extra calculators, extra books and extra papers and pencils. In what way does that support the students becoming independent and taking responsibility? (Elin, email, 22/9/2009)

We learnt during this initial period the importance of how we positioned ourselves in the classroom in relation to each other and to the students. We reflected together on the ways we talked with and answered the students and how responsibilities and expectations were distributed in the classrooms. During periods when the students worked with textbook exercises, usually in contexts rarely connected to the students' everyday life (Andersson & Ravn, in press), they tended to "ask for help" and expected Elin to explain and know the right answers. During project work the students' questions tended to be of other characters. They seemed to be aware of that the teacher did not know all "the right answers" and that there were different ways to reach solutions. They rather wanted to discuss and negotiate and/or get Elin's opinions on their questions (Annica, fieldnotes). In fact, during sessions when the teaching was organised with textbook work the positioning and power relations seemed to be as Elin said "stuck in the walls" (Elin, conversation, 11-2009) "Stuck in the walls" is a Swedish metaphor, indicating that the (school) walls implicated certain ways of talking and acting learnt through prior socialisation processes. As Hargreaves (2000) pointed out, the structures of schooling have become so institutionalized over years that they define the essence of schooling itself for the teachers and the students who work there.

Elin's teaching identities changed during these first weeks. It was a time of uncertainty. She struggled to give students opportunities to take decisions and thus become agentic in their learning of mathematics, as she had planned to do. On the other hand, she became aware of the constraints that the curriculum and examinations put on her teaching. Her "curling identity" as leading the class, and interfering in the students' decisions and work became obvious to her and consequently questioned by her. She became aware of the constraints and that helped her to restructure her understanding of the classroom. But at the same time this might not contribute to her being able to become the teacher she originally anticipated she wanted to be, that is to close a gap between an actual and designated identity. Or in other words, her designated identity was changing as she realised it might be problematic to become a teacher who taught mathematics within curriculum and examination boundaries while at the same time wanting students to become agentic in their learning and experience mathematics lessons as meaningful. Through adapting her designated identity to present circumstances she learns, but not what she anticipated to learn.

The second critical moment: Telling a "Not-being-a-teacher" identity

Inspired by Frankenstein's (2008) work on quantitative numbers we invited the students to a full day workshop in critical mathematical argumentation one week into the second month of our collaboration. The "Newspaper Flyer Project" was themed on the United Nations Convention on The Rights of the Child. This project began as a consequence of a large cross-subject project run in the school that focussed on young peoples' prejudices. Mathematics was not included as one of the subjects that contributed to the project as teachers in charge of the larger project thought that "if mathematics was included, this would complicate the projects so they would take up too much time, and honestly I can't see how mathematics could fit in" (Subject teacher, interview, 09-2009). Another teacher confessed that her experiences of mathematics education when growing up made her "feel awkward if I am expected to do correct mathematics" (Language teacher, interview, 09-2009). This was the case even though the students had to conduct a survey in the larger project. As a reaction to being excluded from the larger project, Elin suggested the theme 'Children's Rights' for our work-shop. It had a direct link to the larger school project and thus would support the students, as well as connect mathematics to an important societal theme. Ironically, the "other-subject" teachers' prior experiences and/or prejudices about mathematics complicated Elin's intentions for cross subject projects.

To complicate the situation even further at this time, Elin was upset by a discussion with the school principal. I received the following email from her:

> On top of everything I got very upset today in a conversation with Karl-Gustav [the principal]. We discussed possible "mattestuga"[8] and work distribution. I have, according to the work distribution, five percent 'remaining time' in my duty. I know that in prior conversations with Karl-Gustav, we discussed that the work I do with Annica should be counted in, as it requires both reflection and time. Surprise, surprise, when Karl-Gustav said that this was nothing that had been put forward as he had understood that this wouldn't take me extra time. "Then we have to rethink". I can't understand how Karl-Gustav can say that development work does not require any time and that it is still important to have possibilities to reflect on ones work. When does he think there will be time for that? You have to excuse me Annika, but my first feeling was to tell Karl-Gustav that we close the project and you instead turn to another school. Not that I don't want to continue, but it is a problem when the work we do doesn't get valued, and yet it SHALL be done. Sigh, will tell you more about this tomorrow... (Elin, e-mail, 25/9/2009)

Her choice of words, short sentences and wrong spelling of my name (compared with earlier e-mails) suggest that she wrote the e-mail when she was upset. In this semester her first hour without teaching duties was scheduled late on Wednesday afternoons. She needed time without teaching obligations for planning, preparations and marking involved in the new way of working, as well as for the other classes she taught. Elin had high teaching responsibilities, too many different classes in different courses and long distances between classroom locations. Yet there did not seem to be anything that could be done about the situation. According to the union

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[8] A "mattestuga", a "maths cottage", is a possibility for students needing extra instructions in mathematics and thus mathematics teachers had weekly scheduled hours for "mattestugor".

representative the school leader was aware of the situation. At this point, school organisational issues almost resulted in the project abruptly ending, despite the school leader's good intentions for teacher developmental work. Subsequently, Elin and I resolved part of the problem by my taking on the task of sketching out the teaching ideas. Elin always made the final decisions, added examination information and remained in charge of the courses. During this particular time Elin reflected seriously on looking for other work places and/or leaving teaching. She applied for further university courses in psychology (Elin, conversation, 10-2009).

Elin expressed her identity at this stage as either "not being a teacher" or "working in another school" (Elin, conversation, 10-2009). Colleagues constrained her wish to teach within cross-subject projects, and issues on school organisation level impacted on Elin's opportunities to organise her teaching in a way she aimed for. As a researcher I learnt about the complexity of issues that impacted on Elin's ordinary school life, and I realised I needed to adapt my intentions if the research project and Elin's change of practice was to proceed.

The next section reveals a critical moment two months later in time when Elin succeeded in organising and commencing a larger cross-subject project.

The third critical moment: Telling a confident identity

A large number of the news headlines during this autumn semester where related to news reports from COP 15, the UN Conference on Climate Change in Copenhagen. The school, situated in southern Sweden, got affected by this conference in different ways and we decided to use the themes of the Copenhagen meeting; climate change and climate sustainability for a statistical project. The environmental science teacher was about to start a project on sustainable development and ecological footprints, so we efficiently joined these projects together. Within this project students had opportunities to make decisions on time and work distribution as well as task contexts and assessment. The project began with the students designing their investigations. At this time, the students worked very intensively with great energy. However, Elin felt that they were working without reflecting on how a good survey was conducted. Elin reflected:

> It was hard for me to feel that I could stand back and not interfere with their work. Especially when they formulated their investigation questions I wanted to start explaining [how to do them better] before they had thought it over themselves (Elin, conversation, 12/2009)

Then, Elin became sick and stayed home from school for two days. We mutually agreed to keep the project going within the given time schedule. Consequently the project continued with me supporting in the mathematics classes during these two days. When Elin came back to school, the students requested her presence in the environmental science classroom because they had assessment questions, which were Elin's responsibility to answer. At this time circumstances meant that her perceptions of the situation changed:

> Annica: Elin, tell me how you experienced the project when you came back to the science class.
>
> Elin: Just then, I was only there to clear some questions... but... everybody was so into what they were doing. Everybody sat working, *independently*, it was awesome to see the difference from the introduction. I had only seen one session with that group, then you had

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had two, but it was any way a very big difference between the first introduction when they threw themselves into [...] it felt as if they had been over a peak and worked down slope [a Swedish metaphor for having flow, working smoothly and efficiently] now. And they were *proud* of what they were doing. (Elin, interview. Italics indicate the teacher's emphasis, /12/2009)

This was a turning point for Elin and her reflections on her teaching. Her identity narratives about her in the classrooms after this experience were clearer and more confident e.g. a relief was sensed in her way of being and engaging with the students (Annica, field notes). An email arriving a month after I left the school confirmed this point:

Reflecting on my personal learning process is difficult. I think I have been influenced in a way that I feel more freedom and more inspiration in working in different ways in relation to the curriculum goals. [We have worked with] projects, group work [and] oral assessment. I have also been thinking quite a lot about my teacher role. I am very concerned that the student shall feel they have enough support from me. I have done extra "mattestugor", given them special exercises […] one can say I have worked quite hard with those things. But, during the [autumn] semester I have been thinking if all these really support the students. How does their independence and autonomy develop if they always get everything served to them? Obviously I, as a teacher, am there for the students, but in what way? During this [statistical] project I have been fighting a bit with letting go of the responsibility to the students. Letting them seek information on their own, ask for help when needed – plainly being responsible for their own learning. Looking back, now when the new spring semester has started I think the students, even when we do book calculations, seem to have learnt a lot by working in the way we have done. With that I mean they use the book and work more together. Not always, and not everybody – but many! (Elin, email, 17-01-2010)

In this paragraph Elin illumines three different important points in relation to her learning. First, she states that she feels "more freedom and more inspiration in working in different ways in relation to the curriculum goals". Second, she questions her role as being a "curling teacher" and reflects on how she serves the students in the best way for them to develop competences as independence and autonomy. Third, reflecting on students' learning there was a tendency for students to adapt a collaborative way of working also during teaching organised with textbook work. Her prior actual identity, characterised by uncertainty, now seams transformed to a new actual identity by words as "feeling more freedom" and "inspiration". When realising the tensions within her prior designated identities; being a teacher teaching within curriculum frames with students becoming agentic and experiencing meaningfulness in mathematics education she now accepts those tensions. However, her learning resulted in new emerged designated identities through the new questions articulated above. Identities continuously change, and her learning continues. Concluding what we learnt when step by step introducing larger project work was that Elin became confident with her different way of organising her teaching and different ways of acting. She continued to organise her teaching with these concerns in mind even after I had left the classroom. There were no more opportunities for her to commence cross-subject projects, but within the mathematics classroom she continued with smaller societal projects and mathematical activities and kept spaces open for students to gain ownership in their learning of mathematics.

Concluding remarks

The focus of this article was on processes of identity change with the aim of understanding issues that supported or hindered Elin to change her teaching and how success in teacher development and sustainability can be achieved. Fullan (1991) talked about stable teacher change and sustainability. In Elin's case we can queer: What is actually being sustained? The question of success and sustainability is questioned. Elin's identity narratives and learning suggests that researchers might have to rethink what we mean with terms such as change and success, and if sustained development is possible. When Elin learnt about the constraints within her prior designated identities; being a teacher teaching within curriculum frames with students becoming agentic and experiencing meaningfulness in mathematics education, she changed her designated identity. However, her learning resulted in new emerged designated identities through the new questions articulated above. I would suggest that the continuous learning Elin is involved in might be a success in itself, even if that was not the learning we originally anticipated as a result of the collaboration.

Elin had initially identified why and in what ways she wanted to change her teaching and she received support from the school. Thus, she had a substantial degree of ownership of the process. Still, other issues, initially not-participating students, problematic time or work distribution and colleagues' attitudes to mathematics, influenced her decisions and actions. These issues, being part of the social, cultural and political constitution of Elin's ordinary school life, were not all possible to foresee and control. The ebbing and flowing of incidents at the school impacted on her teaching identities at particular times, and her designated identities changed when she became aware of the constraints within them. Elin's changing way of narrating herself, as e.g. being the caring, 'curling-teacher' in the beginning of the semester, to the still caring, but less-curling teacher became a balancing act between smoothly giving up some caring duties, recognising the students' voices in the classroom while at the same time acknowledging the risk of students failing their mathematics education. The way Elin's identities developed during this process of change indicated that, for example, her learning resulted in her becoming a mentoring, 'less-curling' teacher when her designated identities became clearer and articulated. Her learning was described by her as sensing freedom in her teaching, reflecting on new possibilities and reflecting on different responsibilities different actors in the (mathematics) classrooms have (Elin, email, 17-01-2010). Compared with Lasky's (2005) findings about teachers experiencing constrained agency in a reform context, Elin's experience of sensing freedom and inspiration are in sharp contrast.

The process of forming a collaborative partnership together with the researcher offered both the teacher and the researcher learning opportunities. The change in Elin's teacher identities through the different stages of the process confirmed that teacher development work requires time to plan and reflect (Hargreaves et al., 2001). The development process almost came to an end in the middle of the semester due to lack of support by the school leadership. In trying to overcome other needs within the school the principal did not recognise the necessity for such processes and the time they take. If changes in mathematics education are to prosper as suggested by policy writers and politicians, then Elin's experiences verify that organisational structures play an important role and need to be taken seriously (Fullan, 1991; Valero, 2007).

As a researcher I learnt about the complexity of issues that impacted on Elin's ordinary school life, and I realised that an adaption was required from me if both the research and Elin's change of practice was to proceed. The learning I as a researcher originally anticipated was to grasp how students' identities are told in mathematics education. However, my learning also resulted in an awareness of the social, cultural and political embeddedness of identity narratives in the multiple contexts where teachers meet. Seeing learning in terms of changes in the way we narrate our experience opens up to consider in serious ways in our analysis the relationships between the participants and contexts of mathematics education.

Acknowledgements

References

Andersson, A. (2010). Making Sense of Critical Mathematics Teaching. In L. Sparrow., B. Kissane,. & C. Hurst (Eds), *MERGA33. Shaping the Future of Mathematics Education. Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia* (pp.51-58) . Fremantle: MERGA Inc.

Andersson, A., & Valero, P. (2011). Negotiating critical pedagogical discourses. Stories of contexts, mathematics and agency. In P. Ernest and B. Sriraman (Eds), *Critical Mathematics Education: Theory and Praxis.* USA: Information Age Publishing.

Andersson, A., & Ravn, O. (In press). A critical perspective on contextualisations in mathematics education. In O. Skovsmose & B. Greer (Eds.), *Critique and politics of mathematics education*. Rotterdam: Sense Publishers.

Atweh, B., & Brady, K. (2009). Socially response-able mathematics education: Implications of an ethical approach. *Eurasia Journal of Mathematics, Science & Technology Education, 5*(3), 267-276.

Biesta, G., & Tedder, M. (2006). How is agency possible? Towards an ecological understanding of agency-as-achievement. Working paper 5. Exeter: The Learning Lives project.

Black, L., Mendick, H., & Solomon, Y. (Eds.), (2009). *Mathematical relationships in education. Identities and particiption.* New York: Taylor and Francis.

Clarke, D. (2007). Ten key principles from research for the professional development of mathematics teachers. In G. C. Leder & H. J. Forgasz (Eds), *Stepping stones for the 21st century. Australasian mathematics education research* (pp. 27-37). Rotterdam: Sense Publishers.

Clarkson, P.C., Bishop, A., & Seah, W.T. (2010). Mathematics education and student values: The cultivation of mathematical wellbeing. In T. Lovat, R. Toomey & N. Clement (Eds.), *International research handbook on values education and student wellbeing* (pp. 111-135). Dordrecht: Springer.

Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of Mathematics Education Journal, 15* (1), 1-16.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Understanding teaching and classroom practice in mathematics. In F. K. J. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 225–256). Charlotte, NC: NCTM.

Fullan, A. (1991). *Leading in a culture of change*. San Francisco: Jossey-Bass.

Hargreaves, A. (2000). Four ages of professionalism and professional learning. *Teachers and teaching: History and practice. 6*(2), 151-182.

Hougaard, B. (2004). *Curlingföräldrar och servicebarn - en handbok i barnuppfostran*. Stockholm: Bokförlaget Prisma.

Hunter, R. (2010). Changing roles and identities in the construction of a community of mathematical inquiry. *Journal of Mathematics Teacher Education*. DOI 10.1007/s10857-010-9152-x.

Kelly, P. (2006). What is teacher learning? A socio-cultural perspective. *Oxford Review of Education 32*(4), 505–519.

Kram, K.E., & Ragins, B.R. (2007). The landscape of mentoring in the 21st century. In K.E Kram & B.R. Ragins (Eds), *The handbook of mentoring at work: theory, research, and practice* (pp. 659-689). Thousand Oaks: Sage Publications Inc.

Lasky, S. (2005). A socio-cultural approach to understanding teacher identity, agency and professional vulnerability in a context of secondary school reform. *Teaching and Teacher Education, 21* (8), 899-916.

Radford, L. (2008). The ethics of being and knowing: Towards a cultural theory of learning. In L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in mathematics education: epistemology, history, classroom, and culture* (pp. 215–234). Rotterdam: Sense Publishers.

Robinson-Pant, A. (2005). *Cross Cultural Perspectives in Education Research*. Maidenhead: Open University Press.

Sfard, A., & Prusak, A. (2005). Telling identities: In a search of an analytical tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4)*,* 2-24.

Skovsmose, O. (2001). Landscapes of investigation. *ZDM 33*(4), 123-132.

Skovsmose, O. (2005). *Travelling through education: Uncertainty, mathematics, responsibility*. Rotterdam: Sense Publishers.

Skovsmose, O. & Borba, M. (2004). *Research methodology and critical mathematics education*. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology* (pp.207-227). Dordrecht: Kluwer Academic Publishers Group.

Skovsmose, O. & Nielsen, L. (1996). Critical mathematics education. In A. J. Bishop, M.A Clements, C. Keitel & J. Kilpatrick (Eds.), *International handbook of mathematics education* (pp. 1257-1288)*.* Dordrecht: Kluwer Academic Publishers.

Solomon, Y. (2009). *Mathematical literacy. Developing identities of inclusion.* New York: Routledge.

Swedish Ministry of Education, (2000). Goals to aim for in mathematics education in upper secondary school. Retrieved from www.skolverket.se

Valero, P. (2004). Socio-political perspectives on mathematics education. In P. Valero & R. Zevenbergen (Eds) *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology* (pp. 5-24). Dordrecht: Kluwer Academic Publishers.

Valero, P. (2007). In between the global and the local: The politics of mathematics education reform in a globalized society. In B. Atweh, A. Calabrese Barton, M. Borba, N. Gough, C. Keitel, C. Vistro-Yu & R. Vithal (Eds.), *Internationalisation and Globalisation in Mathematics and Science Education* (pp. 421-439). New York: Springer.

Valero, P., & Stentoft, D. (2010). The 'post' move of critical mathematics education. In H. Alrø, O. Ravn, & P. Valero (Eds.)*, Critical mathematics education: Past, present and future* (pp. 183-195). Rotterdam: Sense Publishers.

Wedege, T. (1999). To know or not to know - Mathematics, that is a question of context. *Educational Studies in Mathematics, 39*(1-3), 229-249.

14. A PHILOSOPHICAL PERSPECTIVE ON CONTEXTUALISATIONS IN MATHEMATICS EDUCATION

INTRODUCTION

> It is because you teachers walk up front and say: This is how it is – and here you have the book. That's how it has always been since ages back. It's the book that counts, and that is how it is. It's hard to break that pattern. One has always had the book to refer too… (Katja[9], a 16-year old upper secondary student sharing her experiences of mathematics education in an interview with Annica)

Mathematics is often taught as a subject that can be presented in a clear structure and abstract form within the confines of a textbook resembling what Cobb, Wood, Yackel, and McNeal (1992) referred to as the "school mathematics tradition". Katja quoted above is, thus, probably not unique in her description of her experiences in mathematics education. Because of the special characteristics of mathematics there might not initially seem to be any obvious reason for introducing students to the troublesome complexities of contextualisation where mathematics is, for example, used in a practical setting, a cross-disciplinary project or the like. Learning the core skills of mathematical calculations can be more than enough of a challenge for both teachers and students.

Sometimes, however, serious attempts are made to contextualise the mathematics to be learned and this is frequently done for motivational reasons (for an example, see Boaler, 1993). Students in upper secondary mathematics classrooms (at least in the Swedish context) are usually invited, through textbook problem solving exercises, to reflect on more or less real-life situations where the mathematics learned can be applied to a predefined task. This is one way to contextualise mathematics in a school setting. But other forms with other purposes are also tried out in practice from time to time.

In this chapter, we discuss the contextualisation of mathematics[10] in a school setting, focusing on upper secondary students in Sweden that are faced with different types of contextualisations in mathematics education. In the following we explain our approach to, and perspective on, this discussion.

"Context" in mathematics education

There is a rich body of research conducted on context/ contextualising/ contextualisation of mathematics education from a number of different perspectives. To give some examples, first the ethnomathematical movement (D'Ambrosio, 1985, 2006) should be mentioned, that strongly advocates the importance of cultural contexts in mathematics education, understanding mathematics in cultural context as mathematics "which is practiced among identifiable cultural

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[9] All names of students in this chapter are pseudonyms.

[10] Wedege (1999) distinguishes specifically between situation context and task context in mathematics education. The first meaning of context, "situation context" is by Wedege understood as social and historical matters and relations as "contexts of learning". In this paper we understand context in mathematics as "task context" in Wedege's definition: "In this sense the word is often normatively employed, e.g. in curriculum documents as a requirement that teaching and materials shall contain 'real-life context' or 'meaningful and authentic contexts' " (pp. 206-207).

groups such as national-tribal societies, labour groups, children of a certain age bracket, professional classes, and so on" (D'Ambrosio, 1985, p. 45)[11].

Second, a societally oriented research movement is connecting mathematics with societal contexts. Critical mathematics education (see e.g. Gutstein, 2006; Skovsmose, 2005) and the work by researchers like Atweh and Brady (2009), Frankenstein (2009), Valero (2007), Vithal (2003) and many others are examples of researchers advocating for taking such concepts as equity, social justice, empowerment, and democracy seriously in mathematics education. Research presented at the Mathematics Education in Society (MES) biennial conferences foregrounds mathematics in society from different points of view.

In this chapter, we attempt to add a philosophical argument to the many different discussions relating to context and contextualisation in mathematics education. The idea we bring to the fore is not to see the issue of using contextual situations in mathematics education from either an ethnomathematical perspective or from a critical mathematics education perspective. Instead we argue within a philosophical framework for the relevance of contextualising school mathematics education beyond the confines of textbooks. Through a philosophical theoretical perspective, and a case study highlighting the impact of different types of contextualisation techniques in teaching, we build an argument that emphasises the importance of connecting the teaching and learning of mathematics with the problems, visions, conflicts, developments, and different spheres of life beyond the mathematics textbook.

This study is, therefore, drawing on the philosophy of mathematics education, with the particular aim of producing an interpretation of how we can understand the notions of the "core" of mathematics and the "context" of mathematics in relation to mathematics education.

The aim of the chapter

The situation in Sweden, as in many countries, is that students need to take compulsory mathematics courses in connection to their major subject and their choice of study path in upper secondary school. In these required courses, mathematics is not necessarily taught as a subject that clearly involves the surrounding world of affairs or has any close relation to other sciences or subjects in the education programmes. In general, mathematics in Sweden is studied as a subject rarely connected to the students' everyday life (see, e.g., Johansson, 2006) and this rather radical choice of content in mathematics education is what we will discuss.

In this chapter, we aim for a conceptual clarification of why there are no paramount arguments to support the idea that mathematics education is best taught in isolation by being focused on abstract and isolated syntactical approaches and problem-puzzle contextualisation assignments. Instead, we argue, drawing on a human-centred conception of mathematics, that a strong argument can be made for using student-centred contextualised approaches in mathematics education. Not because it may provide a strong motivational force but because it is in line with the nature of mathematics to do so.

After the philosophical, theoretical discussion we focus on a case study relating to two different types of contextualisation approaches in an upper secondary school setting for social science students in Sweden. The first type discusses contextualisations indirectly proposed through a particular mathematics textbook used in the social science students' mathematics course. The second type relates to examples of a student-centred approach to contextualisation in the same mathematics classroom. Building on this case study, we finally discuss how a student-centred contextualisation of mathematics can, in different respects, be advantageous considering the philosophical standpoint developed in the chapter.

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[11] Andersson (2007) has challenged mathematics education with a mathematics teaching grounded in ethnomathematical theories, understanding social science students as a cultural group in line with D'Ambrosio's definition.

THE "CORE" AND THE "CONTEXT" OF MATHEMATICS

In Western philosophy of mathematics there has been a strong tendency to understand mathematics as consisting of objects that can be studied in a parallel manner to the objects of, for example, physics or chemistry. In relation to these special mathematical objects the idea has been pursued – first by Euclid and his contemporaries and, after Euclid, many other mathematicians – that there is a kernel of mathematical knowledge that is fundamental and that only exists in its abstract form. Western mathematics has been preoccupied with axiomatic mathematics in which mathematics is presented as orderly and isolated from the doings of human affairs and any practical use of mathematics. Instead, especially in the European history of ideas, mathematics has been associated with the building blocks of the universe or the principles of nature as well as the universal and logical structures of human reasoning (we refer to Skovsmose and Ravn (2011) for an in-depth development of this argument).

In mathematics education today this persisting ideology about mathematics naturally has implications for the skills that students are expected to learn in the mathematics classrooms. Mathematics is first and foremost centred on abstract reasoning and what we could refer to as the "core" of mathematics, and only secondarily related to practical life and practical problems – what could be referred to as the "context" of mathematics. In education, the "core" is associated with assignments and problems with clear-cut answers and with no fluffy sources of error or lack of information or the like in the mathematical syntax or the problem solving. As Boaler (1993) wrote, historically mathematics has been presented as a subject of "absolute truths" with one correct answer to each problem.

As described above, the presentation of mathematics to students as abstract and essentially isolated from practice hinges, in our understanding, on a particular and ancient ideology about what mathematics is, following the tradition established by Euclid and his contemporaries. In parallel, contemporary writers note that an important force in the general conception of mathematics relates to the view that mathematics consist only in computation and formulas:

> A major inertial force holding back radical reform of mathematics education is the simplistic perception of mathematics prevalent among people in general, including politicians and other policymakers. Mathematics is commonly seen as consisting essentially of computation and formulas, yielding exact and infallible answers, without relevance to everyday life, accessible only by experts, and not open to criticism. Indeed, in many respects mathematics is commonly perceived as the antithesis of human activity – mechanical, detached, emotionless, value-free, and morally neutral. (Mukhopadhyay & Greer, 2001, p. 297)

This dominant idea about mathematics is, however, contested by a more human and socio-culturally oriented conception of what mathematics is, and we intend to highlight the characteristics of a particular position of this sort. We relate this human-centred idea about mathematics to the later Wittgenstein's conception of mathematics. Here we only outline some main characteristics of his conception and refer to Ravn and Skovsmose (2007) for a detailed discussion of the human-centred orientation in Wittgenstein's philosophy of mathematics.

Wittgenstein's ideas about mathematics have their foundations in a philosophy of language as outlined in his principal work *Philosophical Investigations* (Wittgenstein, 1983). He continuously sought an explanation for how our words have meaning – how they make sense to us – and he argues that it is only through the practices of human beings that different types of signs like numbers, equations, letters, words, gestures, outbursts, etc. gain meaning. No sign has a meaning in itself, according to Wittgenstein – only through our joint use of signs in what he refers to as "language games" will a sign be given a meaning. A language game can be about cooking, needlework, politics, scientific experiments but also about finding solutions to mathematical equations, adding two numbers, proving a theorem, calculating in tens, and so on. In Wittgenstein's interpretation, mathematics is to be understood as a family of language games sharing family resemblances in a crisscross network of mathematical practices. This, however, does not mean that mathematics is a game in which you can do as you think or like. It

characterises mathematics that it is exactly the family of language games where the syntax cannot be reasonably questioned. Instead, Wittgenstein refers to mathematics as the language games in which we decide on the measures that we shall use to measure the world: "What I want to say is: mathematics as such is always measure, not thing measured" (Wittgenstein, 1978, p. 201 [III-75]). Mathematics is understood as the language games whereby we negotiate which measures to use in relation to different practices. It therefore stands on a pedestal in relation to other sciences because it is in mathematics that we define what measures – which ways of calculating, which ways of deducing etc. – to use in many areas of life.

In arguing a language game oriented conception of the nature of mathematics, Wittgenstein destroys any notion of there being any mathematics that necessarily comes before other parts of mathematics – for example, in an abstract axiomatic structure. The notion of a "core" of mathematics thereby becomes problematic as referring to an entity that can firstly be reasoned about and secondarily applied in a particular practice. On the contrary, in the Wittgensteinian conception there is nothing that can be claimed to be the real or true core of mathematics. The extreme order and organisation of mathematical concepts, theorems, proofs, vocabulary, number systems etc. that we know today is a construction that make us forget that mathematics is in fact a network of language games wherein the symbols have gained meaning from the use of the mathematical signs in different practices, or in Wittgenstein's terms, language games.

In this human-centred understanding, the idea that mathematics exists in itself is therefore rejected. Mathematics is considered a completely human enterprise in which the mathematical symbols have no other meaning than the meaning different that types of mathematical communities – in research, in the classroom, in everyday life – give them, through practice and endless repetition in the diverse settings in which we use mathematics. It is essential for the argument of this chapter that there is nothing in the nature of mathematics that forces upon us the isolation of a sort of "core" of mathematics. Importantly, this human-oriented conception of mathematics also does not advocate the primacy of the "context" of mathematics. It states that these two poles or perspectives from which mathematics can be conceived are, in fact, intertwined and intimately connected and can only secondarily be divided into different domains.

We will refrain from going deeper into the Wittgensteinian arguments for his conception of mathematics and the many philosophical discussions and questions that revolve around his work. For such discussions, we point to the sources that we have used in this section from Wittgenstein's own work and for further insight into the turning point that we believe occurred in Wittgenstein's ideas on mathematics we also refer to Shanker (1987). However, it should be clear from the above that in this human-centred understanding of mathematics it is obvious that the learning of mathematics would necessarily seem to include acquaintance with practices in which mathematics is used. And the practice of learning the syntax of mathematics from this perspective seems to have a very limited scope, teaching students only a fragment of what mathematics is all about. It resembles a learning situation in which cooking or needlework would be taught only through the principles of these crafts in textbooks and never by engaging with the real thing.

In conclusion, a human-centred philosophy of mathematics imposes the importance of another set of educational practices that are directed towards mathematical reasoning in a given practical setting – contextualisations of mathematics – as opposed to a more abstract and isolated teaching of mathematics. In the following section we present a case study from mathematics education comprising two contrasting types of contextualisation approaches in mathematics education at the upper secondary school level. These two approaches are discussed throughout against the background of the philosophical ideas presented above.

A SWEDISH CASE STUDY ON TWO TYPES OF CONTEXTUALISATION

In Sweden, students choose from different study programs when starting upper secondary school around the age of 15. Different theoretical programs provide students with an academic foundation for further studies. There is also a variety of occupational and crafts programs to choose from. In this particular study, we focus on the compulsory Mathematics A course for students who have selected a three-year theoretical social science program. The research presented in this chapter is a cut-out slice of a larger research project by Annica, a mathematics education researcher. The planning and implementation of the described projects was done in collaboration with Elin, a mathematics teacher and mathematics education co-ordinator in the Swedish upper secondary school where the research took place (Andersson, in press: Andersson & Valero, in press, a).

The Mathematics A course covers mathematical contents such as arithmetic calculation skills, geometry, algebra, statistics, and linear functions (Swedish Ministry of Education, 2000). A problem with this particular course is that the mathematical content is, as we will see, not obviously connected to other subjects in the social sciences, even if the intention as expressed in the national mathematics curriculum states otherwise:

> The subject aims at pupils being able to analyse, critically assess, and solve problems in order to be able to independently determine their views on issues important both for themselves and society, covering areas such as ethics and the environment. (Swedish Ministry of Education, 2000, original translation).

Focussing on the goals for the specific Mathematics A course, we read:

> Pupils should be able to formulate, analyse and solve mathematical problems of *importance for everyday life* and *their chosen study orientation* (Swedish Ministry of Education, 2000, original translation, our emphasis).

Our interpretation of these ministerial guidelines is that mathematics teaching should give students mathematical knowledge and competences for taking well-grounded decisions in everyday life and to interpret the flow of information and thereby follow, understand, and participate in political discussions in society. Specifically in relation to the social science programs, we also understand the curriculum as arguing that mathematics education ought to be deeply connected to, and contextualised in relation to, the social sciences. There are several possible explanations for the Mathematics A course content not being clearly connected to the study programs. In our interpretation, this disconnect hinges on the dominant non-human-oriented idea about what mathematics is, as explained above. From the two ministerial quotations it is clear that the least problem in contextualising mathematics in the Mathematics A course is one of political backing and the laws of government relating to the contextualisation of mathematics.

Working with the ideas inspired by the later Wittgenstein, we now present and discuss two different language games for contextualising the syntax of mathematics in mathematics education in upper secondary schools. First, we address the contextualisation approach used by the students' mathematics textbook. Second, we address a contextualisation approach in which teachers and students are active in forming a contextualisation.

The school mathematics language game of textbook contextualisation

The mathematics textbook used in the school where this case study took place is one of the five most used textbooks in Swedish upper secondary mathematics education. The textbook is, according to the particular textbook editors, designed in a way that it meets the social science students' needs particularly well (Szabo, Larson, Viklund, & Marklund, 2007). According to the mathematics education co-ordinator, Elin, this claim was one of the main reasons for the school to use this textbook. The book covers the mathematical areas to be addressed in the Mathematics A course in accordance with the Swedish national curriculum of mathematics. The tasks and

exercises are on three difficulty levels, mostly related to the three different assessments and grading levels in Swedish mathematics education.

In the Swedish context, mathematics education textbooks are understood as having a unique status (Johansson, 2006). Teachers in Sweden often work close to the textbook. According to Johansson, "Textbooks influence not only *what* kind of tasks students are working with and the examples presented by the teachers but also how mathematics is portrayed in terms of concepts and the features that are related to the subject". This statement supports other findings that the authority in the classroom often resides with the textbook and the teacher (Bishop, 1999), with obvious limitations for the students to influence, and reflect on, their learning of mathematics and mathematics per se – as illustrated by Katja's comments at the very beginning of the chapter. In the first language game of contextualisation by use of the textbook assignments, the contextualised problems illustrate very well what Skovsmose (2001) referred to as "semi-reality problems" in the tradition of exercises. We exemplify this approach by discussing five textbook problems, two from the chapter on "Percentage in society" and three from the "Statistics in society" chapter. These chapters were chosen as their titles indicate a focus on mathematics in society, and thus ought to be the most relevant for social science students and connected to their chosen study program as intended in the curriculum. The problems were picked randomly, thus are neither exceptional in any way, nor different from other problem tasks in the textbook. All five are at the basic level in the book, thus all students are expected to calculate, understand, and complete them.

Percentage in society
We start with three examples from the textbook's chapter on "Percentage in society" (Szabo et al., 2007, p. 121) and discuss their content in order to highlight the characteristics of this type of language game of contextualisation.

> 4202. When Anna went to Cyprus on holiday last year she paid 6530 kr. for the trip. This year the same trip would cost her 7200 kr. By how many percent has the price increased? (ibid., p. 122, our translation)

This contextualisation presupposes a background in which students are acquainted with travelling and going on holiday trips. First, this might not be the case for a number of students in Swedish society today. Second, who, if buying a holiday trip, would reflect on the percentage increase of the price in relation to the previous year? Was this particular trip with the same airline, dates, and accommodation even available the prior year? The characteristics of this example of contextualisation seems to have a strong orientation towards syntactical reasoning and a minimal context that appears to make the mathematical steps more difficult but does not close the gap between syntax and a use of it in society.

> 4207. Arthur has a pond with goldfish. One morning he discovers that a heron has been there. At the end of the week he has lost 40% of his goldfish and only 54 fish are left. How many fish did Arthur have at the beginning? (ibid., p. 123, our translation)

The questions students posed on this task were mostly not mathematical. They rather related to "real reality" or the context of the problem. They wondered: How did Arthur know it was a heron and not another bird or animal? As a teacher it is not possible to answer that question and, as Arthur is figurative, it is not possible to ask him either. The author of the textbook might know the answer, but, as illustrated by Wagner (2010), mathematics textbooks authors' voices are seldom recognized in the mathematics textbooks' content.

There are other questions a critical student might ask: Why didn't Arthur prevent the losses instead of thinking of percentage calculations? How come he does not know how many he had from the beginning when he knows he lost 40%? This example highlights that the notion of "Percentage in society" is taken to its limits and also that the context for percentage calculations can raise many questions from students when it is minimalistic and has underdetermined content.

1106. Lasse is helping his uncle selling fruit at the local market. As he doesn't want to calculate so much (ska slippa räkna så mycket) he asks his uncle to complete a table with what the fruit costs. His uncle starts with a table for grapes:

Weight	1 hg	2hg	3hg	½ kg	7hg	1kg
Price	1,70 kr					17,00kr

Complete the table for Lasse. (ibid., p. 10, our translation)

The contextualisation in this task is remarkable from several points of view. First of all, it is worth noting that Lasse's possibly negative feelings for mathematics are highlighted in an exercise in a mathematics education textbook (second sentence).

Either accepting or ignoring this comment in the task, the students continued wondering: Aren't the grapes very cheap? (Yes, they are in a Swedish context). Does anybody want to buy 8 hg? Or 12 hg? Here the teacher has to make a decision. Either (s)he can answer something along the lines of "you only need to complete what is asked for", or alternatively s(he) has an opportunity to challenge the student's mathematical thinking by asking "reflect on why these particular weights were chosen for the table." These are examples of decisions teachers have to take when students question the contexts of problems. A last example of a conversation in the upper secondary classroom: "Do we have to do the exercise for apples and bananas too? No, Lasse only needs a table for grapes, the rest of the fruit he enjoys calculating", and so on.

This example also highlights how many important elements of a practical scenario are left out and/or taken for granted in a short textbook contextualisation. As a result the discussions in class were not about the use of mathematics but about what were interpreted as peculiar circumstances.

Statistics in society
The following two examples are from the chapter "Statistics in society" and exemplify problem-solving exercises on mean values and arithmetic averages.

5204. A building contractor is going to build a block of rental apartments. He analyzed how big apartments the 10 people first in line for new apartments wanted and got the following answers (in number of rooms): 2,4,1,2,4,2,4,2,2,3. Which value is most appropriate to use in this context? Motivate your answer. (ibid., p.155, our translation)

This example is far from 15-16 year old students' personal context. Actually it could be argued that it is far from anybody's context, even that of building constructors.

5211. When the students asked their mathematics teacher how old he was, he answered: We are five in the family and our average age is 28 years. If we count me out the average will be 21 years old. How old was the teacher? (ibid, p. 156, our translation)

The picture of a mathematics teacher answering a polite question in this, in our view, tricky way can be troublesome. It might have the effect of positioning mathematics teachers in a discourse of "mathematical nerds" which is neither desirable nor relevant in this context. These final two examples in our interpretation show that in order to have the desired syntactical content in a very short contextual assignment the context will tend to be unrealistic and sometimes even tricky. In these two cases it is, for example, not quite obvious why the problems were placed in a chapter on "Statistics in society" as their content only remotely relates to this issue.

All the examples above, in their individual ways, portray the language game of contextualised problem solving in upper secondary mathematics textbooks. It is a language game that we find has some serious flaws in its approach to mathematical practice in different contexts. It works with highly simplified and underdetermined contexts. It does not support a notion of mathematics as a network of language games that is practised in numerous ways in the private, public, or

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global sphere. It raises many questions in student discussions in relation to the problem context that are seemingly not meant to be raised. However, the skilled student will know how to cut all unnecessary information away from the problem (that is, the context) (as discussed by e.g. Verschaffel, Greer & de Corte , 2000; Gellert & Jablonka, 2009; Palm, 2009) and thereby focus on, and resolve, the syntactical "core" of the problem.

The point is that the mathematical calculations requested in the textbook problems above are both challenging and relevant, but hiding them in simplistic and underdetermined contexts just gets silly with the result that they are not recognised by the students as either interesting in themselves as mathematical syntactical problems or interesting as a contextualisation of mathematics.

In conclusion, we find that when the contextualisation is developed in the portrayed manner, far removed from both the use of mathematics in society in its uncountable forms and from the context of students' lives, there is no sense in using textbook contextualisation. Following Skovsmose's (2001) vocabulary, they are semi-reality problems and can easily degenerate into non-reality problems as we have seen above.

The school mathematics language game of student-centred contextualisation

In this section we analyse an alternative language game of contextualisation in school mathematics education. We do this by presenting three examples in similar mathematical fields to those discussed above, namely percentage calculation and statistics for upper secondary school. Subsequently, we discuss this alternative approach to contextualisation. The tasks built on the idea that curriculum objectives could be achieved on all assessment and grading levels while, at the same time, the students could be part of establishing the context.

Annica developed the three examples in collaboration with Elin, the teacher of the upper secondary class in question (see Andersson, in press: Andersson & Valero, in press, a, for further details on the background, development, and assessment of the projects). This language game of contextualisations was guided by the idea that a human-centred conception of mathematics implies that the use of mathematical syntax in particular practices should be taken very seriously. Also, Boaler's emphasis on using open problems has been an important source of inspiration. She states that:

> The teaching of content mathematics within a scheme or textbook and the development of isolated process skills within an investigation is insufficient to encourage a deep and genuine mathematical understanding. There is no reason to expect students who have learned isolated strategies and atomised content throughout school to be able to combine, separate, or integrate what they have learned in "real" and demanding situations. Mathematical activities need to be open enough, not only for students to formulate strategies but to formulate their own meaning. Tasks should require that students develop an understanding of the underlying processes and the way that these link with content; in this way students will appreciate and develop an understanding of the interrelationship of the two. (Boaler, 1993, p. 16)

On the basis of the human centred conception of mathematics and an emphasis on the need for open-ended mathematical activities in demanding situations of practice, the tasks were developed as follows.

Percentage in society
The first task was formulated in the following way:

> Making your dreams come true?
> Reflect on something you would like to do, experience, or buy, for yourself or others, which costs so much that you need to borrow the money to cover the expenses. You have to find out how much money you require to finance the project and what

repayment (including interest) the bank expects you to pay.
We suggest the following: the repayments are made to the bank once a year, and you pay back the loan within five years. If this is not possible for you we will discuss that.

- How much will you be paying back each year in interests? In total over the five years?
- How much do you need to pay-down in total per year?
- What did the total cost add up to?
- Was it worth it? Why/why not?

The students worked in pairs or in groups of three in order to facilitate discussions with peers, both about the mathematical content and the critical contextual reflections. The task was to think about what they might like to/need to borrow money for. They came up with ideas ranging from taking a study year abroad, starting a music band, a holiday trip, a trip for doing volunteer work in Africa, buying hifi equipment. They learnt how to find out about loans and interest and actual rates and calculate the cost of the repayments and the total cost. Discussions taken in the classroom are exemplified by: Was it worth it? What different lending schemes are possible and when is it appropriate to choose one kind of loan in preference to another? And what about the "quick loans" which you can get via mobile phone texts or at shops, are they smart? Why/why not?

The second task on percentage calculations was an exercise inspired by the work of Frankenstein (2008). Besides percentage calculations it also required arithmetic calculation skills relating to fractions, division, and whole and decimal numbers. A larger project on Human Rights was going on in the school. A decision was taken to relate to that topic and connect with a mathematical argumentation assignment themed on the United Nations Conventions on the Rights of the Child. The task was developed in the following way:

Newspaper posters with mathematical argumentation!

The task of today is, in small groups, to create a number of newspaper posters that hit people, engage people, arouse curiosity, reflections and/or emotions – with a mathematical content!

The goal is for you to acquire insight into how big the penetrating power of numbers can be in advertisements and newspaper articles.

There are 54 articles in "Convention on the Rights of the Child". Choose the one that interests you the most and focus on that specific one. Search and find information addressing the special children in your focus – information you consider important and want all people at the school to know about. You might want to start a debate; it might be positive information, maybe information with facts that the article has not presented – or something else. Reflect on how to present the numbers to get the message on your news poster through in the very best way.

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The students found the relevant information from multiple data sources. They creatively composed different ways of showing the numbers – as percentages, fractions, whole numbers etc. Which illustration best covered the issue they wanted to address? For example, does 10 % look more or less than 10 out of 100, 1/10 – or maybe the issue can be illustrated as 90% or in a different way? If the students completed the exercise as intended they would have carried out at least as many exercises as the number of routine exercises in their textbook. The reflective questions posed after this exercise related either to the content of the chosen Childrens' Rights article, or how the numbers were best exposed to get the attention of an audience. These reflections related both to creativity in using colours and pictures as well as discussing the power of the displayed numbers.

Statistics in society
The third, statistical, task was a further development of a prior project (Andersson & Valero, in press, b), that, in collaboration with the school's environmental subject teacher, was now expanding on the theme "Ecological footprints we make on earth" to a larger cross-disciplinary subject project (Andersson & Valero, in press, a).The statistical project covered statistical mathematical goals stated in both the Mathematics A and the Mathematics B courses (Swedish Ministry of Education, 2000). Opportunities were also given to reach objectives in environmental science and computer science courses (ibid.) and thus made it possible to work across subjects. The students were invited to conduct a survey. The survey theme was expected to relate to the "ecological footprints" we all make on earth (e.g., Wackernagel & Rees, 1996). Within this frame, students focused on such topics as transport, food, consumption, or energy and narrowed down their focus within one of these. For assessment, they were asked to create a PowerPoint presentation or poster where they showed the data analysis and their results in appropriate diagrams and tables and with high accuracy. The students were also asked to take a stance in relation to their survey results and argue for it in their diagrams and texts. For that purpose, they were "allowed" to manipulate diagrams (but not fake them) – just as newspaper articles and advertisements do. Within this exercise they reflected on the possibility of seeing through, or at least being skeptical about, diagrams, tables etc. presented in the media that are typically constructed from a particular political or entrepreneurial perspective that may be quite hard to disclose."

DISCUSSION

In the contextualisation examples of the textbook, students were faced with a context that we judge was far from meaningful to them in addressing the issues of their reality from the perspective of mathematics. These tasks were, in our interpretation, built in a way that complicates the syntax of mathematics and which did not succeed in bringing this syntax into connection with issues either of society, as intended in the curriculum, or the students' lives in general. This language game of contextualisation takes as its starting-point the "core" mathematical problem solving, and only secondarily has the goal of constructing a framework that connects the problem solving to real life situations or "context".

In the second language game of contextualisation, the main characteristic is the structuring and formulation of the context by the teacher in a way that ensures that the mathematical topics and syntax to be learned will be in focus while at the same time a complex and relevant context can be the starting-point for doing and learning mathematics. The idea of introducing, for example, percentage calculation within the existing language game of loans and repayments is to teach mathematics in its natural habitat, so to speak, and to intertwine the "core" and the "context" of mathematics. Or, more to the point, to ensure that "core" and "context" become unnecessary terms when working with mathematics.

What are we arguing here, then? That the first language game is superfluous and that we only need the second type in school mathematics? Well, we would say yes, we actually do not need the first type of language game if we adhere to the conception of mathematics presented above. Does this also mean that training in the pure syntax of mathematics is unnecessary? Well, no, it indicates, rather, that focused training of mathematical syntax can be very helpful indeed, but then it should be presented to students as such.

The idea of confusing mathematical syntax with mathematics itself is the mistake that we have tried to use a Wittgensteinian perspective to deconstruct. Mathematics is entangled in the complexities of the other sciences, of the use of mathematics in everyday life and in newspapers, sports events, classrooms and so on. So, what we argue is that a move towards the second type of language game of contextualisations in mathematics education is not a step away from "pure" or "real" mathematics but a step towards a teaching of mathematics that is more in accord with the actual phenomenon of mathematics. Mathematics is not an isolated thing in itself. Mathematical symbols have meaning as a result of what we do with them in various types of practices, and therefore the real and pure focus of mathematics education should be mediated in contextualisations of what we (the students and the teacher) can do with them.

The human-centred conception of mathematics that we have argued here does not stand uncontested. Mathematics can of course be interpreted as an other-dimansional logical unity or even a pure formalism that has no connection to reality at all and much mathematics education is directly or indirectly based on this assumption. Here we argue that this interpretation of mathematics ought to be left behind in order to advance both our conception of mathematics and the teaching and learning of mathematics.

We have tried to present our argument for using more student-centred contextualisations as not being about the motivation of students or the question of equity in society. We have done so, not because motivation and equity are not to be considered extremely important in the education of any topic, but, first and foremost, because we want to emphasise that the nature of mathematics does not force upon us an abstract and isolated approach to the learning of mathematics.

Detached and taught in isolation, mathematics loses many of its attributes as an enormously important part of our society, culture, and science and the students lose their ability to handle complex situations where mathematics is in action. This ability is something entirely different from being able to make correct proofs or develop and solve general systems of equations (the tasks of the professional mathematician). Although governments and authorities often address the ability to handle mathematics in complex situations as a very important dimension for students to learn, we believe it will not be a strong ability in students as long as it is not trained and practised.

In this chapter, we have only started the construction of a second language game of contextualisation by contrasting it with a particular language game of school textbook contextualisations. A lot of work is still needed to develop alternative language games of mathematics education even though many attempts have been made during the years. The educational transition towards this type of alternative language games, however, is unlikely to develop without a general development in the conception of mathematics in which the core and context of mathematics are considered inseparable.

REFERENCES

Andersson, A. (2007). Ett kulturellt besök i matematikundervisning: En aktionsstudie med ett etnomatematiskt perspektiv [A cultural visit in mathematics education: Action research with an ethnomathematical perspective]. Unpublished Masters thesis. Malmö Högskola. Retrieved August 16, 2011, from http://dspace.hmah.se/bitstream/handle/2043/5861/Magister2.pdf?sequence=1

Andersson, A. (in press). A "Curling Teacher" in Mathematics Education: Teacher Identities and Pedagogy Development. *Mathematics education research journal.*

Andersson, A., & Valero, P. (in press, a). Mathematics education giving meaning to social science students. A case from Sweden. Accepted for proceedings at II Congreso Internacional de Investigación, Educación y Formación Docente in Medelin, Colombia 26-28 Aug 2009. [[13]]

Andersson, A., & Valero, P. (in press, b). Examining a critical pedagogical discourse for agency and social empowerment. In P. Ernest and B. Sriraman (Eds), *Critical Mathematics Education: Theory and Praxis*. USA: Information Age Publishing.

Atweh, B., & Brady, K. (2009). Socially response-able mathematics education: Implications of an ethical approach. *Eurasia Journal of Mathematics, Science & Technology Education, 5*(3), 267-276.

Bishop, A. (1999). Mathematics teaching and values education – an intersection in need of research. *Zentralblatt für Didaktik der Mathematik, 31*(1), 1-4.

Boaler, J. (1993). The role of contexts in the mathematics classroom: Do they make mathematics more real? *For the Learning of Mathematics*. I3(2), 12-17.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29*(3), 573-604.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44-48.

D'Ambrosio, U. (2006). *Ethnomathematics. Link between tradition and modernity*. Rotterdam: Sense Publishers.

Frankenstein, M. (2008). Quantitative form in arguments. In J. F. Matos, P. Valero, & K. Yasukawa (Eds). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Universidad de Lisboa/ Aalborg: Department of Education, Learning and Philosophy, Aalborg University.

Frankenstein, M. (2009). Developing a criticalmathematical numeracy through real real-life word problems. In L. Verschaffel, B. Greer, W. Van Dooren & S. Mukhopadhyay (Eds)., *Words and worlds: Modelling verbal descriptions of situations* (pp. 111-130). Rotterdam: Sense Publishers

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. New York: Routledge.

Gellert, U., & Jablonka, E. (2009). "I am not talking about reality". In L. Verschaffel, B. Greer, W. Van Dooren, & S. Mukhopadhyay (Eds.), *Words and worlds: Modelling verbal descriptions of situations* (pp. 39–53). Rotterdam: Sense Publishers.

Johansson, M. (2006). Teaching mathematics with textbooks: A classroom and curricular perspective. Doctoral thesis. Luleå University of Technology. Retrieved September 20, 2010 from Luleå tekniska universitet Nr.:2006:23. Retrieved 16 August, 2011 from http://epubl.ltu.se/1402-1544/2006/23/LTU-DT-0623-SE.pdf

Mukhopadhyay, S., & Greer, B. (2001). Modelling with purpose. Mathematics as a critical tool. In B. Atweh, H. Forgasz & B. Nebres (Eds.), *Sociocultural research in mathematics education. An international perspective* (pp. 295-311). Mahwah, NJ: Erlbaum.

Palm, T. (2009). Theory of authentic task situations. In L. Verschaffel, B. Greer, W. Van Dooren & S. Mukhopadhyay (Eds.),. *Words and worlds: Modelling verbal descriptions of situations* (pp. 3-19). Rotterdam: Sense Publishers

Ravn, O., & Skovsmose, O. (2007). Mathematics as measure. In Revista Brasileira de Historia da Matematica. Brazil: Publicação Oficial da Sociedade Brasileira de História da Matemática, 143-156.

Shanker, S. (1987). *Wittgenstein and the turning-point in the philosophy of mathematics*. London: Croom Helm.

Skovsmose, O. (2001). Landscapes of investigation. *Zentralblatt für Didaktik der Mathematik, 33*(4), 123-132.

Skovsmose, O. (2005). *Travelling through education: Uncertainty, mathematics, responsibility*. Rotterdam: Sense Publishers.

Skovsmose, O., & Ravn, O. (2011); Matematikfilosofi. Aarhus: Systime.

Swedish Ministry of Education (2000). Goals to aim for in mathematics education in upper secondary school. Retrieved September 20, 2010 from www.skolverket.se [[19]]

Szabo, A., Larson, N., Viklund, G., & Marklund, M. (2007). *Origo. Matematik kurs A för samhällsvetenskapliga och estetiska program*. Värnamo: Fälth & Hässler.

Valero, P. (2007). A socio-political look at equity in the school organization of mathematics education. *Zentralblatt für Didaktik der Mathematik, 39*(3), 225-233

Verschaffel, L., Greer, B., & De Corte. E. (2000). *Making sense of word problems*. Lisse, The Netherlands: Swets & Zeitlinger.

Vithal, R. (2003). *In search of a pedagogy of conflict and dialogue for mathematics education*. Dordrecht, The Netherlands: Kluwer.

Wackernagel, W., & Rees, E. (1996). *Our ecological footprint. Reducing human impact on the earth*. Gabriola Island: New Society Publishers.

Wagner, D. (2010). The seductive queen - mathematics textbook protagonist. In U. Gellert, E. Jablonka & C. Morgan (Eds), *Proceedings of the Sixth International Mathematics Education and Society Conference* (pp. 438-448). Berlin: Freie Universität Berlin.

Wedege, T. (1999). To know or not to know - mathematics, that is a question of context. *Educational Studies in Mathematics, 39*, 205-227.

Wittgenstein, L. (1978). *Remarks on the foundations of mathematics (3rd edition)*. Oxford: Blackwell.

Wittgenstein, L. (1983). *Philosophical investigations* (2nd edition, first published 1953). Oxford: Blackwell.

Agency in Mathematics Education

Annica Andersson, Eva Norén

Aalborg University and Malmö University, Stockholm University

In this paper we elaborate on the notion of agency. We relate agency to Skovsmose's and Biesta's frameworks respectively. Both Skovsmose and Biesta are concerned with citizenship education, mathematics education and the purpose of education from a critical position. We explore if and how Skovsmose's and Biesta´s frameworks respectively relate to agency.

Introduction

The purpose of this paper is twofold, to widen our understanding of different approaches to the notion of agency in relation to mathematics education, and to explore the compability of Skovsmose's and Biesta´s frameworks respectively in relation to agency and to each other (Wedege, 2010). As a starting point we understand agency in a dialectic relationship to structure (Roth, 2007) and as a dynamic competence[12] of human beings to act independently and to make choices. Sometimes the choices are conscious; however at some times we act as agents not being aware of our options (Cohen, 1994). Agency is not just individual; it is exercised within social practises. As Holland, Lachicotte, Skinner, & Cain (2003) put it: "Agency lies in the improvisations that people create in response to particular situations" (p. 279).

In this paper firstly we investigate the notion of agency in relation to earlier research addressing agency in mathematics education. We thereafter relate agency to Skovsmose's theories of critical mathematics education (1994, 2005). The third part of the paper discusses Biesta's (2009) and Biesta and Tedder's (2006) theoretical framework for understanding agency in mathematics education. In the last section of the paper we discuss how the different frameworks and agency may add to our understanding of mathematics education practices.

As we both authors use the notion of agency in our research respectively (Norén, 2010, Andersson & Valero, 2009; Andersson, 2010 forthcoming), and both Skovsmose and Biesta theoretically have inspired our different research projects we find it fruitful to explore and elaborate on the notion of agency cooperatively in this paper. One argument is that understandings of agency in Skovsmose's philosophy of critical mathematics education and in Biesta and Tedder's (2006) may enable a way to use the theories and hence a way forward in analyzing agency in discursive practices in mathematics classrooms.

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[12] Not to be seen as something situated in a person from birth

Agency in mathematics education research

During the last years there have been increasing attentions in mathematics education research addressing the notion agency. For example Boaler (2002), Wagner (2007) and Grootenboer and Jorgensen (2010) all refer to Pickering's (1995, p. 21) metaphor *the dance of agency*. Pickering has elaborated on scientific practices and looked at science as practice and culture. To him the 'dance of agency' takes the form of a "dialectic of resistance and accommodation" (p. 22):

> Within an expanded conception of scientific culture, however – one that goes beyond science-as-knowledge, to include the material, social, and temporal dimensions of science – it becomes possible to imagine that science is not just about representation. /…/ /…/. But there is quite another way of thinking about science. One can start from the idea that the world is filled not, in the first instance, with facts and observations, but with *agency*. The world, I want to say, is constantly *doing things*, things that bear upon us not as observation statements upon disembodied intellects but as forces upon material beings (p. 5f).

As people we respond to material agency such as in winds, heating or winter. Pickering goes on describing how humans as agents seem to be different from non human agency like: "the weather, television sets, or particular accelerators" (p. 15). Humans are active and intentional beings. Pickering links Foucault's elaboration on temporal emergence and the displacement of the human subject (Foucault, 1977) via the notion of agency. According to Pickering (1995) human agency has an intentional and a social structure. The 'dance of agency' manifests itself at the human end in the intertwining of free and forced moves in practice.

Boaler (2003) uses the 'dance of agency' metaphor when illustrating the importance for mathematics learners to have an empowering identity in relation to school mathematics. To know when to draw on mathematical ideas and to be able to solve mathematical problems is a critical part of the dance of agency according to Boaler. Grootenboer and Zevenbergen (2007) note that mathematics teachers have to engage in a 'dance of agency' when to decide to encourage students' own agency as mathematicians or rearrange to the requirements of standard procedures or forms of representation. Wagner (2007) investigated students' voice in utterances, he wanted to discuss with the students who had agency in the discourse and who had control in the classroom communication. Grootenboer and Jorgensen (2010) combine the work of Boaler (2003) and the work of Burton (2001) to illustrate how teachers work together to solve mathematical problems. Teachers' sense of agency allowed them to expand their sense of learning and achievement through the solving of mathematical tasks, relying on the members of the group, their individuals' knowing, and the collective knowing of the group.

Powell (2004) uses the notion of agency and motivation to avoid deterministic theories and to resist deficiency explanations of African-American students' failure in mathematics in the US. Powell's research study among 24 sixth graders gave "evidence of the mathematical achievement of students of colour as a byproduct of their engagement of their agency" (p.10). Powell found that the students initiated investigations, reasoned and progressed in building foundational understanding of certain mathematical ideas. He continues saying that understanding agency "is particularly important since both failure and success can be located within the same set of social, economic, and school conditions that usually are described as only producing failure" (p. 6).

The last example we present comes from a Danish context, where Lange (2010) in his study on 10-year old children concludes that children seem to be suspended between two conflicting experiences: from the practical and creative school subjects and the school subjects, like mathematics, not so creative but important for their future. In the practical and creative school subjects students experience they have more space for agency than in mathematics classrooms[13]. From this we infer that the research mentioned above seem to draw on differently theoretical standpoints such as socio cultural ones (research referring to Pickering's 'dance of agency' and Lange) and critical theories (Powell).

Skovsmose's philosophy of critical mathematics education

In this part of the paper we explore the notion of agency in relation to Skovsmose's work on critical mathematics education.

Towards a philosophy of critical mathematics education, including agency

Skovsmose developed his philosophy of critical mathematics education based on the Frankfurter school. The influence of Habermas and the philosophy of Critical Theory can be traced in his work "Towards a philosophy of critical mathematics education" (1994).

Critical Theory has changed its emphases since its beginning with the Frankfurter school. Today it has included contributions from structuralism, feminism and lately postmodernism and post colonialism (Popkewitz, 1999). The different perspectives have various assumptions regarding definition of power and the self. One influence of post modernity is from Foucault and his conception of power as productive and positive, not repressive and negative. Power is then conceptualised as working in two directions and not as a one-way surveillance technique of power. The late Foucault (1980, 1982) saw discourse as a medium through which power relations produced speaking objects – in our view this relates to the concept of agency even though Foucault did not discuss agency, but related to human beings as agents

Critical mathematics education emphasizes social justice issues and students empowerment through mathematics education. In Skovsmose's work, (1994), a basic assumption is that implicit as well as explicit functions of mathematics education are of importance for society and democracy (see also Skovsmose, 1998; Skovsmose & Valero, 2001). According to a thesis of mathematics as a formatting power Skovsmose finds mathematics as "an essential instrument when technological authority is exercised. Mathematics is part of technological empowerment" (1998, p. 201). When he spoke about the formatting power of mathematics Skovsmose says it was a way to try to address the relationship between mathematical knowledge and power (2005). Skovsmose also articulates that mathematics education serves as a gatekeeper, to who will get and who will not get access to the information and communication structures in society (1994, 1998, and 2005). He concludes saying that the learner is a member of society and mathematics can be a source for decision-making and action makes mathematics education a critical feature in society.

When conceptualising Critical Theory "as an interdisciplinary attempt to raise awareness of problematic socio-political states of affairs" (Skovsmose 2005, p. 130) Skovsmose relates to post-modernism. He also relates to Foucault and his description of technologies of the social, the

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[13] Though we don't perceive mathematics as non creative, Lange relates to what students told him

connections between power and knowledge, and to discourse. To Skovsmose it seems obvious that knowledge can be expressed in ways of acting.

Is agency part of Skovsmose's writings?

Certain forms of acting and communicating in the mathematics classroom may support the development of citizenship. According to Skovsmose (1998) citizenship is about to face the "output" from authorities, but also to provide an "input" to authority. Education for citizenship "also presupposes participation" (p. 199). Skovsmose states that "mathematics education could play an important role in developing critical citizenship" (2005, p. 132). To Skovsmose "empowerment" seems to refer to a person in an informal meaning; as to have the capacity to speak for oneself. We believe this reasoning of Skovsmose applies to the notion of agency in terms of students' capacity to act independently and to make personal choices in a situation. Intentionality and action presupposes agency.

Skovsmose (1994) does not use the concept agency explicitly, in his writings it is an evasive concept that conceals behind expressions as empowerment, intentionality, action and choice. He writes:

Actions cannot be described in mechanical or in biological terms; and if a person's behaviour can in fact be described in such a way, then behaviour is not a part of his or her actions. It is not a personal action to breathe or to let one's hair grow. This I see as the first essential condition for performing an act: indeterminism must exist, or, the acting person must be in a situation where choice is possible.

The person acting must have some idea about goals and reasons for obtaining them (p. 176).

Skovsmose writes that it does not make sense to talk about human action when a person is forced to do something or when a person is doing something out of a habit or as a reflex, like when combing your hair. To be called action a person's intentions must be present in what is done. "Intentions are examples of intentionality directed towards action" (p. 177). But a person may not always be aware of her/his intentions. Intentions are grounded in a "landscape of pre-intentions or *dispositions*" (p. 178). Skovsmose divides dispositions in background and foreground. Background belongs to the history of a person, and foreground to the possibilities a certain social situation makes available for the person to perceive as her/his possibilities. The dispositions of a person reveals when a person comes to action. Skovsmose sees learning as caused by the intentions of the learner and learning has to be performed by the learner:

Students will enter school with ideas, hopes and expectations. Intentions are inherent within every human being. /…/. But the demands of the situation in school too often result in *broken* or *ignored intentions*. When students' intentions are ignored, it seems impossible for students to perform actions, which could fulfil negotiated intentions (p. 187).

Skovsmose (2005) articulates that possible structures behind social events may be much more complex than those explanatory principles which are conceptualised within contemporary sociology. He writes:

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Social practices, or collective actions, can appear to be so complex that no 'acting subjects' (a person, a group of persons, an institution, a government, an organisation) can be identified. The very existence of an acting subject may appear impossible. Such actions I will refer to as *happenings*. A happening is certainly not a natural phenomenon and it cannot be explained within a framework borrowed from the natural science. Happenings are social constructions and achievements, which pack together a density of contingencies. In a happening, the involved persons are doing something, but it seems out of control as of what this could imply (p. 135f).

As an example of a happening he addresses Woodstock, the music festival. Persons taking part in a happening may not be aware of their role in it, and they may not have any control. In a happening there is no defined acting subject and a happening cannot be explained as a sum of human actions. But, we inquire, if agency is looked upon as a result of social practices, can a happening then be explained in the terms of agency?

There is a possibility to see happenings and intentions as intersecting. When intersecting the notions converge close to the notion of agency; as the capacity of human beings to act independently and to make choices of their own, though they not always may be aware of it, and as a result of social practices, as stated in the beginning of this paper.

Another way of understanding Skovsmose and the notion of agency is when Skovsmose suggest students and teachers to work within an investigative landscape, in contrast to the exercise paradigm. Skovsmose writes that working within the investigative paradigm provides recourses for working with investigations as a learning milieu. He finishes his article with the following sentence:

> My only hope is that finding a route among the different milieus of learning may offer new resources for making the students both acting and reflecting and in this way providing mathematics education with a critical dimension (2001, p. 131).

We conclude that Skovsmose within a critical paradigm concerns democratic aspects of mathematics education, part of that is the intentions of students and their role as acting and reflecting subjects in mathematics classrooms. To us the notion of agency seems to work well with Skovsmose's critical mathematics education.

Biesta's philosophy of (mathematics) education

In this part of the paper we explore the notion of agency in Biesta's (2009) and Biesta's & Tedder's (2006) writings. Biesta's (2006) philosophy of education is influenced by philosophers such as Dewey and Derrida.

Biesta (2009) discusses the purposes of education against a background of, what he understand as "the new language of learning"(p.6), that is e.g. the rise of theories emphasising teachers more facilitating role in relation to the active role students' play in their construction of knowledge, the shift of responsibility for education turning education "from a right into a duty" (p.5). He concretizes his reasoning with examples from citizenship education and mathematics education and we find these examples interesting to emphasis in relation to Skovsmose's writings and the concept of agency.

Biesta's (2009) way of understanding the purpose of education as such he describes with the qualification, the socialisation and the subjectification functions. The qualification purpose provides students with skills and knowledge required for particular professions, further studies or more general as an introduction to modern culture. Biesta argues that the qualification function is obviously a major function of schooling. The socialisation function has to do with the purposes to "become members of and part of particular social, cultural and political 'orders'" (ibid p.40). Biesta elaborates this purpose further:

> But even if socialisation is not the explicit aim of educational programs and practices, it will still function in this way as, for example, has been shown by research on the hidden curriculum. Through its socializing function education inserts individuals into existing ways of doing and being and, through this, plays an important role in the continuation of culture and tradition – both with regard to its desirable and its undesirable aspects. (ibid p. 40)

The last purpose of education Biesta refers to is the subjectification process. Biesta writes that education has an impact on the processes of becoming a subject. In education newcomers do not only get inserted into existing orders, they also get to know how to become independent of such orders. The subjectification process, understood as a process of becoming thus relates to a way of independence and being agentic. An example of the subjectification process is Biesta's reasoning about a citizenship education taking political agency seriously:

> Political knowledge and understanding (qualification) can be an important element for the development of political ways of being and doing (subjectification), just as a strong focus on socialisation into a particular citizenship order can actually lead to resistance which, in itself, can be taken as a sign of subjectification (p. 42).

We agree with Biesta when he concludes that whether all education actually contributes to subjectification of students is debatable. However, any education worthy of its name should contribute to processes of subjectification that allow those educated to become more autonomous and independent in their thinking and acting; thus becoming agentic in our understanding of agency. He continues discussing the subjectification function in mathematics education as raising possibilities for students becoming a person who "through the power or mathematical reasoning is able to gain a more autonomous or considered position towards tradition and common sense" (p.43). To us the last quote seems to be an argument for agency as a notion relating to Biesta's philosophy. Biesta exemplifies his arguments with e.g. exploring moral possibilities of mathematics, e.g. dealing with division in relation to sharing – suggestions we think connects very well with Skovsmose's theory about critical mathematics education.

Agency in Biesta and Tedder's writings

Biesta and Tedder (2006) put forward two key ideas for understanding agency, theoretically mainly building on the work by Emirbayer & Mische (1998). The first idea is that agency "should not be understood as a capacity, and particular not an individual's capacity, but should always be understood in transactional terms, that is, as a quality of the engagement of actors with temporal-relational contexts of action" (p.18). They refer to an ecological understanding of agency, "i.e. an understanding that always encompasses actors-in-transaction-with-context, actors acting by-means-of an-environment rather than simply *in* an environment" (ibid p. 18). The second key idea is that agency should not be understood as a possession of the individual,

rather that something that is achieved" (ibid p. 18) in relation to the particular context. They continue: "the idea of achieving agency makes it possible to understand why individual can be agentic in one situation but not in another. It moves the explanation away, in other words from the individual and locates it firmly in the transaction (ibid p.19).

Concluding, Biesta suggests that we engage in a discussion about the purposes of education, where he sees the notions of qualification, subjectification and socialisation as important and interrelating components. His examples from citizenship education and mathematics education highlight possibilities for further discussion within these areas. As we understand agency in Biesta's and Tedder's words as achieved in relation to/transaction with time and context, narrow in the focus, from the larger purpose of education to the individual within education. We see it as one way to further elaborate on relations and intersections between the individual, society and mathematics education.

Conclusions

What we intended is to widen our understanding of different approaches to the notion of agency in relation to mathematics education, and to investigate the notion of agency in relation to Skovsmose's and Biesta's writings respectively. We find that Skovsmose, Biesta, and Biesta and Tedder have established cores grounded on basic democratic concerns, citizenship and empowerment. Though Skovsmose's writings are explicitly addressing mathematics education Biesta and Tedder's are not. In line with Skovsmose we believe that certain forms of communication in the mathematics classroom may support the development of citizenship. We think the "certain form of communication" may enhance students' space for agency, and vice versa, students' agency may support the "certain form of communication".

Biesta and Tedder are implicitly concerned with agency when discussing subjectivity and the individual becoming agentic. We find Skovsmose's and Biesta´s frameworks compatible to some extend (Wedege, 2010) but we need to explore this further. One reason for that is that the framework of Skovsmose is grounded in his many writings since a long time back, Biesta and Tedder's work is not. The work we have taken into account for this paper is just two from two papers.

In mathematics education research, agency can be used both as a tool for locating certain forms of communication in the mathematics classroom and for locating students' activity and intentions in the communication. An example is when students' agency change directions of teachers' already planned lessons. Also the empowerment of learners as individuals and as citizens in today's society can be discussed when relating to agency. The notion of agency can add to our understanding of mathematics education practices.

As learners' intentions and their role as acting and reflecting subjects in mathematics classrooms can be discussed when relating to agency some questions arise. The questions are concerned with whether agency is something a learner can attain or achieve? Is agency already there? Can mathematics education enhance agency?

References

Andersson, A. (2010). Can a critical pedagogy in mathematics lead to achievement, engagement and social empowerment? *Philosophy of Mathematics Education Journal (25)*.

Andersson, A., & Valero, P. (2009). Mathematics education giving meaning to social science students. A case from Sweden. Accepted for proceedings at II Congreso Internacional de Investigación, Educación y Formación Docente in Medelin, Colombia 26-28 Aug 2009.

Biesta, G. (2006). *Beyond learning: Democratic education for a human future.* Boulder: Paradigm Publishers.

Biesta, G. (2009). Good education in an age of measurement: on the need to reconnect with the question of purpose in education. *Educational assessment, evaluation and accountability 21*(1) 33-46.

Biesta, G., & Tedder, M. (2006). How is agency possible? Towards and ecological understanding of agency-as-achievement. Learning lives, Working paper 5. University of Exeter. Retrieved from www.tlrp.org

Boaler, J. (2002). *Experiencing School Mathematics. Revised and Expanded Edition. Traditional and Reform Approaches to Teaching and the Impact on Student Learning.* London: Lawrence Erlbaum Associates, Publishers

Boaler, J. (2003). Studying and capturing the complexity of practice. The case of "Dance of Agency". *Paper presented at the 27th International Group for the Psychology of Mathematics Education Conference Held Jointly with the 25th PME-NA Conference (Honolulu, HI, Jul 13-18, 2003), v1* p3-16.

Boaler, J., & Greeno, J. G. (2000). Identity, Agency, and Knowing in Mathematics Worlds. In J. Boaler (ed). *Multiple Perspectives on Mathematics Teaching and Learning.* Westport, London: Ablex Publishing.

Burton, L. (2001). Research mathematicians as learners – and what mathematics education can learn from them. *British Educational Research Journal, 27*(5) 589-599.

Cohen, A. (1994). *Self Consciousness: An Alternative Anthropology of Identity.* London: Routledge

Emirbayer, M. & Mische, A. (1998). What is agency? American Journal of Sociology. 103(4), 962-1023.

Foucault, M. (1977). *Discipline and Punish.* New York: Vintage Books.

Foucault, M. (1980). *Power/knowledge: Selected Interviews and Other Writings 1972 - 1977.* London: Harvester.

Foucault, M. (1982). Afterword: The subject and power. In H.L. Dreyfus & P. Rabinow. *Michel Foucault: Beyond Structuralism and Hermeneutics* (2nd ed., pp. 208-216). Chicago: University of Chicago Press.

Grootenboer, P., & Jorgensen, R. (2009). Towards a theory of identity and agency in coming to learn mathematics. *Eurasia Journal of Mathematics Science and Technology Education 5*(3), 255-266.

Grootenboer, P., & Zevenbergen, R. (2007). Identity and mathematics: Towards a theory of agency in coming to learn mathematics. *Mathematics: Essential research, essential practice. Proceedings of the 30th annual conference of the mathematics mducation research group of Australasia Tasmania, Vol. 1* (pp. 335-344). Adelaide: MERGA.

Holland, D., Lachicotte Jr, W., Skinner, D. & Cain, C. (2003). *Identity and Agency in Cultural Worlds.* London, Cambridge: Harvard University Press.

Lange, T. (2010). "Tell them that we like to decide for ourselves". Children's' agency in mathematics education. In V. Durand-Guerrier, S. Sour

Lavergne & F. Arzarello (Eds.). *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (pp. 2587-2596). Lyon: Institut National De Recherche Pédagogique 2010.

Norén, E. (2010). Discourses and Agency in a Multilingual Mathematics Classroom. *Nordic Educational Research Working-paper series*. Stockholm University: Stockholm.

Pickering, A. (1995). *The mangle of practice: Time, agency, and science.* Chicago: University of Chicago Press.

Popkewitz, T. (1999). A social epistemology of educational research. In T. Popkewitz & L. Fendler (Eds.). *Critcal theories in education* (pp. 17-42). London: Routledge.

Powell, A. (2004). The Diversity backlash and the mathematical agency of students of colour. In M. Johnsen Høines and A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Bergen: Bergen University College

Roth, W.M. (2007). Identity in science literacy. In W. M. Roth and K. Tobin (Eds)*, Science, learning, identity: Sociocultural and cultural-historical perspectives* (pp. 147-185). Rotterdam: Sense Publishers.

Skovsmose, O. (1994). *Towards a Philosophy of Critical Mathematics Education.* Dordrecht: Kluwer Academic Press.

Skovsmose, O. (1998). Linking Mathematics Education and Democracy: Citizenship, Mathematical Archeology, Mathematicy and Deliberative Action. *Zentralblatt fur Didaktic der Mathmatik/International Reviews on Mathematics Education, 30*(6), 195-203.

Skovsmose, O. (2001). Landscapes of Investigation. *ZDM 33*(4), 123-132.

Skovsmose, O. (2005). *Travelling Through Education. Uncertainty, Mathematics, Responsibility.* Rotterdam: Sense Publishers.

Wagner, D. (2007). Students' critical awareness of voice and agency in mathematics classroom discourse. *Mathematical Thinking and Learning, 9*(1), 31-50.

Wedege, T. (2010). Connecting theories in mathematics education: from bricolage to professionalism. *Nordic studies in Mathematics education, 15*(1), 59-78.

Interplays Between Context and Students' Achievement of Agency

Annica Andersson

Aalborg University, Denmark, and Malmö University, Sweden

The purpose with this paper is to explore, within a socio-cultural theoretical landscape, an analytical framework for understanding interplays between contexts and students' achievement of agency in mathematics education. However, it became apparent that account also had to be taken for relationships between students' identities and their achievement of agency as they impact, together on the students' decisions to participate in mathematics education and hence in their learning of mathematics. A case study from an upper secondary critical mathematics innovation setting provides an empirical example of how the analytical framework was used. Keywords: critical mathematics education, agency, identity, context, student.

Introduction

The students in focus of this research are a group of the increasing number of upper-secondary students who either just dislike mathematics, or who objectify (Sfard, 2008) themselves as e.g. "math-haters", or remedial students who are present in classrooms but not obviously participating in the learning activities. The students referred to are not students with specific learning difficulties in mathematics, the concerns are for students who in a mainstream way take compulsory mathematics classes but whose well-being "diminishes when they are asked to engage with mathematics learning" (Clarkson, Seah & Bishop, 2010, p.1), whose attitudes to mathematics has fallen as they progressed through school (Beswick, Watson & Brown, 2006) and who have not experienced feelings of inclusiveness in mathematics education (Solomon, 2009). My concern is for these students who have not had the opportunity to experience a prior mathematics education that made sense to them.

For the empirical part of my Ph.D.-study a critical mathematics innovation was arranged for Swedish upper secondary social science students' first compulsory mathematics course. The pedagogy was deeply inspired by concerns raised in critical mathematics education (Skovsmose, 2005) and connected mathematics to society as intended within the domain of sociomathematics (Wedege, 2010). The innovation was a serious attempt to consider possibilities for a pedagogy acknowledging concerns in critical mathematics education although within the frames of the national curriculum and assessment qualities. A focus on individual's voices in this study provided a way to understand students' shifts in participation and changes in identities during this particular mathematics course. The purpose of this paper is to present the analytical framework which assisted explaining the interplays between contexts and students' achievement of agency in this particular setting. Under consideration are different levels of context that impacted on the classroom work; the socio-political school context (Valero, 2002), the situation

contexts within the classrooms and the task contexts expressed in e.g. textbooks exercises and through the developed pedagogical projects (Wedege, 1999). I need to stress that the research is *not* about evaluating the teaching per se. The developed pedagogy rather became the fond for the research aiming for a further understanding of students' identities in mathematics education. My underlying expectation in this paper is to open up for critique and scrutiny of the analytical framework.

This paper is built on two assumptions. First, mathematics learning can be a good thing and may contribute to empowerment and emancipation, even if that not always seems to be the case today (D'Ambrosio, 2006, Skovsmose, 2005). Second, there is a strong belief that students' expect and enjoy to be able to make decisions on their personal learning, that they want to have their ideas valued, and that they enjoy being treated as responsible young adults.

The theoretical framework

The theoretical framework that guided the Ph.D.-study was grounded within a contemporary social-cultural perspective. Learning was thus viewed as a social activity, implying that learning processes "are constituted in the encounter between contextualised, historically grounded human beings and their activity in particular settings and spaces that are socially structured" (Valero, 2004, p.10). Learning in a social-cultural perspective is not just about getting to know, learning is also about becoming someone (Radford, 2008), thus a movement between actual and designated identities (Sfard & Prusak, 2005; Solomon, 2009). This view of learning provided a lens for understanding relationships between contexts, identities and students' achievement of agency in mathematics education at particular historical times during the course. The coming section is divided into three parts. First, theoretical concerns raised in critical mathematics education are presented. Second, an analytical framework for understanding agency is put forward and third, relationships between achievement of agency, context, identity change and learning are discussed.

Theoretical concerns raised in critical mathematics education

The developed pedagogy was deep inspired by critical mathematics education theory. Critical mathematics education is not to be understood as a special way of teaching or branch of mathematics education, on the contrary it is raised by concerns that need to be accounted for with reference to the particular context where the education takes place (Skovsmose, 2005). Concerns raised by Skovsmose addresses issues as how mathematics education can be stratifying and legitimising inclusion and exclusion. While Skovsmose writes about students' inclusion and exclusion in a wider socio-political context, Solomon (2009) engages in questions about inclusion and exclusion within mathematics classrooms. She points to reasons for students' identities of exclusion to be "a product of their particular educational histories and the ways in which they have responded to the ascribed or designated identities carried in repeated discursive positioning." (Solomon, 2009, p. 137)

Critical mathematics education is also concerned with the nature of the competencies supported in mathematics education, as e.g. if learning mathematics can support empowerment. Students' development of competencies as mathemacy (Skovsmose, 2005), or mathematical literacy

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(Solomon, 2009) in a way that supports critical citizenship can be seen as empowering as expressed by Ernest (2002, p. 1-2):

> Social empowerment through mathematics concerns the ability to use mathematics to better one's life chances in study and work and to participate more fully in society through critical mathematical citizenship. Thus it involves the gaining of power over a broader social domain, including the worlds of work, life and social affairs.

Critical mathematics education also raises concerns and awareness of the students' whole situation. They have different foregrounds (understood as those opportunities that the social, political and cultural situation provides for the individual) and backgrounds in relation to mathematics (Skovsmose, 2005). Regarding the students' intentions to participate in mathematics education, understanding participating and learning as action, Skovsmose (ibid, p. 20) continues:

> Intentions of a person refer not only to his or her background, but also to the way he or she experience possibilities. Intentions express expectations, aspirations and hopes.

Indeed, critical mathematics education raises concerns about inclusion and exclusion, considers the competences that are learnt and take students intentionality seriously in mathematics education. That is why a critical mathematics innovation became the context, the background setting, for the empirical part of my Ph.D. thesis and thus the mathematics education context for the students referred to in this study.

Agency

Learning is a social constructed activity and requires an agent, a committed human being who makes the decision of engaging herself in the activity of learning (Valero & Stentoft, 2010). Human agency denotes the faculty to act deliberately according to one's personal will and by that make free choices (Johnson, 2000). A person's agency can be understood as initiating ideas, agreeing with others, to elaborate and critique, to question or disagree with others (Gresalfi et al, 2009, p. 53). There is an obvious dialectic relationship between agency and structures (e.g. Holland et al. 1998). Concerns about agency in education research relates, from the socio-cultural theoretical perspective, to the empirical conditions of agency as when and in what ways agency is possible (Biesta & Tedder, 2006). Building mainly on Emirbayer and Mische's work (1998), Biesta and Tedder (2006) put forward two key ideas for understanding the agency concept. They first suggest that agency should be understood in an ecological way, i.e. strongly connected to context and second, they implicate that agency should be seen as achieved and not as an individual's capacity:

> … agency should not be understood as a possession of the individual, but rather as something that is *achieved* in and through the engagement with a particular temporal-relational situation. The idea of *achieving agency* makes it possible to understand why individuals can be agentic in one situation but not in another. It moves the explanation away, in other words, from the individual and locates it firmly in the transaction (which also implies that the achievement in one situation does not mean that it will necessarily be achieved in other situations as well). (Biesta & Tedder, 2006, p.18-19)

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Agency is thus not about *how* we act in particular situations; the agentic dimension "lies in the ways in which we have control over the ways in which we respond to the situation" (p. 20-21). Within a socio-cultural theoretical framework, regarding learning as a social activity, a definition of agency in line with Biesta and Tedder's ecological understanding of agency fits particular well.

Biesta and Tedder argue that a dimension of agency can be understood as the way in which actors bring their past experiences and future orientations to bear on the present situation, resonating with Skovsmose's (2005) understanding of students' backgrounds and foregrounds as reasons for students' intentions for engaging in mathematics learning. Another aspect Biesta and Tedder (2006) address relates to "the extent to which people are able to distance themselves from their agentic orientations, i.e. make such orientations the object of reflection and imagination" (ibid, p. 21). This way of reasoning resonates with the definition of identity as the reified, endorsed and significant narratives told about a person as suggested by Sfard and Prusak (2005) and thus firmly connects the concept of achieving agency with changes of identity. This connection will be elaborated further in the next section.

Identity-agency dialectic relationship

The relationship between achieving agency and changes of identity needs to be elaborated further as there are other aspects of importance to emphasise. One aspect relates to objectification processes (Sfard, 2008). As a student, being objectified, labelled from experiences in the past, possibly has an impact on how a student act and behave in the future. The labels originate in what Sfard recognise as objectification processes, initiated by our way of transplanting words from one discourse to another. Sfard identifies *metaphors of object* as a special figurative expression with "roots in our tendency for picturing the perceptually inaccessible world of human thinking in the image of material reality' (ibid, p. 42). These metaphors can be both useful and potentially harmful depending on whose actions is objectified and in what way they are objectified in the mathematics education language discourse.

By comparison, another way of understanding the identity-agency relationship is through Boaler and Greeno's (2000) discussions of students' experiences of agency with reference to the notion of *figured worlds* (Holland, Lachicotte, Skinner & Cain, 1998), that is "places where agents come together to construct joint meanings and activities" (ibid, p. 173). Agency is here conceived in terms of authorship and as a prime aspect of identity. A mathematics classroom may be thought of as a socially and culturally constructed figured world. Boaler and Greeno's (2000) research showed that the figured worlds of many mathematics classrooms are unusually narrow and ritualistic, leading able students to reject the discipline at a sensitive stage of their identity development: "traditional pedagogies and procedural views of mathematics combine to produce environments in which most students must surrender agency and thought in order to follow predetermined routines" (ibid, p. 171) and thus lead to exclusion in the way Solomon (2009) addresses the problem. Boaler and Greeno argue that capable students discard mathematics as the views of mathematic education run counter to the students developing identification as responsible, thinking agents. They argue that students do not just learn mathematics in school classrooms, they also learn to be and thus they move between the actual and designated identities as described by Sfard and Prusak (2005). Boaler and Greeno's results

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suggest that many students find the narrowly defined roles they are required to play within mathematics education incompatible with their developing identities.

A case study

In this section four 'critical moments' from Sandra's[1] course trajectory will exemplify how the analytical framework was used. The information was collected with ethnographic methods throughout my participation in two social science students' mathematics classes. In Sandra's case, the data consisted of several spontaneous conversations (Kvale & Brinkman, 2009) and written documents as classroom blog comments, evaluation sheets, logbooks and my field notes. The data was primarily coded in line with Sfard and Prusak's (2005) suggested analytical framework defining identities as reified, endorsed and significant narratives. To clarify *why* these particular narratives were told/written at specific historical times, all narratives were arranged in chronological order on a timeline that became a 'storyline', one 'storyline' for each student. Events and incidents at the school and the teacher's assessments comments and test results were added on the 'storylines'. At last, students' comments referring to individual or group action as e.g. "I decided to…" or "we went to the bank", or "fucked algebra today" finalised the 'storylines'. In these 'storylines', clusters, which were labelled 'critical moments', emerged in different ways and numbers for different students. The 'critical moments' illumined changes in the students' narrations of themselves and how the transactions with contexts impacted on the students' identities through changes in their expressed narratives at particular historical times[2].

In this paper four out of seven 'critical moments' from Sandra's 'storyline' are chosen as a frame within which to theoretically consider interplays between agency and context. They are chosen in a way that they provided four qualitative different ways of narrating the self, and hence they support the theoretical discussion. They were, in chronological order: 1) Sandra's narratives told in the transition phase between lower secondary and upper secondary schools 2) Sandra's narratives during the project '*Making your dreams come true*' 3) Sandra's narratives during a teaching sequence organised with textbook work and 4) Sandra's narratives after the larger cross-subject project '*Ecological footprints*'[3].

First 'critical moment'. Sandra initially shared with me that she had always disliked mathematics because she had "mathematics anxiety". This label was Sandra's way of objectifying herself (Sfard, 2008), causing her not wanting to spend more time with mathematics than was absolutely needed. That is the reason for why she did not want me to interview her, which would, as she said, result in more "mathematics related time". However, I was very welcome to read her blog comments, evaluation sheets and logbook and to talk with her in the classroom.

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[1] All names are pseudonyms. The data was analysed in Swedish and here translated by the author.

[2] For a more detailed methodological discussion, please see coming publications by Annica Andersson.

[3] I refer to Andersson (2010) for an in-depth description of the development of the teaching sequences and the projects within their societal-, and school contextual background.

Sandra told me she desperately wanted to pass the mathematics course, as it required for her future university studies. Foregrounding herself as a university student became her intentionality for attending and passing the mathematics courses that where required by society at this time. The socio-political context constrained Sandra's achievement of agency; she could not decide to not participate, as her designated identity was to become an university student. Within the situation context, objectifying herself with the label 'having math-anxiety' seemed to impact on her decisions on how to act within the classroom (e.g. spending a minimum of time with mathematics). Sandra's agentic orientation might be characterised as iterative at this time; she effected action consistent with schemas derived from prior personal experiences.

Second 'critical moment'. The second transition occurred during a two-week project where the students got high possibilities for deciding on task contexts, personal time and work distribution. Sandra evaluated her project work in the following way:

> We distributed the time well, I think. […] The group worked well. We were good at different things, and helped each other. I am proud of the work I have done as I felt I could contribute a lot in the beginning when we talked about borrowing money and interest rates. To plan time and content self got me to feel it was related to me. I think mathematics has been a little more fun than usual. […] I feel the project has been meaningful and to look at mathematics from different angles (vända och vrida på matematiken) was positive. But I would have liked more time for explanations from the teacher, as mathematics is difficult for me. (Sandra, evaluation sheet, 10-2009)

During this project Sandra achieved agency in relation to task context and situation context. Her personal influence on content, time and work distribution impacted on her decisions to engage in the classroom activities in a different way than she intended at the course start. In addition she experienced feelings as 'a little fun' and mathematics as 'meaningful'. At this time Sandra took a projective action for learning differently to the initially intended and got rewarded with feelings as "being proud" of her work. However, even if she was proud of her work and actually passed this sequence with distinction (teacher, results sheet), the last sentence indicated that being objectified with 'mathematics anxiety' still implied her wishing for extra help and support from her teacher.

Third 'critical moment'. In the middle of the semester the students were expected to work with textbook algebra exercises during two separate weeks. In contrast to the second transition Sandra's two entries on the blog during these textbook work periods emphasised Sandra's worries and feelings of stress for not passing a coming test:

> I am currently worried about the test. I have received help with things I need help with. Stress. Stress. (Sandra, blog, 07-10-2009).

> Quiet, concentrated and do my best (Sandra, blog, 04-11-2009)

In class she repeatedly asked the teacher about what would happen if she did not pass the test, and she asked for advice on exercises that was 'extra smart to calculate' when preparing for the test (Annica, field notes). She worried, and her achievement of agency seemed to be restricted to do what was required for just passing the test she was so anxious about. Sandra's positive experience of the prior project appeared to have vanished. The interplay between her task

contexts (restricted to advised exercises on given topic), the situation context within the classroom (to pass a test) and her foreground to become a university student was obvious in her actions. Not passing the test would have impacted on her designated identity. Her "math anxiety", imagining herself not passing and thus not becoming what she wanted, became problematic and restricted her achievement of agency at this particular time.

Fourth 'critical moment'. At this time a larger cross-subject project commenced themed "Students' Ecological footprints on earth". At that time Sandra's logbook was rich with comments regarding hers and her work-friend's collaborative work. This excerpt exemplifies her reflections on her mathematics learning during the project:

> During the project I have learnt about different diagrams. E.g. I did not know about histograms before the project. I think it has been really interesting with manipulated diagrams and results – now I will be more observant when reading newspapers etc!
>
> What surprised me most though was how important role mathematics plays when talking about environmental issues. With support of mathematics we can get people to react and stop. […] I am so interested in environmental questions and did actually not believe that maths could be important when presenting different standpoints. I have probably learnt more now than if I had only calculated tasks in the book. Now I could get use of the knowledge in the project and that made me motivated and happy! I show my knowledge best through oral presentations because there you can show all the facts and talk instead of just writing a test. To have a purpose with the calculations motivated me a lot. (Sandra, logbook, conclusions).

The teacher assessed her with the highest grade, implicating she presented her statistical investigation with correct mathematical language using appropriate concepts and terminology, that she had chosen appropriate diagrams and arithmetic mean values and could argue for her choices, and that she reflected on possible sources of error and how these could have been prevented. Sandra was convinced that she had not been able to account for these criteria on a written mathematics test. However orally she clearly in a correct and convincing way presented her results and answered questions in front of an audience of 50 students, two teachers and one researcher (Annica, fieldnotes).

Sandra's actions voiced as e.g. "I will be more observant…", "I have learnt more than…" and "I could get use for…" evidently expressed her achieved agency and the relationship between her present identity as a responsible and thinking agent, achieved agency and her learning of mathematics at this particular transition. Her expressed narratives, oriented both to the past, future and present is an important factor for actual agency according to Biesta and Tedder (2006). Her learning is expressed both in relation to the subject mathematics and the power of mathematics in society. There seems to be an indication of a changing agentic orientation in this particular situation context; e.g. her 'mathematics anxiety' identity is not expressed at all at this point (however, this identity is still present in coming stories, especially when preparing for the national mathematics test at the end of the course).

Concluding remarks regarding Sandra's achievement of agency

Sandra's change of telling about herself, thus her identities expressed different qualities of her achieved agency that impacted on her classroom engagement and her learning of and accountancies for mathematics. The implications of this case suggest that Sandra's possibilities for achieving agency is an important component for her learning of mathematics. Maybe Sandra's stories indicate that possibilities for achieving agency are an important feature when constructing mathematics learning environments.

Concluding remarks

The purpose of this theoretical paper was to present the analytical framework which explained the interplay between different contexts and students' achievement of agency in this particular setting and thus open up the framework for critique. The analytical framework put forward by Biesta and Tedder as understanding agency as achieved in-transaction-with-context at particular historical times opened up possibilities to consider Sandra's different narratives and actions in the figured world of a particular mathematics classroom. When analysing Sandra's achievement of agency the framework suggested by Biesta and Tedder (2006) was combined with Sfard and Prusak's (2005) operational definition of identities and Skovsmose's (2005) concept of intentionality understanding learning as action. This was done in parallel with considering the relationships between achieved agency and different levels of context. Analysing Sandra's achievement of agency in relation to the obvious task-context could have given information on how this student achieved agency when conducting project work, compared with mathematics textbook work. However, by also taking the wider socio-political context and situation context into consideration gave a deeper understanding of Sandra's achievement and indicated why she acted, or achieved agency in some situations but not in others. Further elaboration of this framework might open up possibilities to recognise students as agents of their learning of mathematics in mathematics education research.

References

Andersson, A. (2010). Can a critical pedagogy in mathematics lead to achievement, engagement and social empowerment? *Philosophy of Mathematics Education Journal (25)*.

Beswick, K., Watson, J., & Brown, N. (2006). Teachers' confidences and beliefs and their students' attitudes to mathematics. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Identities, cultures and learning spaces* (pp. 68-75). Sydney: Mathematics Education Research Group of Australasia.

Biesta, G., & Tedder, M. (2006). How is agency possible? Towards and ecological understanding of agency-as-achievement. Learning lives, Working paper 5. University of Exeter. Retrieved from www.tlrp.org

Boaler, J., & Greeno, J.G. (2000). Identity, agency and knowing in mathematics worlds. In J. Boaler (Ed) *Multiple perspectives on mathematics teaching and learning* (pp. 171-200). USA: Greenwood Publishing Group, Inc.

Clarkson, P.C., Bishop, A., & Seah, W.T. (2010). Mathematics education and student values: The cultivation of mathematical wellbeing. In T. Lovat, R. Toomey, & N. Clement (Eds.). *International research handbook on values education and student wellbeing* (pp. 111-135). Dordrecht: Springer.

D'Ambrosio, U. (2007). Peace, social justice and ethnomathematics. In Sriraman, B. (Ed) *International perspectives on social justice in mathematics education* (pp. 25-34). Charlotte: Information Age Publishing, Inc.

Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of mathematics education journal. 15* (1)1-16.

Emirbayer, M., & Mische, A. (1998) 'What is agency? *American Journal of Sociology. 103* (4) 962-1023.

Holland, D., Lachicotte, W., Skinner, D., & Cain, C. (1998). *Identity and agency in*

cultural worlds. Cambridge: Harvard University Press.

Johnson, A.G. (2000). *The Blackwell dictionary of sociology. A user's guide to sociological language, second education.* Cornwall: TJ International.

Radford, L. (2008). The ethics of being and knowing: towards a cultural theory of learning. In L. Radford, G. Schubring, & F. Seeger (Eds.) *Semiotics in Mathematics Education: Epistemology, History, Classroom, and Culture*, (pp. 215–234). Rotterdam: Sense Publishers.

Sfard, A., & Prusak, A. (2005). Telling identities: In a search of an analytical tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4) 2-24.

Sfard, A. (2008). *Thinking as communicating.* New York: Cambridge University Press.

Skovsmose, O. (2005). *Travelling through education: Uncertainty, mathematics, responsibility*. Rotterdam: Sense Publishers.

Solomon, Y. (2009). *Mathematical literacy. Developing identities of inclusion.* New York: Routledge.

Valero, P. (2002). The myth of the active learner. In P. Valero & O. Skovsmose (Eds.) *Proceedings of the Third International Mathematics Education and Society Conference, Second edition* (pp. 542-553). Copenhagen: Centre for Research in Learning Mathematics.

Valero, P. (2004). Socio-political perspectives on mathematics education. In P. Valero & R. Zevenbergen (Eds) *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology* (pp. 5-24). Dordrecht: Kluwer Academic Publishers.

Wedege, T. (1999). To know or not to know mathematics, that is a question of context. *Educational Studies in Mathematics (39)* 205-207.

Wedege, T. (2010). Sociomathematics: A subject field and a research field. In U.Gellert, E. Jablonka., & C. Morgan (Eds). *Proceedings of the Sixth International Mathematics Education and Society Conference* (pp. 171-200). Berlin: Freie Universität Berlin.

"I Am [not Always] a Math-hater"
Students' (dis)engagement in Mathematics Education

Annica Andersson, Tamsin Meaney and Paola Valero

Abstract

The objective with this study was to explore individual students engagement with mathematics in relation to different specified contexts. The students' narrated identities during the their first compulsory, upper secondary, mathematics course provided a way to understand students' shifts in participation and why the engagement shifts occurred at particular times. The analysis showed that students' identity narratives such as that of being a "math hater" are bounded to the opportunities that the students participate in offers. Our research give evidence for that identities are not consistent in the way literature often suggest. The connections found between identity narratives and contexts invited us to rethink how students are categorised or labelled in mathematics education and the impact these objectifications have on students' agency and learning of mathematics.

Keywords: students, identity narratives, agency, resistance, mathematics learning, contexts.

Introduction

Mathematics, as the gatekeeper to a variety of education and career opportunities, is judged to be an important study subject by society. However, in Sweden, as in other countries, there is trouble in mathematics education. A number of students struggle with compulsory mathematics courses in Swedish upper-secondary schools (Ministry of Education, 2010). A quality report from the Schools Inspectorate (2010), written by mathematics education researchers, documented the reasons for this; mathematics teaching is not connected clearly to students' chosen study program as stated in the national curriculum, and individual work dominates mathematics lessons. The report concludes that the observed teaching seems to result in under-stimulated students, who experience mathematics as a boring and even "stupidizing" (fördummande, p. 8) subject. Elsewhere it has been noted that boredom has an impact on students' engagement in mathematics (Ingram, 2009).

In this article we focus Malin and Petra, two 15-year old girls who labelled themselves "maths-anxious" and "maths-hater" respectively. Their identity narratives are examples of students' shifts in engagement in mathematics learning activities. We argue that Malin and Petra are two of the large number of young people who dislike or get bored with mathematics. It may be that, like

the Australian students reported by Clarkson et al (2010), these are students whose well-being diminishes when they are asked to engage in mathematics education.

Unlike previous research that seems to suggest that students' dislike of mathematics is fixed and as a consequence also their likelihood of engagement, we argue that contextual changes to the way that mathematics is presented can alter students' perceptions and engagement. To illustrate these changes, we analysed the stories that the students told about themselves and their relationship with mathematics. Different levels of contexts affected their decision-making on whether to engage at specific points in time. Four sequences are provided to show how these students became aware of themselves and their decisions to engage or not in mathematics education.

Affect, context, and identity narratives

Traditionally, discussions about boredom or dislike in mathematics have been covered under the umbrella of affect. Research on students' affective responses (McLeod, 1992) have included considering beliefs (Leder et al, 2002), feelings (Ball, 1988), and values (Bishop, 2001; Lovat et al, 2010). Being 'good or bad at maths' is closely linked with valued features (e.g., Ernest, 1998, Black et al, 2009), hence causes strong emotional reactions in people, as documented by affective research (e.g., Leder & Grootenboer, 2005). Recently, research within the affective domain has turned to socio-constructivist perspectives that highlight the situatedness of students' affective responses or emotions in relation to one-off contexts, usually specific classroom activities. Consequently, interactions between cognitive and psychological factors when learning mathematics are the focus of much of this work (Op 'T Eynde et al, 2006). For example, in order to formulate a theoretical framework for affect in the context of individual mathematical problem solving, DeBellis and Goldin (2006) describe affect "windows" (p.140), understanding affect "functioning as an internal representational system exchanging information with cognitive systems" (p. 132). However, developments in educational, psychological and social-psychological research have encouraged a wider variety of multi-disciplinary theoretical perspectives into the study of affect (Zan et al, 2006). Within this work on affect, there are many attempts to understand the influence of the individual and the collective in relationship to differences in students' socio-cultural backgrounds (Ingram, 2009). As well, it stresses the importance of "classroom culture as a determinant for students' under-participation in schooling" (Sullivan et al, 2006, p. 81).

Boredom is one of the most debilitating emotions used by students as a description for mathematics according to a number of large-scale quantitative reports (e.g., Goetz et al, 2006). Brown et al (2008) showed that *bored* was the word used most often by 16-year old students to describe their attitude in a questionnaire on students' affective relationship to mathematics. The questionnaire was distributed to 1997 students in the same age as students enrolled in the first mathematics course in Swedish upper secondary schools. One of the reported reasons for boredom was students' lack of creativity. Similar to the findings of the School Inspection (2010), Bibby (2008), although working with students at the end of primary school, suggested that boredom indicated lack of stimulation and challenge. She also found that students' lack of control over tasks and direction was connected to feelings of boredom.

On the whole in affect research, students' opinions of their relationships to mathematics seemed to be missing, especially when contexts and discourses changed. In mathematics education research, context tends to be restricted to the immediate context of a particular classroom or studied activity episode (Morgan, 2006). Researching affective responses within only one context, like problem solving, means that students are seen as retaining the same set of affective responses, which consequently become objectified (Sfard, 2008). Subsequently, we recognise a need to explore individual students' relationships with mathematics in different contexts, which can be considered in a number of ways. We recognised *task contexts* including contexts expressed in textbooks exercises and through developed pedagogical projects (Wedege, 1999). Research reported by César (2007) where students initially drew mathematics, and Gutstein (2003) and Stocker & Wagner (2007) who introduced tasks influenced by critical education (e.g., Freire, 1992) exemplify research addressing task contexts. There are *situation contexts* (Morgan, 2006; Wedege, 1999), which in this study encompasses "the current activities, the other participants, the tools available and other aspects of the immediate environment" (Morgan, 2006, p. 221). Researching teaching organised as project work (e.g., Brown, 2009; Gutstein, 2003) or individual mathematical problem solving (DeBellis & Goldin, 2006) addresses situation contexts. *School contexts* refer to school structures such as teachers, timetables, school leadership and so on, pinpointed by Martin (2000) as being of importance for African-American youths' achievement in mathematics education. For Wedege (1999), situation contexts are the social contexts within the classrooms, including wider socio-political contexts that refer to other aspects of students' life possibilities, wishes and concerns. We use the notion *socio-political context* (Nieto, 2010; Valero, 2004) when referring to contexts outside classrooms that influence what occurs within classrooms. We divided the *socio-political context* into two components. First, there is the *socio-political school context* (Valero, 2004), which is operationalized through governmental policies on schools, curricula, ideologies and school policies. Biesta's (2009) analysis of purposes with schooling and the societal talk about "lifelong learning" through the critical concept "learnification" and Popkewitz (2009) analysis of governmental documents and argumentation about how children become "fabricated" within schools depending on how education steering documents are formulated are examples of research within the socio-political school context. Societal discourses, as "specialness" when being "good" at mathematics (Mendick et al, 2009), or "discourses of belonging, being accepted, or rejected, and succeeding or failing" that often contribute to people giving up mathematics' studies (Black et al, 2009, p.29) are examples discourses within the *socio-political societal context*. A different perspective on the societal context is ethnomathematical research reported by e.g., Knijnik (2010). Within the "Movimento Sem Terra", a Brazilian national social movement, the teaching practice became grounded in the socio-political societal context.

Given the lack of research on understanding the relationship between students' relationship with mathematics and different contexts, we used the notion of identity as an analytical tool. There are a number of reasons why this was useful way to address this research area. Students need to be recognised as social and historical beings with multiple motives for learning (Valero, 2004) and so learning is not just about objectification of knowledge but it is also about subjectification or becoming someone (Radford, 2008). When considering how students affective responses impact on their willingness to engage then the notion of identity became particularly important because

it provided ways to understand the complexity of students' decision making. Solomon (2009) showed the potential of using identity as a research tool when discussing mathematical literacy, students' engagement and inclusiveness in mathematics education. We adopted Sfard & Prusak (2005) operational definition of identities: "[We] equate identities with stories about persons. No, no mistake here: We did not say that identities were finding their expression in stories—we said they were stories." (2005, p. 14)

Instead of seeing identity as the core of the true self of a person, a person's identity is considered to change with the stories that the person accepts about him/herself. The identity stories adapt as a result of the interactions between the person and the environment in which the person operates. To reinforce the idea that identities are narratives about a person, rather than talking about stories we use the phrase "identity narratives". *Identity narratives* are stories that the participants, including the girls themselves, teachers, parents and other students told about the girls at a particular moment in time.

Given the concerns raised by the School Inspectorate (2010) about individualised work and issues raised by Biddy (2008) about the lack of choices about learning, the concept of students' agency needed to be incorporated. A larger number of research articles on students' agency in mathematics education (e.g., Wagner, 2007) draw on Pickering's (1995, p. 21) metaphor of the "dance of agency". This dance metaphor illustrates the tension between individuals' initiatives and structures or conventions in relation to mathematics per se. Pickering differentiated between different forms of agency. Conceptual agency allowed space for choosing methods when solving set mathematical problems or textbook exercises. On the other hand, disciplinary agency referred to the use of established solution methods and thus turned the agency concept to the discipline of mathematics itself rather than being seen as the sole responsibility of the person. However, we would argue that other aspects of agency are engaged in the 'agency dance' in mathematics classrooms. Students' agency might involve making decisions about tasks, time allocation and work distribution, assessment and accountability, who to work with and so on (Andersson & Valero, 2011). It might concern what sorts of decision-making students' can make in relation to their learning. The agentic dimension "lies in the ways in which we have control over the ways in which we respond to the situation" (Biesta & Tedder, 2006, p. 20-21) or in what ways students can act within classroom culture (Sullivan et al., 2006). Students decide to act or resist according to expectations or classroom discourses. Resistance can be defined as certain behaviours, for example not bringing pens to lessons or regularly question, "Why do we have to do this?" (Martin, 2000). Alternatively, resistance can be defined in a relational way, as students who do not accept teachers' invitations to learning activities and/or who challenge school authorities (Alrø & Skovsmose, 2003). For us *resistance* are ways of acting, talking, engaging or participating in mathematics learning activities that neither are proactive for the student's learning or corresponding with the present classroom discourse and teacher's expectations.

Methodology

The research context

The data for this article comes from a one-year study exploring upper secondary students' learning mathematics within a social science program. Students commonly complete this program because it provides entry into university studies in the social sciences and language faculties. As well, students, who do not enjoy mathematics and thus do not want to take the alternative natural science or technical programs, see it as a good option.

Annica, in collaboration with Elin, a mathematics teacher, introduced teaching sequences that, enabled students' mathematics learning to be connected to societal topics. From critical mathematics education, three different aspects inspired the teaching. The first concerned how the mathematical topics related to societal contexts: "understanding mathematics as a tool for identifying and analysing critical features in society" and "seeing mathematics both as a tool for critique and an object of critique" (Skovsmose & Nielsen, 1996). These aims matched curriculum objetives, which asserted that mathematics education for social science students should "provide general civic competence and constitute an integral part of the chosen study orientation" (Ministry of Education, 2000). The second aspect concerned the epistemological point that an educational practice was considered to involve learning and becoming, rather than a simple transmission of knowledge (Skovsmose, 2005). The third aspect involved how power relations between the actors supported a classroom environment where students could become agentic in a positive way towards their learning. Annica and Elin aimed for classrooms structures where students had access to and contributed to the discourse between participants.

Collecting information

In order to understand students' relationships with mathematics from their perspectives, ethnographic methods were used for data collecting (Hammersley & Atkinson, 2007). Annica, as the main researcher, established a trustful environment through engaging with the students in both formal and informal settings. In this way, she interacted closely with the students and experienced the contexts and discourses. The data-collection included interviews, spontaneous conversations, a blog and students' project logbooks. Annica's research-diary described the contexts and allowed the students' stories to be related to what went on in school and society at particular times. There were obviously differences in the kind of stories that were told in interviews, blog and logbooks. The interviews provided reflective data about the different context levels. The blog was a course activity and provided data mainly about task contexts. Students' actions and behaviours, and hence their reflections of their agency (including resistance) also appeared in the blog. The logbooks provided data about the students' learning, task contexts and situation contexts.

Data analysis

The data used Sfard and Prusak´s (2005) proposal to "equate identities with stories about persons" (p. 14). For them, an identifying story could be considered *reifying* if the words *be,*

have, can, always, never, usually were used*; endorsable* if the identity-builder (the person the story is about) accepted the stories as being about themselves; and *significant*, that is if any change to the story was likely to affect the storyteller's feelings about the identified person, particularly with regard to membership of a community. The stories told or written by the students' about themselves or stories told by others about them were analysed by locating the verbs and adverbs that suggested a reified quality. Stories with expressions such as "I am worried about maths-tests …", or "I have always hated maths…" indicated reification. The students were the significant narrators of these identities and they drew of stories from their parents and their mathematics teacher (Andersson, 2011).

Petra and Malin's identity narratives

We start this section with an introduction of Petra and Malin. When they began upper secondary school, the students were asked to write an introductory letter to Annica about their previous relationships with mathematics education and their view of the Mathematics A course, the first out of two compulsory mathematics courses they would complete at this school. The two girls' identity narratives from these letters are discussed before we look at their engagement during four teaching sequences, which show their engagement shifts in relation to different contexts.

Petra lived on a small farm. She had trained horses all her life. Petra wrote about herself: "I am a very happy and extroverted person and I love meeting new people" and "language subjects are what I love most, I love language, I think it is interesting and I have always loved them" (Petra, letter, 08-2009). However, her relationship to mathematics was different as seen in an interview with Annica later in the semester.

Annica:	Please, tell what you said when I asked if I could interview you.
Petra:	I said, "I would not be of any use".
Annica:	Why would you not be of any use?
Petra:	Because I hate mathematics so that would be the only answer you would get from me […]
Annica:	Can you describe how the maths-lessons were at your prior school?
Petra:	Well, I was never messy and jumped around like the others. But I wasn't motivated and the last half year, I was only so sick tired of school and maths so I was there physically but not mentally. I listened to music and disappeared in the music… […]
Annica:	Well, if you were to decide, what would a good mathematics course look like?
Petra:	I do not know how I want it; I just want to pass the course. I don't care. I just want to pass. […]
Petra:	It is you teachers who decide in the end that maths is this way. […]
Annica:	But don't you need to take any responsibility yourself for your learning? [With a teasing voice]
Petra:	[Laughing] It is because you teachers always say, well we do not understand each other here. It is because you teachers always stand up front and say: This is how it is, and here you have the book. That's how it has always been since ages back. It's the

> book that counts, and that is how it is. It's hard to break that pattern. One has always had the book to refer too. Even if I know I learn best in other ways and never understand anything of the book-problems, I have never been able to change that (sighs). (Petra, interview, 09-2009)

Despite describing herself as extrovert and easy-going, in the context of mathematics education research Petra identified herself as "not being of any use", not even for an interview about herself and her relationship with mathematics. Petra classified herself "I am a math-hater", in interviews and repeatedly in class. Her actions in previous mathematics education were a form of resistance. She attended but did not engage because the situation context, "only textbook work" was "boring" (interview, 09-2009). The societal context of needing mathematics courses for university made her feel she had to take the Mathematics A course "because I really need this course for my further studies". Yet her dislike of mathematics meant that her engagement was precarious: "but do as little as possible so I just pass the course because I hate maths" (interview, 09-2009).

Malin lived in a residential area. She stated that she was ambitious in school and strove for high grades. School had always been an important part of her life. She chose the social science program because she wanted a theoretical program that prepared her for university. However, she excluded mathematics from her future studies:

> What I am sure of is that I will not study anything containing mathematics. I would much rather study something within the social sciences, as for example Human Rights. (Malin, letter, 08-2009).

Malin took a deep interest in equity and social justice issues; "I have always loved discussing those questions, maybe it is because I am adopted" (Malin, interview, 09-2009). She saw her future occupations as "maybe working abroad or doing something in the third world, like for SIDA [Swedish International Development Cooperation Agency]" (Malin, interview, 09-2009).

Malin described her previous experiences from mathematics education with the word "obehagligt" (unpleasant), which is a strong word in Swedish to describe an educational context. Malin's father endorsed her story with even stronger words:

> It is as if mathematics has become something contaminated or toxic at home. It has become so associated with difficulties and anxiety so one hardly dares to talk about it. I usually say that what is wrong with Malin's thinking is that she believes that mathematics is about being intelligent. She believes one has to understand intuitively, and if one does not do that one shall not do mathematics - it is only geniuses that should do it. (Malin's father, interview, 12-2009)

Malin had passed her previous mathematics courses, yet Malin's father's analysis of her mathematical relationship associates with strong emotional reactions, documented in affective domain research (Leder & Grootenboer, 2005) and verifyes societal discourses about being good in matheamtics, described by Black et al (2009). However, both Malin and her father told about periods where Malin had not been present in mathematics classes. Malin's thus recognised that this course might be challenging:

> During lower secondary, well, sometimes I struggled but usually I did not have the energy to really work with it. I feel there is a wide gap that I now have to fill and I need to catch up. But I don't really know how to do that! [sighs] (Malin, interview, 08-2009)

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Words, such as "unpleasant", "toxic" and "anxiety" that peppered Malin and her father's comments about her prior experiences, together with her recognition of a knowledge gap, indicated that the Mathematics A course might become problematic and require hard work even though she had no intention to pursue mathematics further.

Although neither of the girls recognised any personal meaningfulness in learning mathematics, the societal requirement of mathematics to become university students or for future occupations resulted in the girls deciding to participate, resonating with factors like future aspirations found by Black et al. (2009) for students of the same age engaging in learning.

Identity narratives in the context of group work

The first project, "Making your dreams come true", allowed opportunities for students deciding on task and situation contexts. In addition to the mathematical content - interest rates and percentage calculations - the students were asked to reflect on what they would be interested in borrowing money for and how to find out which loan offers were smart to take. The news headlines at this time indicated that an increasing number of young people (18-25 years old) were in financial trouble after taking loans received through, for example, mobile phone texts. If youngsters were placed on Swedish tax enforcement or police registers for not repaying debts, their possibilities to get bank loans for further studies or a house later in life is decreased. Thus, the task context was related to a contemporary societal context, but the specific context, what to borrow money for, was decided by the students themselves. The situation context involved the students working in smaller groups deciding within the groups on time and work distribution and with access to the internet, books and poster materials for their presentation.

Due to contracting flu, a number of students stayed home for the first week. Consequently, Petra initially worked alone. She repeatedly informed the class that this was problematic for her. She told that she neither could nor wanted to start the project and/or to take decisions on her own. Petra suggested several times that she could go home instead of starting the project work (Annica, research-diary). Then Petra also got the flu and stayed at home. Fortunately, both Petra and her work-partner were back for the third lesson and they presented their project on time. Petra wrote about her engagement:

> I do not know. I am neutral. Maths isn't fun, but it goes forward. […] Percentage calculations were actually useful for me to learn, because we could decide ourselves what we wanted to do. (Petra, evaluation sheet, 09-2009)

Petra did not talk about herself as a "math-hater" but rather as "neutral" at this point in time. She stated that the mathematical content could "actually" become useful. Although it took time before Petra engaged, she did work well and proactively with her friend and completed the project. Her initial resistance changed when she chose the task contexts with her friend and realised, although hesitantly, that the project but also mathematics could become meaningful. Similar to the Portuguese 9th grade boy with disruptive behaviours presented by César (2008), Petra engaged in mathematics learning when the mathematical content could be achieved differently than through text book work.

Malin has similar reactions and talked about her relationship with mathematics from the task-context perspective:

> This is new and interesting for me. It's a good task because it's real. It can be good for me to know later on in life (Malin, blog, 18-09-2009)

She indicated that it was "ok" for her to attend mathematics classes during this project. The task-context allowed her to experience meaningfulness. Nevertheless the situation context of doing group work made her state:

> This was a good exercise because we had to find out stuff ourselves and thus to become independent. I tried to calculate the interest rates but realised we have to be better and more efficient to help each other with the mathematics tasks in the group. I will try to get the others to be better at that, so we help each other ☺ (Malin, blog 23-09-2009).

Malin explained that she had never experienced group work in mathematics before and at times she worried she did not learn enough mathematics, or learnt mathematics the wrong way (Malin, conversation, 09-2009). Her comment about the group's mutual responsibilities resonates with students who during project work communicated 'explaining' as being a shared practice in contrast to an individual responsibility (Brown, 2009).

In this situation, both girls used verbs indicating action. Verbs such as *choose, decide, become* and *will try* indicated agency related to the situation context of project work. The "we" expressed in e.g."we have to" and "we help each other" are also attributes of students' agency (Wagner, 2007). The task contexts, decided by the students within given mathematical frames, were described with adjectives with positive connotations as *good* or *useful*, indicated that the girls experienced meaningfulness. Both Petra, however sceptically, and Malin, worrying, were proactively engaged in learning during this group work.

Identity narratives in the context of textbook work

During two weeks the teaching was organised with teachers' instructions and textbook work, a situation context the girls were used to from previous schools. The mathematical content the students were expected to learn were "algebraic expressions, and formulae required for solving problems in everyday life and in other subjects in their study orientation" (Ministry of Education, 2000).

Petra resisted during this teaching sequence. Either she listened to music, looked out of the window and did not disturb anybody, or she stated regularly in a loud voice that she hated mathematics and tried to gain the teacher's attention for each new exercise by asking "Elin, help me" (Annica, research-diary). Elin commented:

> Petra did some exercises today, but was not focused. [...] She demands a lot of attention both from her classmates and me. She regularly and loudly states that she does not like maths and asks all the time "Why do I have to do this?" It is becoming quite disturbing. (Teacher, interview, 10-2009).

Petra made two blog comments during these weeks, both written sarcastically. For example "Today I really liked maths. I was very focused and calculated loads of exercises in the book" (Petra, blog 10-2009). That was neither the teacher's nor Annica's experience during that particular lesson. In this case Annica kindly asked Petra after class: "I didn't get the impression

that you loved maths today?" Petra answered with a laugh "Off course I didn't…" Neither task nor situation context appealed to Petra at this time.

Malin attended the first of these mathematics lessons; then she was absent the rest of the time. Like Petra, Malin acted in similar ways to how she described her previous engagement with mathematics. In this situation context, the girls' agency was resistance or passivity. Petra either acted as a "noisy" student, being loud, complaining and regularly seeking assistance, or with passivity. Malin was simply not present.

Identity narratives in the context of mathematical argumentation

Annica and Elin invited the students to *"Newspaper Flyer Project"*, a full day workshop on how to use mathematics to produce powerful arguments and contextually themed on the United Nations Convention on The Rights of the Child. The mathematical content included calculations using percentage, fractions, whole numbers and decimal numbers. . The situation context invited students in pairs to decide on which article of the convention to focus and issues to highlight and discuss in relation to their selected article. The students had access to the Internet, the town library and newspapers to find the numbers, statistics and other data to support their arguments.

However, the students were not aware of the types of activities that would take place. Their schedule simply read "Mathematics day". This day coincided with a higher number of sick students, dentist and doctors' appointments than usual (Teacher, interview, 10-2009).

> First I thought, a whole day of mathematics, I can't do it; I just can't be there the whole day. However, when I got there it was actually quite fun and now, afterwards, I read and look in the newspapers in a different way. So I actually learnt something and that was really unexpected of a math-day. (Petra, interview, 12-2009)

Petra's evaluation of the workshop indicated within itself two different identity narratives; the "maths-hater" who "can't be there a whole day" and one who found it "quite fun" and "I actually learnt something". Petra referred to the societal context as so meaningful that she now reads newspapers "in a different way". Hence she indicated that the critical context enabled her to engage in learning, similar to students' experiences reported by Gutstein (2003).

Resembling Petra, Malin told that she did not look forward to a whole mathematics-day. Malin arrived late and did not participate in the morning group work sessions. Malin articulated her experience:

Annica: What were your thoughts before the 'math-day'?

Malin: Oh no, I don't want to be there. I didn't believe it would be good for me. […]

Annica: And during the day?
Malin: Ok, at least it was a good theme. But I had a doctors' appointment so I came in late, after lunch.
Annica: What were your thoughts after the day?
Malin: I am so happy this day is over. And I am happy that I have it so much better compared with these people (referring to the children's situations addressed on the flyers).
Annica: I remember you talked a lot and that you were very engaged in the discussions in the afternoon?

Malin: Yes, I remember talking a lot, but I got so touched. Like for example that woman in Sudan who got whipped because she wore trousers [...]. How can one use mathematics, to have impact for women, to prevent discrimination? (Malin, interview, 12-2009)

Malin told that being present during a whole mathematics day would decrease her well-being, thus she decided to participate only in the afternoon sessions. Malin engaged in the discussions and presented several analytical comments to the class about the societal content in the other students' flyers. She made critical suggestions for presenting the mathematical content in more accessible ways. The contextual societal and critical framing of the project related to Malin's interests, and she got emotionally involved. She reflected on how mathematics could be used as a tool to gain social justice for marginalised groups in society and talked about experiencing mathematics as meaningful at this time. Her experiences from the day resonates with students described as "wanting to be treated as if they mattered and as if they were agents of change in the world" in an American context after working with topics as domestic abuse, racial profiling and poverty during mathematics lessons (Stocker & Wagner, 2007, p.19).

In this situation context, neither of the two girls looked forward to take part in a whole day workshop in mathematics. Rather they told about diminishing well-being if they had to participate. Malin was present only part of the day; Petra initially talked about resisting but engaged when realising the societal and critical context. The possibilities for using mathematics as a tool for social justice appealed to both girls. During interviews eight weeks after the argumentation day they both queried how mathematics can be used in society. They searched for potentials rather than complaints when talking about mathematics in relation to a critical and societal context at this time.

Identity narratives in the context of a larger societal project

A number of news headlines during the autumn semester related to the United Nations Conference on Climate Change in Copenhagen (COP 15), so its themes, climate change and sustainability, became the societal context for a statistics project on sustainable development and "ecological footprints" in collaboration with the environmental-science teacher. Within this wider project, students made decisions about task contexts and situation contexts such as time and work distribution and assessment. The project ran for three weeks, with a whole day of displaying results with power points, posters, presentations and interactions in the fourth week. The project was designed as a "three-stage rocket". First, the students decided on working groups, chose topics, and designed surveys in parallel with gathering information. Second, they conducted, analysed and reported the survey in a descriptive way. However, they were also asked to argue for their view with support of statistical data and specifically designed diagrams. In that way they learnt how to use statistical information to strengthen arguments. Thus they experienced how different ways of presenting statistical information affect people's judgements. Third, all collected information was summarised into a website, "My Planet", with the goal to calculate how many planets would be needed if the entire population lived like these students. We compared and problematized data from different countries and examined critical issues about different ways of living.

Petra engaged in mathematics learning at this time. She worked in a group of three which seemed to suit her particular well. In her logbook, there were statements such as:

> […] today our collaboration worked well. I have participated in discussions and believe this project will be something really good and fun!" (Petra, logbook, 11-2009).

She reflected on her learning and how she best accounted for mathematical knowledge in relation to assessment and grading:

> One of our line-charts was wrong. […] Our oral presentation could have been much better if we had not put all the texts in our power point pictures. But if one looks carefully at the content I thought we did well. […]

> This was a much better way for me to show my mathematics knowledge than on those tests we always get, because here I can explain how I think […] (Petra, evaluation sheet, 11-2009).

Petra engaged with both the situation context and the task context and decided on work and time distribution. She accounted for her knowledge in a way that suited her well. Petra passed the statistical mathematical topic with distinction (teacher, evaluation sheet).

Malin's identity as a societal interested student became obvious during this project:

> This is really interesting. I only talk about environmental issues with my friends. I have become an environmental nerd (Malin to teacher, 11-2009).

However, with mathematics her identity narratives were different:

> Malin: The others were so clever and I thought it was really interesting to see their presentations. I became more aware when I saw the different diagrams […] but ours was not so good.
>
> Annica: Why do you say that?
>
> Malin: I don't think I did so well, it is difficult. I don't know. It is the mathematics that troubles me. And then I became unsure and decided not to say so much in mathematics. I rather focused on the environmental science stuff. (Malin, interview, 11-2009)

Malin engaged in mathematics because she was interested in the contextual topic and passed assessment. However, she did not recognise her mathematics learning. Instead, her identity narrative acknowledged her learning of environmental science, which resonated with her interests.

In Malin's case it was the contextual framing, a topic that critically related to society that impacted on her decision to participate. Mathematics was still experienced as problematic even if she reached the mathematical objectives and passed this sequence. Malin's experiences are similar to students' stories reported by Gutstein (2003, 2006). Gutstein's students also indicated that they realised the importance of mathematics in society however their view on mathematics as a subject to dislike remained. It also resembles students' experiences as provided by Brown (2009, p.183) that "explaining and arguing may be viewed by students as being social resources that have the potential to promote mathematical understandings and an awareness of the 'self' as acting in a classroom community". In this situation, the girls again talked about their relationship with mathematics in more positively active ways. The verbs indicated action, such as "*participated", "talked about*", and they both critically discussed their mathematical learning and the best way for accounting for their knowledge. Petra did not mention being a "math-hater" during this project.

However, when related to task contexts and situation contexts in the long term, we conclude that these resources (Brown, 2009) might not be sustainable. As Petra told Annica in the last interview:

 […] hm, these projects have been good. But, then I know that we will go back to textbook work and then that is, like I just do not want to do it. I don't want to go back to the book because then I will do nothing. […] There is never anything in the book that interests me. (Petra, interview, 12-2009)

This statement indicated that Petra had experienced a different relationship with mathematics education however she did not believe it would be sustainable. The case was rather the opposite. This statement made by Petra leads us into the final section of this paper and supports us exploring students' engagement in and relationships with mathematics and how individual students' engagement are connected to different mathematics education contexts.

Concluding discussion

The temporal analysis of students' identity narratives in relation to contexts highlights the complexity of students decision making when deciding to engage in mathematics learning – or not. Relationships between identity narratives and the contexts the narratives referred to indicated why students acted in particular ways at specific times, and why they became agents in their learning of mathematics in some situations but not in others. It was not the mathematics per se, it was rather the context in which the mathematics was introduced, that allowed students' space to act in for them qualitatively different ways. The view on 'contexts' in mathematics education research tends to be restricted (Morgan, 2006). We agree and conclude that there are needs to further explore individual students' school mathematics relationships in affiliation with contexts.

We let the "Mathematics day" from Malin's point of view illustrate our argument: Similar to the case with British students (Black et al., 2009), it was the requirement of mathematics within the *socio-political societal context* that made the girls decide to participate in the course. The societal view of mathematics as a difficult subject (Ernest, 1998), that "only geniuses should do" (Malin's father) impacted negatively on Malin's relationship with mathematics. The governmental steering within the *socio-political school context* required that the mathematics teaching should be connected to the students' study program (School Inspectorate, 2010). When this requirement became realised Malin engaged in learning. The *school context* impacted on Malin's talk about diminishing wellbeing when the school administration labelled a workshop "mathematics day" and resulted in half-day participation. The *situation* and *task context* with the societal and critical framing allowed Malin to experience meaningfulness. We argue that contextual changes to the way that mathematics is presented can alter students' perceptions and engagement.

Our research focus also gave evidence for with what qualities students achieved agency, hence decided to act and engage, or resist and disengage, in learning activities. Boredom is experienced during mathematics lessons and a reason for students' disengagement (Brown et al, 2008). Reported reasons for boredom as students' lack of creativity, stimulation and challenge (Bibby, 2008; School Inspectorate, 2010), but also students' loss of control over tasks and direction

(Bibby, 2008) became verified in this study. In addition, this research show that when students themselves took decisions on task and situation contexts, they did not talk about being bored. The case was rather the opposite, they influenced classroom discourses and took responsibility for learning. When the students' themselves decided on task and situation contexts neither of them asked: "Why do we have to do/learn this?"

Petra and Malin's identity narratives, chosen as examples of students talk in the larger research study, indicate that identities are not consistent in the way literature often suggests. These results indicate problematic issues when students become objectified (Sfard, 2008). In Petra's case her way of (sometimes, but not always) expressing herself as a "math-hater" could easily be interpreted as she *had* an identity as a "math-hater", or as *always being* a "math-hater". However, this study showed that this not always is the case. Identities fluctuate and are less stable when connected to the available contexts. Stories with resemblances as these girls told were also narrated by students that could be labelled boys, immigrants, coming from certain social-economic backgrounds, and so on. It was their stories as such that allowed us researchers to understand the importance of different contexts in which mathematical topics were presented. The changes of identity narratives as such rather supported us researchers to understand the roles contexts play when students decide to engage or not in learning of mathematics.

References

Alrø, H. & Skovsmose, O. (2003). *Dialogue and learning in mathematics education: intention, reflection, critique.* Boston: Kluwer Academic Publishers.

Andersson, A. (Accepted for publication). A "Curling Teacher" in Mathematics Education: Teacher Identities and Pedagogy Development. *Mathematics education research journal.*

Andersson, A., & Valero, P. (2011). Negotiating critical pedagogical discourses. In P. Ernest and B. Sriraman (Eds), *Critical Mathematics Education: Theory and Praxis.* USA: Information Age Publishing.

Ball, D. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics 8*(1), 40–48.

Bibby, T. (2008). The experience of learning in classrooms: moving beyond Vygotsky. In T. Brown (Ed), *The Psychology of Mathematics Education: A Psychoanalytic Displacement* (pp. 37-60). Rotterdam: Sense Publishers.

Bishop, A.J. (2001). What values do you teach when you teach mathematics? in P. Gates (ed.), *Issues in Mathematics Teaching* (pp. 93–104). London: Routledge.

Biesta, G. (2009). Good education in an age of measurement: on the need to reconnect with the question of purpose in education. *Educational Assessment, Evaluation and Accountability 21*(1), 33-46.

Biesta, G., & Tedder, M. (2006). How is agency possible? Towards an ecological understanding of agency-as-achievement. Working paper 5. Exeter: The Learning Lives project.

Black, L., Williams, J., Hernandez-Marritnez, P., Davis, P., Pampaka, M., & Wake, G. (2009). Developing a 'leading identity': the relationship between students' mathematical identities

and their career and higher education aspirations. *Educational studies in mathematics 73*, 55-72.

Black, L., Mendick, H., Rodd, M., Solomon, Y. (2009). Pain, pleasure and power: selecting and assessing defended subjects. In L. Black, H. Mendick, & Y. Solomon (Eds), *Mathematical relationships in education* (pp. 19-30)*. New York: Routledge.

Brown, R. (2009). Teaching for social justice: Exploring the development of student agency through participation in the literacy practices of a mathematics classroom. *Journal for Mathematics Teacher Education 12*, 171-185.

Brown, M., Brown, P., & Bibby, T. (2008). 'I would rather die': reasons given by 16-year-olds for not continuing their study of mathematics. *Research in Mathematics Education 10*(1), 3-18.

César, M. (2007). Dialogic identities in students from cultural minorities or students categorized as presenting SEN: How do they shape learning, namely in mathematics. In ScTIGGroup (Eds.), *Second Socio-Cultural Theory in Educational Research and Practice Conference Proceedings*. Manchester: Manchester University.

Clarkson, P., Bishop, A., & Seah, W.T. (2010). Mathematics education and student values: The cultivation of mathematical wellbeing. In T. Lovat, R. Toomey, & N. Clement (Eds.). *International research handbook on values education and student wellbeing* (pp. 111-135). Dordrecht: Springer.

De Bellis, V. A. & Goldin, G.A. (2006). Affect and meta-affect in mathematical problem solving: A Representational Perspective. *Educational Studies in Mathematics 63*(2), 131-147.

Ernest, P. (1998). A postmodern perspective on research in mathematics education. In A.Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain. A search for identity* (pp. 71-85). Dordrecht: Kluwer Academic Publishers.

Freire, P. (1992). *Pedagogy for the oppressed*. New York: Seabury Press.

Goetz, T., Frenzel, A.C., Pekrun R., & Hall, N.C. (2006). The domain specificity of academic emotional experiences. *Journal of Experimental Education 75*(1), 5–29.

Gutstein, E.(2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*(1),37-73.

Hannula, M. (2002). Attitude towards mathematics: Emotions, expectations and values', *Educational Studies in Mathematics* 49(1), 25–46.

Hammersley, M., & Atkinson, P. (2007). *Ethnography : principles in practice*. New York: Routledge.

Ingram, N. (2009), Engagement in the mathematics classroom. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In Search of Theories in Mathematics Education: Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol2, pp. 233-240). Thessaloniki: PME.

Knijnik, G. (1998). Ethnomathematics and Political Struggles. *ZDM 30*(6), 88-194.

Leder, G., & Grootenboer, P. (2005). Affect and mathematics education. *Mathematics Education Research Journal, 17*(2), 1-8.

Leder, G., Pehkonen, E., Törner, G. (Eds.) (2002). *Beliefs: A Hidden Variable in Mathematics Education?* Dordrecht: Kluwer Academic Publishers.

Lovat, T., Toomey, R., & Clement, N. (Eds.) (2010). *International research handbook on values education and student wellbeing*. Dordrecht: Springer.

Martin, D. (2000). *Mathematics success and failure among African-American youth.* New Jersey: Lawrence Erlbaum Associates, Inc.

McLeod, D. (1992). Research on affect in mathematics education: a reconceptualization. In D.Grows (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp.575-596). New York: McMillan Publishing Company.

Mendick, H., Moreau, M.P., & Epstein, D. (2009). Special cases: Neoliberalism, choise, and mathematics. In L. Black, H. Mendick, & Y. Solomon (Eds), *Mathematical relationships in education* (pp. 71-81)*.* New York: Routledge.

Ministry of Education, (2000). Goals to aim for in mathematics education in upper secondary school. Stockholm: Frizes.

Ministry of Education, (2010). *Gymnasieskolans kursprov vt 2010. En resultatredovisning.* Available at: www.skolverket.se

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational studies in mathematics 61*, 219-245.

Nieto, S. (2010). *Language, culture and teaching*. New York: Routledge.

Pickering, A. (1995). *The mangle of practice: Time, agency, and science*. Chicago: University of Chicago Press.

Popkewitz, T. (2004) The alchemy of the mathematics curriculum: inscriptions and the fabrication of the child. *American Educational Research Journal 41*(4), 3–34.

Radford, L. (2008). The ethics of being and knowing: towards a cultural theory of learning. In L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in Mathematics Education: Epistemology, History, Classroom, and Culture* (pp. 215–234). Rotterdam: Sense Publishers.

School Inspectorate (2010). *Skolinspektionens Kvalitetsgranskning Rapport 2010:13.* Stockholm: Skolinspektionen.

Sfard, A. (2008). *Thinking as communicating.* New York: Cambridge University Press.

Sfard, A., & Prusak, A. (2005). Telling identities: In a search of an analytical tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4)*,* 2-24.

Skovsmose, O. (2005). *Travelling through education: Uncertainty, mathematics, responsibility.* Rotterdam: Sense Publishers.

Skovsmose, O., & Nielsen, L. (1996). Critical Mathematics Education. In A. J. Bishop (Ed), *International Handbook of Mathematics Education* (pp 1257-1288). Dordrecht: Kluwer Academic Publishers.

Solomon, Y. (2009). *Mathematical literacy. Developing identities of inclusion.* New York: Routledge.

Stocker, D., & Wagner,D. (2007). Talking about teaching mathematics for social justice. *For the Learning of Mathematics*, 27(3), 17-21.

Sullivan, P., Tobias, S., & McDonough, A. (2006). Perhaps the decision of some students not to engage in learning mathematics is deliberate. *Educational Studies in Mathematics 62,* 81–99.

Valero, P. (2004). Postmodernism as critique to dominant mathematics education research. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 35–54). Greenwich: Information Age Publishing.

Wagner, D. (2007). Students' critical awareness of voice and agency in mathematics classroom discourse. *Mathematical Thinking and Learning, 9*(1), 31–50.

Wedege, T. (1999). To know or not to know mathematics, that is a question of context. *Educational Studies in Mathematics (39),* 205-207.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*(2), 113–121.

Identity Narratives of [dis]engagement in Mathematics Education Contexts

Annica Andersson and Paola Valero

Abstract

We have aimed to start filling the research gap concerning the relationships between individual students' engagement in mathematics and different contexts in and outside the classrooms that impact on what occurs in the education practice. The identity concept provided ways for us to understand the complexity of individual students' decision making for engaging in the classroom activities at certain points in time. The concept of agency needed to be incorporated in our study as the students accounted for agency as an important feature in their stories. Seven different narratives related to contexts and agency, here told by 15-year old youngster Henrik, are provided to illustrate how upper secondary students became aware of themselves and their decisions to engage or not in their mathematics education. The relationships between the identity narratives and the contexts the narratives referred to indicate why students acted in particular ways at specific times, and why they became agents in their learning of mathematics in some situations but not in others. The students' identity narratives also made us quire the way research often concludes, with objectifying or categorising labels, how particular groups of students, teachers or parents are expected to act or behave in certain ways.

Keywords: identity narratives, engagement, agency, learning, contexts, mathematics.

Introduction

In this chapter we focus on Henrik, a 15-year old youngster and his reasons for participating in mathematics education. His reasons for (dis)engagement with learning were brought to live in the stories he told and expressions he used in his relationships with Annica (the researcher), Elin (the teacher) and the other students in the mathematics classrooms. We argue that Henrik experience is not unique, but is representative of the large number of young people who either just dislike mathematics and/or experience mathematics as a boring subject. It may be that, like the Australian students reported by Clarkson, Seah and Bishop (2010), he is a student whose well-being diminishes when asked to engage in mathematics education. Similar students' identities are increasingly scrutinized in international mathematics education research. For example, Black, Mendick and Solomon (2009) illustrate how socio-cultural, discursive or psychoanalytic theoretical perspectives help to "better understand patterns of participation in mathematics" (p. 1). Greer, Mukhopadhyay, Powell and Nelson-Barber (2009) focus on students with different cultural and/or disadvantaged backgrounds and their engagement in mathematics learning. Lovat,

Toomey and Clement (2010) address the issue of students' values and well-being in mathematics education. Henrik might also be like one of the students that has not experienced inclusive identities in mathematics education, as was the situation for the British students identities reported by Solomon (2009), even if he indeed has attended and passed years of prior mathematics education. Solomon (2009, p. 27), defines an inclusive identity as "particular beliefs about oneself as a learner and about the nature of mathematics, an identity of engagement in mathematics and a perception of oneself as a potential creator of, or participant in, mathematics". She concludes from her research in the UK, probably extendable at least to Western countries, that

> […] the issue of an inclusive mathematics literacy seems all the more pressing within its climate of audit, assessment and ability discourses. Perhaps the biggest challenge to these discourses comes from the fact that even successful students feel disempowered with respect to mathematics (p. 200).

Even if we find Solomon's research illuminating, the results presented seem to bring forward the idea that students' dislike of mathematics is a permanent feature of some types of students, and as a consequence their (dis)engagement in mathematics education is also seen as a constant characteristic. In our research we found that students' identity narratives fluctuate as changes in the classroom discourses make possible different opportunities for engagement. To illustrate these changes, we analysed the stories that individual students told about themselves and their relationship with mathematics (e.g., Andersson, in press; Andersson, Meaney & Valero, under submission). As different contexts for mathematical activity are made available, students' decision-making on whether to engage in learning changes at specific points in time. Seven different moments and their corresponding identity narratives are provided to illustrate how Henrik became aware of himself and his decisions to engage or not in mathematics education.

In the context of mathematics education in Sweden, students are struggling with passing the Mathematics A course, the first compulsory mathematics course for all students in Swedish upper secondary schools. A newly published quality report from the Swedish School Inspectorate (2010) documented reasons for this being the case: Mathematics teaching is not obviously connected to students' chosen study program as stated in the national curriculum, and students' individual work dominates mathematics lessons thus resulting in mainly mechanical procedural calculations with lesser time for discussions, collaborations and problem-solving. As a consequence, students do not develop central competences such as creativity, critical reflections and understanding of mathematical relationships. The report concludes that teaching seems to result in under-stimulated students, who experience mathematics as a boring and even "stupidizing" subject (p. 8). Elsewhere it has been noted that boredom has an impact on students' engagement in mathematics (Ingram, 2009). With a decreasing number of students choosing to go on to take mathematics and science-related subjects at university (Valero, 2011), it is clear that there needs to be a better understanding of students' responses to their mathematics learning.

Theoretical landscape

The theoretical framework that guided the study is grounded in the social-cultural-political turn that has emerged in mathematics education in the last decades (Lerman, 2000, 2006; Valero, 2004a; Gutiérrez, 2010). We recognise students as social and historical beings with multiple motives for learning. Thus we see learning not just as a matter of objectification of knowledge but also as subjectification understood as the complex process of becoming in the meeting with others in the institutional frames of mathematics education (Gutiérrez, 2010; Radford, 2008). Within this overall approach, three particular notions have been important: identity narratives emerging in discursive practices, contexts for learning, and agency.

Identity narratives

The notion of identity has become particularly important and has been extensively used in recent years. Danny Martin's thesis from 1998 (Martin, reprinted, 2000) was one of the earliest work proposing the idea of students' mathematical identities, in his study mathematics socialization and identities among African-American youth. Short after, Boaler and Greeno (2000) used the concept of identity showing that the figured worlds as cultural constructs (Holland et al, 1998) of many mathematics classrooms are narrow and ritualistic and thus leading students to reject the discipline at a sensitive stage of their identity development. Lerman (2000) argued for research potentialities in understanding identities in a sociological way and hence recognizing learning as transformations of identities. In this understanding, the concept of identity refers to the processes of participation and enculturation, labelled for example "identity development" (e.g., Cobb, Gresalfi & Hodges, 2009; Hodgen & Marks, 2009), "identity work" (e.g., Mendick, 2006) or "identity formations" (Lerman, 2009). Hence, instead of talking about who we are in an essentialist way, identity on the contrary becomes recognised as a process, a movement or a destination. Recently identity research has moved further towards discursive sociological perspectives (e.g., Black, Mendick & Solomon, 2009; de Freitas, 2010; Sfard & Prusak, 2005; Solomon, 2009; and Valero & Stentoft, 2009). In this understanding identity intersects with language use as discursive practises in context. Thus they write about identities as multiple subject positions in discursive practises. Stentoft and Valero (2009) draw our attention to the "fragility identities" that together with classroom discourses interrelate in the landscape in which learning take place. They point to the fact that "there are obstacles pertaining to immediate discourse and identities to overcome before the mathematical content and learning mathematics will appear relevant" (p. 104).

Sfard and Prusak (2005) propose identities to be collectively narrated reified, endorsed stories about a person, which are produced through engagement with and connected to available discourses:

The key move was to equate identity-building with storytelling. The difference between identity as a "thing in the world" and as a discursive construct is subtle. The kind of data that the narrative-minded researcher analyses in her studies is the same as everybody else's: These are stories that people tell about themselves or about others to their friends, teachers, parents, children, and bosses, as well as to researchers. The only distinctive feature of the present narrative approach is that, rather than treat the stories as windows to another entity that stays

unchanged when "the stories themselves" evolve, the adherent of the narrative perspective is interested in the stories as such, accepting them for what they appear to be: words that are taken seriously and that shape one's actions. (Sfard & Prusak, 2005, p.21)

However, in a number of research articles we find the contradiction that, even if assuming theoretically that identities change, that identities are constructed and reconstructed, or moving and fluent, research often concludes with objectifying or categorising that certain groups of students, teachers or parents are, act or behave in certain ways. Swanson (2005) gives examples of black students in South Africa that are labelled as disadvantaged and as a result being offered different possibilities for learning mathematics than other students. Martin (2009) addresses likewise important issues concerning African American children in the US, and contends "mainstream research and policy contexts have served as particularly fertile grounds to aid in socially constructing African America children as learners with particular kinds of deficiencies in relation to other children" (p.4). As Cobb and Hodge (2002, p. 258) wrote: "A crucial limitation of these institutionalized categories in our view is they do not necessarily correspond to people's own sense of identity". The issues at stake might be that other learning qualities or processes; available through mathematics classroom discourses, remains hidden or overlooked as suggested by Stentoft and Valero (2009).

Being aware of this risk of objectifying students, and the possible vulnerability of the identity construct, in this research we adopt a discursive understanding of identities where the key to the students' relationship with mathematics is their evolving stories that they tell about themselves, in the context of particular discursive practices in the mathematics classroom. .We see identity as a construction, relational and in dependence with contexts. In order to make the definition operational for analysing learning, we adopted Sfard and Prusak's (2005, p. 14) proposal of "equat[ing] identities with stories about persons." A person's identity was considered to change as we could document changes in the stories that the person told and accepted about him/herself, in the context of changed classroom discourses and their corresponding activities. We use the term *identity narratives* to refer to those stories that Henrik himself, his teachers and parents told about Henrik at particular moments in time.

Contexts for learning

In mathematics education research, context tends to be restricted to the immediate context of a particular classroom or studied activity episode (Morgan, 2006). Consequently, there is a need to explore students' identity narratives in relationship with the different contexts. However, the concept "context" is complicated to grasp as a single concept, but also in its relationship with discourses. According to the Oxford Dictionary of English, the noun "context" refers to "the circumstances that form the setting for an event, statement, or idea, and in terms of which it can be fully understood" and "the parts of something written or spoken that immediately precede and follow a word or passage and clarify its meaning" (Stevenson, 2010). Hence, the word is a reference to circumstances, but in our language use it also refers to and makes discursive spaces possible. Context comprises the network of relationships and available recourses in the social practises in which we act, but at the same time contexts are forming the ways and spaces where

we act. So, "context" is by us understood as a reference to circumstances, but also with an understanding that contexts influence discourses.

Contexts can be considered in a number of ways. In what follows we will try to capture and define the different contexts that the students in this study referred to in their identity narratives. First, we recognised task contexts as the referents to which a particular task appeal in order to invite students to engage in mathematical activity. Task contexts are expressed in textbooks exercises and through developed pedagogical projects (Wedege, 1999). Research reported by César (2007) where students initially drew mathematics, and Gutstein (2003, 2006), and Stocker and Wagner (2007) who introduced tasks influenced by critical education (e.g., Freire, 1992) exemplify research addressing the contexts in which exercises and tasks are presented and thus situated. Second, there are situation contexts (Morgan, 2006; Wedege, 1999), understood as the array of "current activities, the other participants, the tools available and other aspects of the immediate environment" in the classroom (Morgan, 2006, p. 221). A situation context thus also refers to the communicative understanding of contexts. Researching teaching organised as project work (e.g., Brown, 2009; Gutstein, 2003, 2006) focus situation contexts where students are invited to become (more) engaged in learning mathematics. Third, we recognised a wider socio-political context of schooling, referring to contexts outside classrooms that influence what occurs within the mathematics classrooms, which is operationalized through governmental policies on schools and the national curriculum, ideologies and school policies. (Nieto, 2010; Valero, 2004a) This school context refers to layers of school organization that shape possibilities for engagement. These include, for example, school structures such as timetables, school leadership and so on, as elaborated by Martin (2000) when addressing the complexity of reasons behind African-American youths' achievement or failure in mathematics education. Research by Biesta (2009) that questioned the purposes with schooling as such and societal talk about "lifelong learning" through the critical concept "learnification", and Popkewitz (2004) analysis of governmental documents and discussions of how children become "fabricated" within schools and societies depending on how education steering documents are formulated, are examples of research within the school context. Fourth, we recognised a societal context as the impact of societal discourses in mathematics classrooms. To give some examples, "specialness" when being "good at mathematics" (Mendick et al, 2009), or "discourses of belonging, being accepted, or rejected, and succeeding or failing" that often contribute to people giving up mathematics' studies (Black et al, 2009, p.29) are examples discourses within the socio-political societal context that impact on what occurs within the classrooms. For Wedege (1999), situation contexts also include the societal contexts that refer to aspects of students' life possibilities, wishes and concerns. However, in this study we refer to these aspects as socio-political societal contexts.

Agency

Given the concerns raised by the School Inspectorate (2010) about individualised work and issues raised by Bibby (2008) about the lack of choices about learning, the concept of students' agency needed to be incorporated in our study. A larger number of research articles on students' agency in mathematics education (e.g., Boaler & Greeno, 2000; Cobb, Gresalfi & Hodge, 2009; Wagner, 2007) draw on Pickering's (1995, p. 21) metaphor of the "dance of agency". This dance is a metaphor for the tension between individuals' initiatives and structures or conventions in

relation with mathematics per se. Pickering differentiated between different forms of agency: conceptual agency addressed how to develop meanings and relations between mathematical concepts, theories and principles. For example, conceptual agency allowed space for choosing methods when solving set mathematical problems or textbook exercises. On the other hand, disciplinary agency referred to the use of established solution methods and thus turned the agency concept to the discipline of mathematics itself rather than being seen as the sole responsibility of the person. However, we would argue that other aspects of agency are engaged in the 'agency dance' in mathematics classrooms. Students' agency might involve making decisions about tasks, time allocation and work distribution, assessment and accountability, who to work with and so on (Andersson & Valero, 2011). It also might concern what sorts of decision-making students' can make in relation to the learning of mathematics. The agentic dimension "lies in the ways in which we have control over the ways in which we respond to the situation" (Biesta & Tedder, 2006, p. 20-21) or in what ways students can act within classroom culture (Sullivan et al., 2006). Resistance can be defined as certain behaviours, for example not bringing pens to lessons or raising regularly questions such as "Why do we have to do this?" (Martin, 2000). Alternatively, resistance can be defined in a relational way, as students who do not accept teachers' invitations to learning activities and/or who challenge school authorities (Alrø & Skovsmose, 2003). In adult education, resistance is often explained as responses to learning situations with students' "lack of motivation and the symptom as non-learning" (Wedege & Evans, 2006, p.35). Gutiérrez (2010) points out that

> … resistance exists in forms that are not easily unearthed in interviews or classroom observations and, perhaps more important, that exercising agency does not necessarily mean choosing to fail. Students can knowingly play the game without letting the game define them. They do not do this individually, but rather as part of a larger group of people who are (re)writing society and education.

Here we pragmatically define *resistance* as a way of choosing, acting, talking, engaging or participating in mathematics classroom activities that are neither proactive for the student's mathematics learning or corresponding with the present classroom discourse and teacher's expectations.

Methodology

The study

The data for this article comes from a one-year study exploring Swedish upper secondary students' learning of mathematics within a social science program. Students commonly complete this program because it provides entry into university studies in the social sciences and language faculties. As well, students, who do not enjoy mathematics and thus do not want to take the alternative natural science or technical programs, see it as a good option.

Annica, in collaboration with Elin, a mathematics teacher, introduced teaching sequences that, enabled students' mathematics learning to be connected to societal topics, with the aim to bridge the gap between students' experiences in society and the mathematics classroom. From critical

mathematics education, three different aspects inspired the teaching. The first concerned how the mathematical topics related to societal contexts: "understanding mathematics as a tool for identifying and analysing critical features in society" and "seeing mathematics both as a tool for critique and an object of critique" (Skovsmose & Nielsen, 1996). Thus, these aims matched curriculum goals, which asserted that mathematics education for social science students should "provide general civic competence and constitute an integral part of the chosen study orientation" (Ministry of Education, 2000). The second aspect concerned the epistemological point that an educational practice was considered to involve learning and becoming, rather than a simple transmission of knowledge (Skovsmose, 2005). The third aspect involved how power relations between the actors supported a classroom environment where students could become agentic in a positive way towards their learning (Biesta & Tedder, 2006). Annica and Elin aimed for classrooms structures where students had access to and contributed to the discourse between participants. Annica and Elin wanted to promote a classroom discourse valuing students' ownership in their learning of mathematics, and hence opening spaces for students to achieve agency. Such orientation fitted very well with the national curriculum objectives for the Mathematics A course (Ministry of Education, 2000) and also addressed the critique raised by the Schools Inspectorate (2010) mentioned above.

Collecting information

In order to understand students' relationships with mathematics from their perspectives, ethnographic methods were used for data collecting (Hammersley & Atkinson, 2007; Willis & Trondman, 2000). Annica, as the main researcher, established a trustful environment through engaging with the students in both formal and informal settings. In this way, she interacted closely with the students and experienced the contexts and discourses. The data-collection included interviews, spontaneous conversations, a blog and students' project logbooks. Annica's research diary and field notes described the contexts and allowed the students' stories to be related to what went on in school and society at particular times. There were differences in the kind of stories that were told in interviews, the blog and the log-books. The interviews provided reflective data about the different context levels. The blog was a course activity and provided data mainly about task contexts. Students' actions and behaviours, and hence their reflections of their agency (including resistance) also appeared in the blog. The logbooks provided data about the students' learning, task contexts and situation contexts.

Data analysis

The analysis of the information was carried out in three stages. First, Sfard and Prusak's (2005) operational framework was used when analysing the data. They proposed to relate identities with reified, endorsed and significant stories about an individual. More operationally, they defined an identifying story to be *Reifying* through the use of words as *be, have, can, always, never, usually; endorsable* with the identity-builder and *significant* - if any change in it is likely to affect the storyteller's feelings about the identified person particularly with regard to membership of a community. The stories told or written by the students' about themselves or stories told by others about them were analysed by locating the verbs and adverbs that suggested a reified quality.

Stories with expressions such as "I am worried about maths-tests …", or "I have always hated maths…" indicated reification. The students were the significant narrators of these identities and they drew of stories from their parents and their mathematics teacher (Andersson, in press). Second, to clarify *why* these particular narratives were told or written at specific points in time, the data was organized in chronological order on a timeline that became a 'storyline'. One 'storyline' was produced for each student. Events and incidents at the school were added on to the 'storyline', as well as the teacher's assessments comments and examination grades. The purpose with this exercise was to connect the students' identities as narratives with occasions in other aspects of the contexts for these narratives. Third, stories that indicated action and aspects/qualities of achieved agency as e.g. "today we went to…" or "I decided not to do…" or "today I talked a lot and probably disturbed others" where added on the timelines. Sentences with the word 'we' were accepted in those cases they indicated action and were clearly referring to a working group. The "we" expressed in for example sentences as "we decided to…" and "we had to …" are attributes of students' agency (Wagner, 2007).

When analysing these storylines it became evident how the words the students' used to express their experience changed in relation to different contexts and what the present discourse allowed them to tell. In other words, the analysis made visible the change in the ways the students told stories about themselves when there were shifts in the discourses in the classroom.

Henrik's identity narratives in context

We decided to explore Henrik's identity narratives, generated from multiple sources, in his own words, side-by-side a contextual description of the circumstances in which they were narrated, through Annica's research narrative and comments from Elin and Henrik's father. We decided to communicate the narratives in a double column because this form allows to clearly connecting Henrik's identity narratives with the different contexts and available discourses in the mathematics classroom. We strive for a thicker account of the data that had not been possible with the presentation of short excerpts from Henrik's words. As Roth and McRobbie argued (1999, p. 518 in Nolan & de Freitas, 2008, p. 6) "w/ri(gh)ting research that "allows us to resist epistemic violence of traditional master narratives by encompassing heterogeneity" is a challenge that we try to take here.

In each of the following seven moments we introduce both Henrik's identity narratives with Annica's research narrative in the context of classroom discourses. We also put forward an analysis of the changes that we see happening in the characteristics of Henrik's stories about his engagement in mathematics education.

Identity narratives in the context of his previous mathematics education

Let us start with an introduction of Henrik. When the students began upper secondary school, the students were asked to write an introductory letter to Annica about their previous relationships with mathematics education and their view of the Mathematics A course, the first out of two compulsory mathematics courses they would complete at this school.

Hi, I am Henrik and I have lived my whole life in [a middleclass area in a Swedish city] except for one year in Cambridge, USA, when I was ten years old. I am interested in a lot of things; graphic design, photographing, fishing at our summerhouse together with my brother, and of course to be with friends. I am also interested in sports like tennis, soccer and badminton.

In general I have always disliked mathematics. It has never felt meaningful for me. The problem was not that I didn't understand mathematics. I was usually quick on that. The most difficult was probably that I did not experience mathematics as meaningful. I could not relate this knowledge to something I would need further on. To sit down, and focus, calculating the same type of exercises again and again felt so meaningless. I still remember the book from my first years in school; when I got this thick book where one should fill in number after number, exercise after exercise. All lessons were the same, we only calculated textbook tasks, and raised our hands when we needed help. Group work in mathematics? No, I have never experienced that. Is it possible?

I think I ought to have had higher grades in mathematics, but it became so boring. I tried to calculate but then I talked to friends instead. I felt disappointed, like in ninth grade, the last year of lower secondary, I got G^1 (pass), and I thought can't I do better than this? When I asked teachers and friends; why do we actually study mathematics? What am I going to use this knowledge for? I usually received the answer; you need it for upper secondary school, you will need it in your coming work life. But, I wonder, do I really have to calculate a ball's volume if I become an artist or a photographer?

My friends think mathematics is boring, as most of us young people do. However, they do what the teachers tell them to do but I think one should reflect more personally about what is really meaningful.

My thoughts about my future: When I was younger I always said I wanted to become an architect. Later, when I got older I started to use Photoshop on my computer, and then Illustrator, and After Effect. I did a poster for a festival and did a practise period on an advertisement office. Now, lately, my thinking has been more about industrial design. I want to design things that can become a reality and that you can use with your hands. I have become interested on Apple products, and those who have created the IPod, IPhone and MacBook. I became a fan of Jonathan Ive who works as a designer in Apple. Now my goal is to study industrial design at university. I found a program at Umeå University that seems good.

Henrik walked into the classroom, together with all his classmates, and sat down in the middle of the room at a table for five students. He was wearing jeans, a blue t-shirt and trainers, similar as the other boys in the class. His black school backpack was hanging loosely on his left shoulder. He gave a thin and gangly impression, with long arms and legs, just as teenage boys usually do. His glasses gave him an intellectual look. Overall, he gave the impression of being a well-behaved youngster. He paid attention to what the teacher said in a polite way. His books were opened on the advised pages, and his pencils were in place in front of him.

- Well, as a parent I believe it is important that they get an education that will give them a job after graduation. Concerning Henrik, I think he will become, hm, maybe an architect, not a doctor like me. Henrik has so many good qualities; his calm and mature way of being… not like I am… his calmness is really pleasurable. My wish for him is that his own personal dreams will come true.

- About mathematics education, well, Henrik always needs to se a point in what he is doing. To learn or babble knowledge by heart, to learn something not anchored in reality does neither make sense nor work for Henrik. He almost honours not to learn the multiplication tables or mathematical formulas; for him that is like dead knowledge. It seems as if mathematics teachers never have reflected on these things (Henrik's

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[1] In Sweden we have four grading levels in upper secondary school: IG (not pass), G (pass), VG (pass with distinction) and MVG (pass with excellent distinction).

The first stories illumine Henrik's reflections on his mathematics education background and his designated identity in far future to become an industrial designer. His intention to participate in the compulsory Mathematics A origins in the requirement in the larger socio-political context of mathematics education; in other words, mathematics as the gatekeeper for further education. Black et al. (2009) showed that British students of the same age as in this research accounted for factors as future aspirations, e.g. exchange for value for becoming university students and vocational 'use values' for possible coming workplaces, as motives for students to engage in mathematics education. These findings were confirmed by Henrik's reasons to participate in this course.

Henrik told he experienced his previous mathematics education, based on a situation context of teacher instructions, and task context as textbook work within what Skovsmose (2001, p.123) labelled the "exercise paradigm", as boring and disappointing. Henrik was dissatisfied with his prior results in mathematics, and specified the boring way of organising the teaching and learning of mathematics as the reason for this, as discussed by Bibby, (2008). His identity narrative at this particular moment in time indicated that he did not enjoy mathematics, but recognised within the socio-political societal context the importance of learning mathematics and passing the course to reach his designated identity, namely to become an industrial designer.

Identity narratives in the context of the first group work

It was very comfy to choose and decide oneself; we brainstormed together and came up with good ideas about what we could do. At the same time it felt very abstract that this could be possible, but it still was. Fun!! (Henrik, blog, 16-09-2009)

Our goal was to become bakers and then start a bakery. We searched for the world's best and most famous baker education and found one in France. I mean that this did not seem so trustworthy that it would be a real possibility... far from the position I am in today as an upper secondary student. But I really got the opportunity to live myself into the situation through this project, to step by step plan out what needed to be done to reach that goal. In that sense it became much more than only a mathematics task ☺ (Henrik, blog response, 17-09-2009, original smiley)

Today Pelle and I checked on the information about the baker education and checked the costs. We have not chosen what bank to use yet. We will do that tomorrow. Everything works as planned ☺. (Henrik, blog, 22-09-2009, original smiley)

I wonder how the idea about the baker education came up; it was very different to the contextual choices made by the other groups. However, the boys seemed to enjoy what they were doing. They laughed a lot and there were a lot of "bakery" jokes going on at their table. On the other hand they also seemed to have the intention to work effectively to finish the project on time. "No more time spent than absolutely required" seemed to be the working discourse in this particular working group. They made decisions on how to find information from different sources and collaborated well in a sharing way. I never heard them ask for "help", they rather posed questions to the teacher as "What do you think about this...?" I did not hear them at any time ask "Why do we have to do this?" The three boys enjoyed presenting their becoming-bakers-project to the

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I was most content with the way we got everything together and with our poster. Our poster was very distinct; one got a clear picture about the costs for becoming bakers. Obviously there are things we could have done better. We could e.g. have calculated on costs for living. Buying an apartment in Paris and then sell it with a profit after our education could have been smart. (Henrik, blog, 30-09-2009)

This project was very interesting because it was… interesting. We decided to not use the book. So it was reality for me. This I might use in my future. (Henrik, interview)

whole class. They made a large colourful poster where their percentage calculations, interest rates and results were presented, together with pictures of Paris and lovely bread. They answered the other students' and the teacher's questions in a clear and confident way. They showed they knew the mathematical topic well. They also made critical reflections on their work and their collaboration. (Annica, fieldnotes)

In this situation, the boys used verbs indicating action. Verbs such as *choose, decide, become* and *will do* indicate agency related to the situation context, which at this time was the "Making your dreams come true"-project, relating to interest rates and percentage calculations where the students had access to the internet, books and poster materials for their presentations. The "we" expressed in "we could have…" and "we decided to…" are also attributes of students' agency (Wagner, 2007). The analysis of adjectives and adverbs Henrik chose in relation to the task context revealed positive connotations (fun, possible, real content), indicating that he experienced learning mathematics as more meaningful at this time when the students were asked to reflect on what they would be interested in borrowing money for and how to find out which loan offers were smart to take. The news headlines at this time indicated that an increasing number of young people in Sweden (18-25 years old) were in financial trouble after taking "quick-money" loans received through, for example, mobile phone texts. If youngsters were placed on Swedish tax enforcement or police registers for not repaying debts, their possibilities to get bank loans for further studies or a house later in life is decreased. So, Henrik ascribed meaning to the whole educational situation, not only meaning in the conceptual sense Skovsmose (2005, p.86) criticized in the following way:

> […] the discussion of meaning in mathematics education cannot be structured by the priorities of conceptism. Instead, the basic discussion of meaning has to do with the meaning of the activities in which students are involved as part of an educational process.

At this time Henrik participated in a different way than what he had expressed in the letter about his previous mathematics education. He was proactive and worked well. During this project Henrik took on the different pedagogical discourse. He did not position himself as a help-needer, he rather asked for advice or discussed mathematics or the contextual framing of the project in a reflective manner with the teacher and his friends.

Identity narratives in the context of textbook work

Today I have learnt new techniques on how to simplify equations. What this has meant for me personally I do not know. I have been concentrated so I am tired (Henrik, blog, 07-

The three boys sat together, calculating the algebra textbook tasks assigned by their teacher. The first project "Making your dreams come true" was now followed by two

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- What's up, Henrik? Annica asked the question one particular day when Henrik was leaving the classroom looking weary and low with hanging shoulders. Henrik answered:
- Sometimes one comes here and knows one only has to do them, all the boring exercises, to learn this stuff the teacher tells you to do.
- What do you mean?
- Well, somebody has to take the decisions for one, because I don't know what or how I have to do to learn this stuff or when to use it (with a resigned voice)
- Do I understand you right here; do you mean it is boring because you can't take any decisions on what to do yourself?
- Yes, exactly. (Henrik, conversation)

weeks of textbook work focusing on one-grade equations and algebraic simplification, complemented with problem-solving tasks. This book chapter was considered by the teacher to be a repetition from prior mathematics education, and thus should not be too complicated for the students to complete. There where spaces for students to discuss the mathematical content and tasks with each other, however most students raised their hands and asked for Ein's attention, positioning themselves as requiring "help" in stead of discussing challenges with each other. Usually at least one of the students at Henrik's table had forgotten the calculator or the textbook at home. They either borrowed books or calculators from each other or used the calculator in their mobile phones when the teacher did not notice. (Annica, fieldnotes)

During these weeks of teaching organised with teachers' instructions and textbook work, there were fewer entries from the students in the blog than usual. The entries were predominantly written with negative comments or in a sarcastic way, disparate from prior entries during the percentage project. Henrik's number of entries on the blog decreased during these weeks to only one comment, which indicated tiredness and meaninglessness through adjectives and adverbs such as e.g. "boring" and "tired". However, the designated identity as completing the course was strong so he decided to act and follow Elin's (and the book's) instructions even if he did not recognise the meaning of the tasks: "algebraic expressions, and formulae required for solving problems in everyday life and in other subjects in their study orientation" (Ministry of Education, 2000). The stories Henrik told during these weeks, in a situation context he told he recognised from his previous school, were told with phrases such as "has to do", "the teacher tells you to" and "take the decisions for you" indicating resistance or passivity. In conversations Henrik reflected on restrained possibilities for achievement of agency that resulted in tiredness feelings but he also acknowledged that this some times has to be the case. The actions he decided to take during these weeks were related to his intention/designated identity to pass the course, hence the socio-political societal context. The quality of his participation could be discussed: Do we want tired students walking out with low shoulders or active creative students walking out with proud feelings and energy? Brown, Brown and Bibby (2008) showed that bored was the word used most often by 16-year old students to describe their attitude in a questionnaire on students' affective relationship to mathematics. Henrik would probably agree with these findings. The questionnaire was distributed to 1997 students in the same age as students enrolled in the first mathematics course in upper secondary schools in Sweden. One of the reported reasons for boredom was students' lack of creativity. Similar to the findings of the School Inspectorate (2010), Bibby (2008), although working with students at the end of primary school, suggested that boredom indicated lack of stimulation and challenge. She also found that students' lack of control over tasks and direction was connected to feelings of boredom. Perceived dislike, lack of

relevance was also important reasons for students not continuing with mathematics (Brown et al, 2008). Henrik would probably have answered in the same way as the British upper secondary students who were asked for reasons for their feelings of failure in mathematics. They suggested that "they do not understand parts of what they have been taught, and point to the predominance of routine and formal work on algebra and multiplicative reasoning" as reasons for feelings of failure (Hodgen, Küchemann, Brown & Coe, 2010).

Identity narratives in the context of mathematical argumentation

- What did you think of the flyer day the days before, when it said 'math-day' in your time plan?
- Honestly? I did not have any positive thoughts about it at all. I thought it should be a tiresome [jobbig] day, like the traditional mathematics a whole day.
- What did you think during the day?
- I felt, hm, content, I had learnt something new [with a smile].
- Have you thought about the day afterwards?
- Yes, actually I have done that; about how one can influence and have impact on other people with support of mathematics. When we have done these special things mathematics has like become broader, one can really use it for something, which is interesting. Like e.g. when I watch the news, how they represent some things there. And, yes, I saw 'The age of the stupid' and the diagrams they showed when explaining the difference between different countries was amazing… [Continued talking about the film in this way] (Henrik, interview, 11-2009)

Communication trouble on school level, labelling our work-shop themed on the United Nations Convention on The Rights of the Child simply "Mathematics day" resulted in a higher number of absent students than usual. And a number of those who came arrived late, as if it was a drop-in day. So the start of the day could have been better from the teachers' and my point of view.
Yet, Henrik was there. He and his friend chose the contextual topic children and AIDS in Africa. They sought contextual information on the internet. For their presentation they created four colourful different flyers, all computer made with a graphical design that supported their message about the high numbers of children suffering from AIDS in Africa. Their News paper flyers hang in the school cafeteria for a couple of weeks after the "mathematics day". (Annica, fieldnotes)

This day coincided with a higher number of sick students, dentist and doctors' appointments than usual (Teacher, interview, 10-2009). Henrik told two different narratives about his relationship with mathematics at this time when he was invited to a full day workshop, *"Newspaper Flyer Project"*, on how to use mathematics to produce powerful arguments. The first narrative concern the wider school context where communication problems resulted in that he, as the rest of the students, was not aware of the types of activities that would take place. He talked about negative expectations of a whole day of mathematics and not feeling well if he had to participate. The second narrative refers to Henrik becoming critical of mathematics and mathematics role in society and referring to that as learning of mathematics. The teaching was inspired by

Frankenstein's (2008) work on numerical information and quantitative argumentation. It seams as if it was the critical discussions and realising what impact mathematics has in society that made him engage in mathematics learning during the day. The phrases "how one can influence and have impact on other people with support of mathematics" and "one can really use it for something, which is interesting" indicated again meaningfulness to the whole educational situation, not only for the mathematical concepts, percentage, fractions, whole numbers and decimal numbers, per se. His experiences from the day resonates with students described as "wanting to be treated as if they mattered and as if they were agents of change in the world" in an American context after working with topics as domestic abuse, racial profiling and poverty during mathematics lessons (Stocker & Wagner, 2007, p.19). The possibilities for using mathematics as a tool for social justice appealed to Henrik. During an interview eight weeks after the argumentation day he queried how mathematics could be used in society. He searched for potentials rather than complaints when talking about mathematics in relation to a critical and societal context at this time.

Identity narratives in the context of a larger societal project

We could not agree about one of the survey questions. "Do you believe that ecological products are more sustainable for earth in the long run?" I wrote it, but Pelle thought it was obvious and thus superfluous. I asked him what proof he had that the answer was obvious? How do we know that? I get annoyed when he only criticises without coming up with other suggestions instead. Andreas and I have all ready made so many compromises with him.

Today I was home from school sick but I had contact with Pelle and Andreas on Google Docs. We wrote the introduction and tried to formulate our project objectives. We also wrote our time plan together on Google Docs.

Today we reflected on diagrams and decided which types of diagrams would suit our purposes and questions best. We also discussed how we could manipulate them to our advantage ☺ (Henrik, logbook)

Today I learnt that PowerPoint does not do what I want the program to do. (Henrik, logbook)

Tough discussions and negotiations went on at the boys' table. Henrik and Andreas wanted to continue with the prior bakery project, however now in relation to ecological sustainability. They intended to create an "Ecological bakery" and look at cost differences between conventional and ecological bakery products. But the third boy, Pelle, did not like this idea, and he was very stubborn. Henrik was irritated and wanted to proceed further and focus on the project instead of using ("wasting") time on collaboration issues. It took the group some time to compromise and decide what to focus their project on. In the end they decided on a food topic, hence to work with 'Kravmärkt' (organic) food. After the topic was settled the group worked well and focused even if they did not all agree on all matters.

There were some concerns about how to formulate survey questions, but these discussions were more solution related with a purpose to pose the 'best' questions. They sometimes asked me: Annica, what do you think is smartest…? I always asked them what possible answers they expected the other students to give to their suggested questions. Hence we discussed for example how the questions could be interpreted in different ways. So we rather examined different possibilities than me giving my opinion, explanation or 'correct answer'. This was the way we intended to work within the different discourse.

What amazed me most, but also Henrik's father and the other teachers was the groups' decision to work with their project on the platform Google Docs. This was the first time the other math teachers and I had experienced a

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I have learnt much more this way than if I only had done book calculations. I believe I receive more valuable (värdefull) knowledge because this way of working is more real with a stronger connection to the 'real' reality (Henrik, logbook).

I learnt new mathematics in the last project we had, the investigation. How to make ones interests to have impact on others through mathematics and still learn, that has been fun. It was harder, and required more work but was more fun (Henrik, interview, 12-2009).

The teacher wrote: I evaluate that Henrik has reached the highest level […]. Wow, I got rewarded, I can do maths, it is recognised!!! I can't believe it, MVG in maths, I am so happy!!

group of 15-year old social science students that, without anybody else suggesting or requiring them to, took the initiative to work on a platform like Google docs, in mathematics, to create a good project and be able to collaborate outside school hours.

- I am astonished, through Google, in real-time, they sit in different locations and rectify their writings. So their project is very much alive and in progress. I believe they have developed (personally) and that they learn a lot through this way of working (Henrik's father, interview, 11-2009)

Henrik received the highest grade, MVG, implicating that he presented the statistical investigation with correct mathematical language using appropriate concepts and terminology, that he had chosen appropriate diagrams and arithmetic mean values and could argue for his choices. He reflected on possible sources of error and how these could have been prevented. Henrik told me was convinced that he would not have been able to account for these criteria on a written mathematics test. However orally he clearly in a correct and convincing way presented his results with a well done power-point, discussed his results and answered questions in front of an audience of two teachers, one researcher and 50 students.

A number of news headlines during the autumn semester related to the United Nations Conference on Climate Change in Copenhagen (COP 15), so its themes, climate change and climate sustainability, became the societal context for a statistics project on sustainable development and "ecological footprints" (Wackernagel & Rees, 1996) in collaboration with the environmental-science teacher. In this situation, Henrik again talked about his relationship with mathematics in more positively active ways – in spite of the disagreements that arose within the group. However, it is important to recognise that socialisation processes are an important part of schooling (Biesta, 2009) and also an important part of students' identity work (Lerman, 2000).

The project ran for three weeks, with a whole day of displaying results with power points, papers, posters, presentations, discussions and interactions in the fourth week. The project was designed as a "three-stage rocket". First, the students decided on working groups, chose topics, and designed surveys in parallel with gathering information. Second, they conducted, analysed and reported the survey in a descriptive way. The students were also asked to argue for their view with support of statistical data and specifically-designed diagrams. In that way they learnt how to use diagrams and statistical information to strengthen arguments and thus experienced how different ways of presenting statistical information affect people's judgements. Third, all collected information was summarised into a website, My Planet, with the goal to calculate how large each student's "ecological footprints" was on earth and how many planets would be needed if the entire population lived like these students. We compared and problematized data from different countries and examined critical issues about different ways of living. The verbs Henrik used during the focused working sequences were again indicating action: we wrote, we decided, we reflected etc. Within this wider project, he made decisions about task contexts and situation

contexts such as time and work distribution and assessment together in the group. He told that this way of working "was more fun" even if it "required more work" and time than what he calls a "traditional" way of working with textbook exercises.

Henrik reflected more on his personal learning at this time. He stated that the knowledge was "more valuable" as it connected to his "reality". Henrik initiated collaboration work on a web-based platform that allowed the group to work on the project outside scholl hours. His narratives resembles students' experiences provided by Brown (2009, p.183) that "explaining and arguing may be viewed by students as being social resources that have the potential to promote mathematical understandings and an awareness of the 'self' as acting in a classroom community".

Identity narratives in the context of preparing a geometry report

The sixth sequence we share are two e-mails that Annica received after she left the school. As Annica does not account for any research narrative at this point, Henrik's e-mails this time stands alone.

> Hi Annica
> I am very sorry that I have not responded to your e-mail until now. The reason is that there has been some mathematics schoolwork that has not felt meaningful and thus I lost some of my study motivation. Before the Easter holiday the topic in maths was functions. That was 'usual' mathematics however with some practical laboratory tasks that made a few of those lessons more interesting. We then got examined with a typical math test.
> Now I have got back some of my study motivation because now we do some interesting and meaningful stuff, in maths it is geometry. We could choose, for the geometry sequence, to get examined through a "usual" math test or, if one wanted, to suggest another way. I decided to do a report about Tetra Pak's[2] legendary milk package.
> I will calculate volume, angles, area etc. that I then will use to create a smaller miniature milk package model. After that I will reflect on usefulness, how much material is required and a conclusion about how it can get better and improve. If I have time I will create a personal variety with a miniature milk package containing 1 dl. That feels meaningful! ☺
> Cheers Henrik (e-mail, 24-04-2010).

> Hi again
> I finished the Tetra Pak-project some time ago and got response from Elin yesterday. It was a fun project to do and I got a very good grade too. It was interesting to try to do a project in mathematics and with support of mathematics investigate and strengthen out a problem and a question. I am very proud of my work. […]
> Cheers Henrik (e-mail 14-05-2010)

These two e-mails showed Henrik's different ways of talking about himself as a mathematics learner in relation to the situation contexts and task contexts. In Henrik's two e-mails we notice a swift in Henrik's way of narrating his identities as a mathematics learner. During periods with textbook work he acted in similar ways to how he described his previous engagement with mathematics. He lost his motivation to study when mathematics became as "usual" and examined

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[2] Tetra Pak, grounded by R. Rausing, is a legendary industry in this part of Sweden, famous for inventing the milk package Tetra Pak. So Henrik connected his geometry project to society and a geographically important invention for the region.

through a "typical math test". Even writing e-mail to Annica became tiresome for him at this point in time. He waited two months before he responded. However, he became proactive when realising opportunities for taking personal decisions on contextual content, learning and assessment strategies in the geometry sequence. The contrast between Henrik's ways of talking about himself during the functions sequence and geometry sequence is obvious. Phrases as "has not felt meaningful", "I lost some of my study motivation" and "we then got examined with" indicating passivity, meaninglessness and lack of power during the functions sequence contrasted "we do some interesting and meaningful", "I decided to do a rapport", "I then will use", "how it can get better and improve" and "if I have time I will". The latter all indicated qualities of action and agency achieved in relation to task and situation context. They also give evidence for the impact agency achieved in relation to contexts had on his decision to participate and learn mathematics. He accounts for meaningfulness in relation to the educational situation and personal ownership during geometry work.

We also note that Henrik received higher grades in mathematics when he had space to account for his knowledge in other ways than to sit a written mathematics test. Writing a mathematics document or presenting orally allowed him to take discussions and show competencies he told he could not account for during ordinary mathematics tests. These findings relate to the "system of competence", constructed by the participants in a practise when negotiating what students are supposed to be accountable for and in what ways they need to be accountable to become successful mathematics students (Gresalfi et al, 2009). Or, as Gutiérrez (2010, p.7) reflects:

> Rarely do our definitions of success include self-actualization—the idea that we
> should be allowed to become better people by our own definitions, not just those
> prescribed by schooling. That is partly why discussions of identity and power are
> so important—because the goals we have for students may be disconnected from
> the ways in which they see themselves now or in the future.

Identity narratives in the context of the National Mathematics A Examination

The last narrative origins from an e-mail Annica received at the end of the teaching year, after Henrik completed the Mathematics A course. This can be perceived as Henrik's epilogue where he summarised his experiences through the course. Henrik's mail stands alone as Annica does not account for any research narrative at this point.

Hi Annica
I am very content with the semester and feel I have done as best as I could. The most interesting and worthwhile have been the projects and the theme works. Then it has felt real and meaningful. We have not only repeated old mathematical facts – we have also really used it to shape something new and creative.
Last week I had the national test, and that is not exactly my favourite. I think it went rather poorly – and I felt again I lost my interest in and motivation for mathematics. I received extra time, as I am dyslectic, but did not manage to do all the tasks within the time limit. I have learnt that there is a connection between dyslexia and difficulties in learning mathematics. Mathematics is as learning a new

language, with its "grammar", however it is lucky that dyslexia has nothing to do with intelligence!
I got a much larger interest in mathematics after the projects we did when you were here, and it is interesting to see the possibilities that mathematics create in problem solving. That has been good.
Henrik. (Henrik, e-mail, 05-06-2010)

At this particular point in time Henrik talked about himself as "being dyslectic". During all circumstances Annica and Henrik talked in and outside the classrooms, in informal or formal situations, or shared written information, Henrik never ever indicated that he was diagnosed with dyslexia. Subsequently, it was never of importance for him to talk about his dyslectic problems until he sat the National Examination Test and found himself in a situation he recognised from his previous mathematics education as "not exactly my favorite". His teacher stated that this was the first time during the course year that Henrik had required and received extra time in a test situation due to his dyslectic diagnose (Elin, conversation, 05-2010). Not once was this particular identity; being objectified as a "dyslectic student", important or significant for him to talk about – until it was required to pass a national test reminding of prior identities of exclusion in mathematics education. His identity narratives told with a sense of pride and experience of meaningfulness, achieved primarily through projects and critical reflections on mathematics per se, faded away when he performed the national test. He also thought he performed badly on the exam. However, he passed with distinction (Elin, assessment, 06-2010).

Concluding discussion

The analysis of the storyline of Henrik's identity narratives in relation to the diversity of contexts highlighted the complexity and variability of his engagement in mathematics learning. The assertion that students' narrative identities are discursive and emerge in the discursive practices in which they participate is only an empty assertion, if the analysis of the way in which students produce identity narratives is not accompanied by a visible analysis of the contextual practices where the narratives are expressed. In this paper we showed that identity narratives are contextually bounded to the pedagogical discourses in which they are made possible. Relationships between identity narratives and the contexts the narratives are bounded to open a window to why Henrik narrated himself as a learner in particular ways at specific times, and why he became agents in his learning of mathematics in some situations but not in others. It was not the mathematical contents per se, but rather the array of contexts in which the activities for learning mathematics took place, what allowed him a space to act and tell stories about himself and his experience in qualitatively different ways. We took the challenge that the understanding of 'contexts' in mathematics education research tends to be restricted (Morgan, 2006; Valero, 2004b). We can document in this study that the task, situation, school and societal contexts intermesh as referents and groundings for the discursive practices in classrooms, including that one of students constructing identity narratives.

Our study also focused on students' agency, hence with what qualities they decided to act and engage, or resist and disengage, in learning activities. Agency, here defined as achieved in an

ecological understanding (Biesta & Tedder, 2006) was an important feature for Henrik at those times he talked about experiencing meaningfulness in mathematics education. Hence the impact he had on the contextual framing made the mathematical contents interesting for him at some points in time. In Henrik's case the spaces were he could act and take personal decisions became important for the qualities with which he acted and also for assessment and finally his grades.

Boredom, so often experienced by students during mathematics lessons, is one reported reason for students' disengagement (Brown et al, 2008). Reported reasons for boredom as students' lack of creativity, stimulation and challenge (Bibby, 2008; School Inspectorate, 2010), but also students' loss of control over tasks and direction (Bibby, 2008), and perceived sense of meaninglessness (Lange, 2009) became verified in this study. In addition, we showed that when students themselves took decisions on task and situation contexts, they did not talk about being bored. The case was rather the opposite. They engaged, influenced classroom discourses and took responsibility for learning. When the students themselves decided on task and situation contexts neither of them asked: "Why do we have to do/learn this?" On the contrary, Henrik's repeated identity narratives of confidence, and meaningfulness show that the experience of mathematics education and learning was interesting and challenging for him.

Henrik's identity narratives, chosen as examples of students talk in the larger research study, indicate that identities are not always as consistent as they often appear to be portrayed in literature. These results indicate problematic issues when students become objectified (Nieto, 2010; Sfard, 2008) as result of our research categorizations. As researchers, if we had only studied Henrik's identities in relation to situation contexts as project work or societal task contexts we could have concluded that Henrik was a young man who engaged in mathematics learning and performed well with high grades in mathematics, who understood mathematics as a meaningful subject and reported inclusive identities in mathematics education. Doing the same analysis during situation contexts with allocated individual work or with task contexts as assigned textbook exercises would have reported Henrik's identities as a young man who found mathematics boring and tiresome, performed poorly and acted with resistance during mathematics lessons. These results illuminate problematic issues when we as researchers generalise students learning of mathematics and conclude that certain labelled groups of students act or behave in certain ways, or that certain pedagogies are to prefer in front of others. Henrik's case emphasises that you are not always a bored student, or your objectifying dyslectic diagnose does not implicate certain ways of acting in mathematics classrooms.

The changes of the students' identity narratives as such supported us researchers to understand the roles contexts play when students decide to engage or not in learning of mathematics. The importance of these findings for education practices supports Gutiérrez (2010) elaboration of students' identity work. If teachers recognize that students positioning of themselves is identity work, hence that students' resistance is "connected to identity-in-the-making: resisting narratives that position the student as inferior, unworthy, abnormal, or on the margins of the local (e.g., classroom) culture" (p.17) possibilities for teachers taking action occurs. As Gutiérrez (2010, p.17) suggests:

Savvy teachers can then use this understanding to think about how to be empathetic toward students who are doing this self- protective work and to think about how to better support

students negotiating the mathematics classroom (e.g., opening up the range of identities), without prescribing the kinds of identities that are seen as valid.

That is to say, we believe most mathematics education researchers and teachers not only reflect on the social and political discursive practices and subsequent consequences of mathematics education but also want to take action through their work as researchers and teachers. We need to be considering the forms of mathematics and in what contexts they are introduced to students. We also need to consider how power is negotiated in the mathematics classrooms, and what opportunities we have for allowing space for students to achieve agency in their learning and accountancy of mathematics. When bearing in mind the importance of agency and how individuals made sense of their surroundings with/through/in mathematics, we began to open up possibilities for rethinking what mathematics and mathematics education research does along the way. It is when we as researchers and teachers pose the question "What identity narratives do we want the students to tell in relationship with their mathematics education?" with the emerging sub questions our findings indicate: "What are the possibilities for students' achievement of agency in my mathematics classroom?" and "How can we as teachers and researchers support students identity work?" that possibilities for students telling different stories occur.

References

Alrø, H. & Skovsmose, O. (2003). *Dialogue and learning in mathematics education: intention, reflection, critique.* Boston: Kluwer Academic Publishers.

Andersson, A. (in press). A "Curling Teacher" in Mathematics Education: Teacher Identities and Pedagogy Development. *Mathematics Education Research Journal.*

Andersson, A. (in press,b). Interplays between context and students' achievement of agency. Accepted for presentation and proceedings at Cerme 2011, Working Group 10 in Poland.

Andersson, A., Meaney, T., & Valero, P. (under submission). "I am [not always] a math hater" Students' [dis]engagement in mathematics education".

Andersson, A., & Valero, P. (in press). Examining a critical pedagogical discourse for agency and social empowerment. In P. Ernest and B. Sriraman (Eds), *Critical Mathematics Education: Theory and Praxis*. USA: Information Age Publishing.

Bibby, T. (2008). The experience of learning in classrooms: moving beyond Vygotsky. In T. Brown (Ed), *The Psychology of Mathematics Education: A Psychoanalytic Displacement* (pp. 37-60). Rotterdam: Sense Publishers.

Biesta, G.J.J. (2009). Good education in an age of measurement: on the need to reconnect with the question of purpose in education. *Educational Assessment, Evaluation and Accountability, 21*(1), 33-46.

Biesta, G. & Tedder, M. (2006). How is agency possible? Towards an ecological understanding of agency-as-achievement. *Working paper 5*. Exeter: The Learning Lives project.

Black, L., Mendick, H., & Solomon, Y. (Eds) (2009). *Mathematical relationships in education*. New York: Routledge.

Black, L., Williams, J., Hernandez-Marritnez, P., Davis, P., Pampaka, M., & Wake, G. (2009). Developing a 'leading identity': the relationship between students' mathematical identities and their career and higher education aspirations. *Educational studies in mathematics, 73*, 55-72.

Boaler, J., & Greeno, J. G. (2000). Identity, Agency, and Knowing in Mathematics Worlds. In J. Boaler (ed), *Multiple Perspectives on Mathematics Teaching and Learning*. London: Ablex Publishing.

Brown, R. (2009). Teaching for social justice: Exploring the development of student agency through participation in the literacy practices of a mathematics classroom. *Journal for Mathematics Teacher Education 12*, 171-185.

Brown, M., Brown, P., & Bibby, T. (2008). 'I would rather die': Reasons given by 16-year-olds for not continuing their study of mathematics. *Research in Mathematics Education 10*(1), 3-18.

César, M. (2007). Dialogic identities in students from cultural minorities or students categorized as presenting SEN: How do they shape learning, namely in mathematics. In ScTIGGroup (Eds.), *Second Socio-Cultural Theory in Educational Research and Practice Conference Proceedings*. Manchester: Manchester University.

Clarkson, P.C., Bishop, A. & Seah, W.T. (2010). Mathematics education and student values: The cultivation of mathematical wellbeing. In T. Lovat, R. Toomey & N. Clement (Eds.). International research handbook on values education and student wellbeing (pp. 111-135). Dordrecht: Springer.

Cobb, P., & Hodge, L. L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical Thinking and Learning, 4*(2&3), 249-284.

Cobb, P., Gresalfi, M., & Hodge, L.L. (2009). An interpretative scheme for analyzing the identities that sudents develop in mathematics classrooms. *Journal for research in mathematics education 40*(1), 40-68.

Frankenstein, M. (2008). Quantitative form in arguments. In J. F. Matos, P. Valero, & K. Yasukawa (Eds). *Proceedings of the fifth international mathematics education and society conference.* Lisbon: Universidad de Lisboa and Department of Education, Learning and Philosophy, Aalborg University.

Freire, P. (1992). *Pedagogy for the oppressed*. New York: Seabury Press.

de Freitas, E. (2010). Regulating mathematics classroom discourse: text, context and intertextuality. In M. Walshaw (Ed). *Unpacking pedagogy: new perspectives for mathematics classrooms* (pp.129-152). USA: Information Age Publishing.

Greer, B**.,** Mukhopadhyay, S., Powell, A. B. & Nelson-Barber, S. (Eds), (2009). *Culturally responsive mathematics education.* New York: Routledge.

Gresalfi, M., Martin, T., Hand, V. & Greeno, J. (2009). Constructing competence: an analysis of students' participation in the activity systems of mathematics classrooms. *Educational studies in mathematics 70*(1), 49-70.

Gutiérrez, R. (2010). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education,* (Equity Special Issue) 1-32.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*(1), 37-73.

Gutstein, E (2006). *Reading and Writing the World with Mathematics. Towards a pedagogy for social justice*. New York: Routledge.

Hammersley, M., & Atkinson, P. (2007). *Ethnography: principles in practice*. New York: Routledge.

Holland, D., Lachicotte Jr, W., Skinner, D. & Cain, C. (1998, 2003). *Identity and Agency in Cultural Worlds*. London, Cambridge: Harvard University Press.

Hodgen, J., Küchemann, D., Brown, M., & Coe, R. (2010). Lower secondary school students' attitudes to mathematics: evidence from a large-scale survey in England. *Research in Mathematics Education, 12*(2), 155-156

Hodgen, J., & R. Marks. (2009). Mathematical 'ability' and identity: a socio-cultural perspective on assessment and selection. In L. Black, H. Mendick and Y. Solomon (Eds), *Mathematical Relationships in Education: Identities and Participation* (pp. 31-42). London: Routledge.

Ingram, N. (2009), Engagement in the mathematics classroom. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In Search of Theories in Mathematics Education: Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol 2, pp. 233-240). Thessaloniki: PME.

Lange, T. (2009). Difficulties, Meaning and Marginalisation in Mathematics Learning as Seen through Children's Eyes. Doctoral thesis. Aalborg: Aalborg University.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (ed.), *Multiple Perspectives on Mathematics Teaching and Learning* (pp.19–44). Westport: Ablex.

Lerman, S. (2006). Cultural psychology, anthropology and sociology: the developing 'strong' social turn. In J. Maasz & W. Schloeglmann (Eds.), *New mathematics education research and practice* (pp. 171-188). Rotterdam: Sense.

Lerman, S. (2009). Pedagogy, discourse and identity. In L. Black., H. Mendick & Y. Solomon (Eds), *Mathematical relationships in education. Identities and participation* (pp. 147-156). New York: Routledge.

Lovat, T., Toomey, R., & Clement, N. (Eds.) (2010). *International research handbook on values education and student wellbeing.* Dordrecht: Springer.

Martin, D. B. (2000). *Mathematics success and failure among African-American youth.* New Jersey: Lawrence Erlbaum Associates, Inc.

Martin, D. B. (2009). Mapping a liberatory research and policy agenda. In D.B. Martin (Ed), *Mathematics teaching, learning, and liberation in the lives of black children* (pp. 3-38). New York: Routledge.

Mendick, H., Moreau, M-P., & Epstein, D. (2009). Neoliberalism, choice and mathematics. In L. Black., H. Mendick & Y. Solomon (Eds), *Mathematical relationships in education. Identities and participation* (pp. 71-82). New York: Routledge.

Ministry of Education, (2000). *Goals to aim for in mathematics education in upper secondary school.* Stockholm: Fritzes.

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational studies in mathematics 61*, 219-245.

Nieto, S. (2010). *Language, culture and teaching. Second edition.* New York: Routledge.

Nolan, K., & de Freitas, E. (2008). Foreword to the research text: Mathematics education under cross-examination. In E. de Freitas & K. Nolan (Eds.), *Opening the research text: Critical insights and in(ter)ventions into mathematics education* (pp. 1-11). New York: Springer.

Pickering, A. (1995). The mangle of practice: Time, agency, and science. Chicago: University of Chicago Press.

Popkewitz, T. (2004). The Alchemy of the Mathematics Curriculum: Inscriptions and the Fabrication of the Child. *American education ressearch 41*(1), 3-34.

Radford, L. (2008). The ethics of being and knowing: towards a cultural theory of learning. In L. Radford, G. Schubring, & F. Seeger (Eds.), Semiotics in Mathematics Education: Epistemology, History, Classroom, and Culture (pp. 215–234). Rotterdam: Sense Publishers.

Roth, W. M., & McRobbie, C. (1999). Lifeworlds and the "w/ri(gh)ting" of classroom research. *Journal of Curriculum Studies*, *31*, 501-522.

School Inspectorate (2010). *Skolinspektionens Kvalitetsgranskning. Rapport 2010:13.* Stockholm: Skolinspektionen.

Sfard, A. (2008). *Thinking as communicating.* New York: Cambridge University Press.

Sfard, A. & Prusak, A. (2005a). Telling identities: In a search of an analytical tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 2-24.

Skovsmose, O. (2001). Landscapes of investigation. *ZDM 33*(4), 123-132.

Skovsmose, O. (2005). *Travelling through education: Uncertainty, mathematics, responsibility.* Rotterdam: Sense Publishers.

Skovsmose, O. & Nielsen, L. (1996). Critical Mathematics Education. In A. J. Bishop. (Ed), *International Handbook of Mathematics Education* (pp 1257-1288). Dordrecht: Kluwer Academic Publishers.

Solomon, Y. (2009). *Mathematical literacy. Developing identities of inclusion.* New York: Routledge.

Stentoft, D. & Valero, P. (2009). Identities-in-action: Exploring the fragility of discourse and identity in learning mathematics. *Nordic Studies in Mathematics Education*, *14*(2).

Stevenson, A. (Ed). (2010). *Oxford Dictionary of English.* Oxford: Oxford University Press.

Stocker, D., & Wagner,D. (2007). Talking about teaching mathematics for social justice. *For the Learning of Mathematics* 27(3), 17-21.

Sullivan, P., Tobias, S., & McDonough, A. (2006). Perhaps the decision of some students not to engage in learning mathematics is deliberate. *Educational Studies in Mathematics 62,* 81–99.

Swanson, D. M. (2005). School mathematics: Discourse and the politics of context. In A. Chronaki & I. M. Christiansen (Eds.), *Challenging perspectives on mathematics classroom communication* (pp. 261-294). Greenwich, CT: Information Age Publishing.

Valero, P. (2004a). Socio-political perspectives on mathematics education. In P. Valero, & R. Zevenbergen (Eds), *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology* (pp. 5-24). Dordrecht: Kluwer Academic Publishers.

Valero, P. (2011). Re-interpreting students' interest in mathematics: Youth culture resisting modern subjectification. Paper presented at the 63 CIEAEM. Barcelona, July 24-29.

Valero, P. & Stentoft, D. (2010). The 'post' move of critical mathematics education. In H. Alrø, O. Ravn, & P. Valero (Eds.), *Critical Mathematics Education: Past, Present and Future* (pp. 183-195). Rotterdam: Sense Publishers.

Wackernagel, W., & Rees, E. (1996). *Our ecological footprint. Reducing human impact on the earth.* Gabriola Island: New Society Publishers.

Wagner, D. (2007). Students' critical awareness of voice and agency in mathematics classroom discourse. *Mathematical thinking and learning 9*(1),31–50.

Wedege, T. (1999). To know or not to know - mathematics, that is a question of context. *Educational Studies in Mathematics* 39, 205-207.

Wedege, T., & Evans, J. (2006). Adults' resistance to learn in school versus adults' competences in work: The case of mathematics. *Adults Learning Mathematics - an International Journal, 1*(2), 28-43.

Willis, P., & Trondman, M. (2000). Manifesto for Ethnography. *Ethnography 1*(5), 5-16.

Closing Down and Moving Forwards…

In these sections I aim to clarify and discuss the overarching research question: How do identity narratives, of students and the teacher, illustrate the complex relationships between mathematics education contexts, agency and engagement? I do this through addressing the three sub-questions that guided this research separately. I then finalise the thesis with some reflections on subjectification processes in mathematics education as a way to move this research forward. I also reflect on some ideas concerning pedagogy development before concluding through talking about the voices that have been present in this thesis.

What are the supports and hindrances for changing teaching approaches in mathematics in a Swedish upper secondary school?

Together with a mathematics teacher, Elin, a pedagogical discourse was imagined, arranged and reflectively analysed, while acknowledging the existing practices at Ericaskolan. This "disturbance" of the teaching needed to be achieved, for me to be able to study students' narratives in a different mathematics education than they expected or imagined. Through identifying supports and hindrances during my collaboration with Elin, I learnt about the complexity of issues that impacted on Elin's ordinary school life.

To change a social practice within mathematics education, and move between discourses required support from different parts of the mathematics education network (Valero, 2010) or levels (Martin, 2000) at different points in time. Locating the experiences in the socio-cultural context of the school gave an understanding of the complex situations and processes that required to be illuminated further. This research question, concerning support and hindrances when moving discourses and arranging pedagogy is mainly answered in Andersson (in press, a) and in Andersson and Valero (in press).

The support we required when changing the mathematics teaching, but also the hindrances, were located within different social practices of the network. To give some examples: concerning the pedagogical development per se, Elin and I found support in the level of Swedish governmental steering of schools. The national mathematics education curriculum emphasis in several places, that mathematics teaching should be connected to the students' chosen study program (Ministry of Education, 2000a, 2000b). These connections are not usually done in Swedish upper secondary mathematics teaching, according to critique raised by the Swedish school inspectorate (2010). The pedagogical discourse aimed for connecting mathematical contexts to social and societal concerns while appreciating students' possibilities for achieving agency both in relation to their mathematics learning and in relation to mathematics per se (see e.g., Skovsmose & Nielsen, 1996; Skovsmose, 2010). In this way Elin and I intended to connect the mathematics teaching with societal questions and hence meet the requirements formulated in the curriculum.

However, the government policies also put boundaries around our work. The number of topics that needed to be covered within a limited time space squeezed the possibilities, for example, to develop project work and interdisciplinary learning of mathematics further. As the Mathematics A course finished with a national test, which was completed by all students in Sweden,

independent of their chosen study program. This meant that not all parts of the teaching could be conducted in connection to the study program or with different teaching models.

At the classroom level, the teacher's openness and willingness for learning supported the changes in the classroom. Issues at the school level both supported and hindered the change at different points in time. Introducing change and pedagogical development within classroom practices were supported, and also wished for, from the school leadership. However, at certain times, issues mainly concerning the teachers working conditions, workload and timetables complicated Elin's ability to implement these changes. At one point, issues regarding wishes for interdisciplinary collaborations complicated Elin's school life. The textbooks did not support us either when connecting mathematics to society even if the chosen mathematics textbook for this group of students was intended to do so (elaborated further in Andersson & Ravn, in press).

Lastly, at the family level, these students' parents supported the change in a way that they did not question what we did, or interfere with the teaching. They rather seemed content with the implemented teaching approach.

My learning during the collaboration with Elin resulted in an awareness of the social, cultural and political embededness of identity narratives in the multiple contexts where teaching occurs. Finding the answer to this first research question made me query research literature on teacher change that talked about stable change as sustainable (e.g., Fullan, 1991). Simplistic models which suggests that if teachers are provided with professional development support, that contains certain characteristics such as being able to choose it themselves and having a supportive network, does not recognise how the wider circumstances impact on the sustainability over time. In the case of Elin I query: What is actually being sustained? What is sustainable change? Elin's identity narratives and learning suggests that researchers might have to rethink what we mean with terms such as change and success, and if sustained development is possible. I would suggest that the continuous learning Elin is involved in might be a success in itself, even if that was not the learning we originally anticipated as a result of the collaboration. Consequently, the notions of success and sustainability of teacher change and pedagogical development became questioned in this study.

What are the relationships between identity narratives, communicated by individual students, and the different contexts that they were experiencing at particular points in time?

The way in which the analysis was conducted, with the individual students' chronological story lines, allowed me to explore how individuals' identity narratives changed in relation to different contexts. I could show, through the cases of Henrik, Sandra, Malin and Petra, that the ways, in which individual students' identity narratives changed during the course, indicated that identities told in relationships with mathematics and mathematics education are not consistent. Different task contexts, situation contexts, school contexts and societal contexts intermesh as referents and groundings for the discursive practices in classrooms, including those that the students drew on to construct their identity narratives. Identity narratives are built in available discourses. Given that I was interested in these questions, the outcome allows me to address two further research issues.

As pointed out by Gutiérrez (2007), Morgan (2006) and Valero (2002b, 2004, 2007), research literature tends to focus on students at one particular time, or one aspect of classroom activities

or students' relationships or positionings in one particular context. Researching only one type of context might give a misleading impression of a students' relationship with mathematics, as I showed in the cases of Henrik (Andersson & Valero, under submission) and Petra and Malin (Andersson, Meaney & Valero, under submission).

The ways in which I was able to connect students' identity narratives to contexts was a methodological research outcome from this study. Qualitative research and small-scale case studies can provide insights into the complexity of education, which large-scale quantitative studies might not do. A contemporary critical example is the backwash of the "No child left behind" act in the US (e.g., Popkewitz, 2004), which now reinforces quantitative large-scale research studies that show research results without nuances. The findings in this study point at problematic issues when research outcomes generalise students' learning of mathematics and conclude that specific groups of students act or behave in certain ways, or that certain pedagogies are to be preferred. It seems that researchers often label generalisations as realities. Swanson (2005) gives examples of black students in South Africa that are labelled as disadvantaged and as a result being offered different possibilities for learning mathematics than other students. Martin (2009, p.4) addresses likewise important issues concerning African American children in the U.S., and contends "mainstream research and policy contexts have served as particularly fertile grounds to aid in socially constructing African America children as learners with particular kinds of deficiencies in relation to other children". The analytical tool used in this study was developed by Sfard and Prusak (2005a, 2005b) in order to understand the cultural differences between "new-comers" and "old-timers" in relation to their performance in mathematics education. I argue that even these labels might reinforce the identification of a student by particular characteristics. This is as problematic as if the focus of mathematics education research remains on cognition because it fails to value students as having whole lives (Valero, 2002b, 2004).

Labels allow researchers to generalise principles, but it becomes problematic when the principles are applied to specific cases. The principle as such is not wrong, but when they are applied in this way then the labels can impede learning rather than supporting it. Students, and teachers are individuals and human beings with whole lives. The choices we make about methodology, and the ways we interpret and communicate our findings, can, and often do, reinforce certain characteristics on students as being the only parts of them to which attention should be paid., This raises the importance for positioning, who has or is given a voice, and who is recognised as a whole human being in mathematics education – or not.

To conclude, individuals' identities, such as those of the students' or the teacher's, but also that of the researcher, fluctuated and were less stable than prior research literature suggested, Rather they fluctuated according to the contexts in which they were narrated. The assertion that students' narrative identities are discursive, and emerge in the discursive practices in which people participate requires an identification of the contextual practices where the narratives are expressed.

What is the relationship between agency and students' engagement in learning of mathematics at particular points in time?

The elusive concept of agency, in this study defined as achieved in an ecological understanding (Biesta & Tedder, 2006), became important during the process of analysis. The reason for this was that students' talk about experiencing senses of meaningfulness, when engaging in mathematics learning, was tied to "being able to decide for yourself", hence to achievement of agency. The relationships that emerged between students' identity narratives and the different contexts in which these identities were narrated, indicated that allowing space for taking personal responsibility and achievement of agency opened up for students' opportunities to sense and talk about meaningfulness when participating in mathematics education. The notions of meaning and meaningfulness in mathematics education are in my understanding not only connected to the mathematical conceptual meaning. It rather concerns the students' whole experience of and relationships with learning mathematics, as theoretically suggested by Skovsmose (2005b). This study adds to prior research by describing this issue empirically. Experiences of meaningfulness and engagement, as told by students in mathematics education, relate not only to pure mathematics. These experiences connect to, for example, what students encounter outside the classrooms, how they view their current and future situations, and what spaces are open for decision-making and negotiations.

Valero (2002b, p.5) provoked the mathematics education research community with her metaphorical discussion of the "myth of the active learner in mathematics education" as a student that would

> … look like a sci-fi being, an outer-space visitor, with a big head, probably a bit of heart, and a bit of body. That being would be mainly alone and mostly talk about mathematics and its learning, and would see the world though his school mathematical experience. It would be a "schizo-being" since he has a clearly divided self: that one that has to do with mathematics, and the one that has to do with other unrelated things.

This sci-fi being was not present in the classrooms where this study was conducted. Instead, a number of students' identity narratives included references to positive relationships between their achievement of agency and when they could make connections to other aspects of their lives, and this resulted in talk about meaningfulness when learning mathematics. Receiving opportunities to decide on task contexts, to influence the situation contexts and assessment seemed to make mathematics more meaningful for students to learn. At other times students' stories were told as meaningless and powerless experiences and at those times students acted with resistance. These reflections open up the question about what human beings do we want students to become in mathematics education, what stories do we want them to tell about their relationship with mathematics and what is to be learnt?

Davies (2006) highlighted our responsibility to examine the possibilities that the "taken-for-granted" discourses in education as a discursive practice are creating for teachers and students:

> The social, psychic and intellectual work is, rather, emergent. Our responsibility lies inside social relations and inside a responsibility to and for *oneself in relation to the other*- not

oneself as a known entity, but oneself in process, unfolding or folding up, being done or undone, in relation to the other, again and again. (p. 436, original emphasis)

The temporal analysis of students' identity narratives in contexts illuminated complexities of students' decision making, about engaging in mathematics learning – or not. Relationships between identity narratives and the contexts the narratives referred to, indicated why students acted in particular ways at specific times, and in what ways they became agents in their learning of mathematics in some contexts but not in others. Agency was an important feature for a number of students at those times they talked about engagement and experiencing meaningfulness in mathematics education. This study thus supports an understanding of students as being whole human beings, and supports Skovsmose's (2005b) theoretical assumptions about students experiencing mathematics education as a whole when talking about learning mathematics as meaningful. The research question could be answered shortly or simplisticly. However, when looking at the research question from a meta-level, the importance of agency for experiencing meaningfulness became clear in considering why students decided to engage in learning activities. Consequently, there is a need for further research on what it means to have students as whole human beings in mathematics classrooms.

About Subjectification

This PhD-study was my first research step towards understanding identity narratives told in mathematics education. My next step will be to address students' subjection or subjectification processes in mathematics education. I initially chose to position this study within a socio-cultural theoretical paradigm but here I take the opportunity to move towards a more socio-political stance, than the "little p" research allowed for.

The socio-cultural theoretical perspective supported my view that (mathematics) classrooms are spaces of socially organised practices that, in different ways, shape how individuals are expected to, allowed to and/or required to act. With this perspective, I could start to untangle what students' subjectification processes might look like and why students told the stories about mathematics education the way that they did. I also learnt, together with the teacher, about a teacher's (and researcher's) learning experiences during a process of changing teaching practise. Learning is not just about *objectifying* understandings; it is also about becoming a subject, or subjectification. Subjectification is processes in which we as individuals become who and what we are in relation to others in a sociological understanding (Radford, 2008). Radford (2006a) wrote extensively about the process of subjectification, which occurs simultaneously with objectification in relation to mathematics education.

In this context, and in light of the previous fundamental concepts, learning is defined as the social process of objectification of those external patterns of action fixed in the culture. Objectification entails another process, the process of subjectification—i.e., the becoming of the self. Subjectification has received very little attention in the literature on mathematics education [...] However, its importance is easy to grasp as soon as we resist the temptation to reduce mathematics to its technical dimension and become aware that learning is much more than constructing logico-mathematical mental structures or picking up ready-made

> knowledge, that is to say, if and when we become aware of the fact that learning is about knowing and being. Objectification and subjectification should in fact be seen as two mutually constitutive processes leading to students' engagement with cultural forms of thinking and a sensibility to issues of interpersonal respect, plurality, inclusiveness and other main characteristics of the communitarian self. (Radford, 2008, p.230)

However, while learning mathematics we also learn other stuff, which I would argue, is a part of subjectification processes. This "other stuff" is included in the identity narratives we tell when we talk about ourselves in relationships to mathematics and mathematics education. during the analysis process The point Radford made about the mutual objectification and subjectification processes when learning mathematics was recognised in the students' talk. As well as learning the mathematics content, researchers have shown the impact of some of this "other stuff". For example, Meaney (2007) showed that task contexts affect what students perceive and what level of mathematical literacy they reach. Lange and Meaney (2011) suggested that some children learn that their parents are insufficient when they cannot provide learning support in mathematics. Abreu and Cline (2005) gave evidence for similar findings and also Swanson (2007) discusses what black children in South Africa learn, more than mathematics, in mathematics education.

What I found was that it did not make sense for me to research what the students talked about without relating to the different contexts and available discourses in which the stories were narrated. Bauchspies (2009) suggested the need to "study in between and to attend to the relationality of subjects" (p. 243) and take into account the tensions between objectivity and subjectivity in science studies. I claim that this ought to be the case also in mathematics education studies. What I missed in much of the prior mathematics education research was the understanding of the complexity of students' reality. This complexity is, from my personal experience, an issue which teachers grapple with every day in their working life.

Knowledge from this study indicates the need for further research that contributes to understanding the complexity of students' becoming whole human beings, engaging with mathematics education, as a result of the subjectification process. The empirical data in this research showed the changes in how students' identities were narrated in relationships to different contexts at specific points in time. The students' identity narratives changed in relation to different levels of contexts. It seemed that identity narratives are contextually bound to the pedagogical discourses in which they are made possible. Over the course of the study, students, and the teacher, told different stories about themselves in the different contexts and available discourses. The obvious complexity behind the identity narratives made me question the sorts of contexts in mathematics education that propagated different ways of narrating identities. It was not the case someone was always a math-hater, or objectifying a diagnosis of dyslexic ensured a certain way of acting in mathematics classrooms. When the pedagogical discourse changed, different contexts were addressed and this also changed narrated identities.

Moving Pedagogical Discourses

This piece of research also suggests further reflections in a very different direction, namely on pedagogical development, by discussing the possibilities and hindrances for changing pedagogical discourses in mathematics education. It was the students' stories that allowed me to understand the importance of different contexts in which mathematical topics were presented. I have struggled through the research process to not evaluate the pedagogical work, the pedagogy per se or the pedagogical discourse but to stay focussed on the identity narratives. Exploring the theoretical ideas about changing teaching that emerged when I worked at Ericaskolan and collaborated with Elin would be interesting within the research area of sociomathematics. Wedege (2010b, p. 479) defined *sociomathematics* in the following way:

> First, sociomathematics is a field to be researched (a subject field) where problems are formulated bringing people, mathematics and society together. Second, sociomathematics is a research field where the societal context of mathematics education is taken seriously into account, e.g. scientific studies with society in the foreground.

According to Wedege, sociomathematical studies relate, to the concept of mathematical literacy, from a perspective that acknowledges the tensions between pure mathematical knowledge and the way mathematics is used out of school in society. Sociomathematics can be denoted as an umbrella covering prior research within the research areas of for example ethnomathematics (D'Ambrosio, 1985; Gerdes, 1996; Knijnik, 2007), folk-mathematics (Mellin-Olsen, 1987) and adults learning of mathematics at the workplace (Wedege, 2000; Wedege & Evans, 2006). *Sociomathematical research problems* concern "people's relationships with mathematics (education) *in* society and vice versa", or "functions of mathematics (education) in society and vice versa" or "people learning, knowing and teaching *in* society" (Wedege, 2010b, p. 483). It would be interesting to analyse the data set from this study, in a way that theoretically moved forward parts of the socio-mathematical research field.

I also recognise the possibilities for interdisciplinary research with the research area of socio-scientific issues (SSI) within natural sciences subjects' research. Within the frames of SSI-teaching natural science topics strongly connect the natural science teaching to features in society, as for example health and medicine, sustainable development and contemporary news media (Ekborg, Ideland & Malmberg, 2009; Ratcliffe & Grace, 2003). Sociomathematics and SSI-research ought to have rich opportunities for mutual learning and experience sharing, which could be fruitful for both research areas.

Regarding the pedagogy as such, we need to be considering the forms of mathematics and in what contexts they are introduced to students. We also need to consider how power is negotiated in the mathematics classrooms, and what opportunities we have for allowing space for students to achieve agency in their learning and accountancy of mathematics. When bearing in mind the importance of agency and how individuals make sense of their surroundings with/through/in mathematics, we begin to open up possibilities for rethinking what mathematics and mathematics education research does along the way. It is when we as researchers and teachers pose the question "What identity narratives do we want the students to tell in relationship with their mathematics education?" with the emerging sub questions my findings indicate: "What are the

possibilities for students' achievement of agency in my mathematics classroom?" and "How can we as teachers and researchers support students identity work?" that possibilities for students telling different stories in mathematics education occur.

Allowing Space for Other Voices

This study aimed to give a voice to students whose identity narratives indicated that for different reasons, they did not feel well in mathematics education. This group of students became my focus because the stories that they told about their prior experiences of mathematics education had disturbed me during my years as an upper secondary mathematics teacher. Skovsmose (2005a, 2005b) wrote that the "noisy" students are seldom the focus of mathematics education research. Rather, he concluded, the case was the opposite. In this study, the particular students in focus were those students whose stories indicated that they often acted with resistance in their previous mathematics education. However, it would be interesting to revisit the data and allow space for the voice of students who felt well in mathematics education. The "well-being" students' identity narratives would most certainly also give researchers deeper insight into students' subjectification processes in mathematics education and contribute to further understanding of relationships between identity narratives and mathematics.

The Researchers Voice

Finally, I feel a need to address the researcher's voice. Working with this thesis has been a substantial part of my life for the last three years. It has been characterised by my curiosity of other people, especially young people and how they talk about and experience their world. I have conducted this research in interplays between students, Elin and other school staff. However, also my supervisors, my family, colleagues and others have influenced my thinking and pushed this thesis forward. My name is the author's name on the front page, and I take full responsibility for everything written in this thesis. Nevertheless, I want to recognise that this thesis would not have eventuated if it had been only my efforts as an individual that influenced the research process and the writings.

I recall my very first meeting with SMERG, the Science and Mathematics Education Research Group at Aalborg University and the discussions I took about wanting to change the world, using words as equity, social justice, empowerment and so on for all students in the world who have not-so-good experiences from their mathematics education. During these three years also my own identity narratives have changed, sometimes dramatically, when reading, experiencing, analysing and learning about the complexity of mathematics education practices, the power relations between different levels and the complicated relationships between the social practices that influence what takes place in mathematics classrooms. I have learnt and received new knowledge in all the relationships with others and today I tell very different stories about myself as a researcher in mathematics education than I did three years ago.

My voice is obviously the dominant one in the writings; however I have strived for opening spaces for other voices to come forward. My researcher attitude has required me to take

decisions, which I regarded as ethical as suggested by Goodchield (2009) and Pring (2000). I found a path that caused me to make choices about theory, methodology and ways of writing up the thesis. In this way I have aimed for carrying out quality research in every aspect. My view is that research quality considers not only the immediate effects of the research in the classrooms; it also takes into account the long-term effects of the research activity. One path I see for future research within this domain would be to follow up and interview these participating students and the teacher and listen to their future stories about their experiences of this particular course and the impact it might have given on their learning of mathematics. *What stories would they tell about their relationships with mathematics, and mathematics education and this research activity then? What if?*

References

Abreu, de. G., & Cline, T. (2005). Parents' representations of their children's mathematics learning in multiethnic primary schools. *British Educational Research Journal, 31*(6), 697–722.

Alrø, H., & Skovsmose, O. (2003). *Dialogue and learning in mathematics education: intention, reflection, critique.* Boston: Kluwer Academic Publishers.

Alrø, H., Skovsmose, O., & Valero, P. (2009). Inter-viewing Foregrounds. In M. César & K. Kumpulainen (Eds.), *Social Interactions in Multicultural Settings* (pp. 13-37). Rotterdam: Sense Publishers.

Andersson, A. (2007). *Ett kulturellt besök i matematikundervisning: En aktionsstudie med ett etnomatematiskt perspektiv*. Masters thesis. Malmö: Malmö Högskola.

Andersson, A. (2010a). Making Sense of Critical Mathematics Teaching. In L. Sparrow., B. Kissane,. & C. Hurst (Eds), *MERGA33. Shaping the Future of Mathematics Education. Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia* (pp.51-58). Fremantle: MERGA Inc.

Andersson, A. (2010b). Can a critical pedagogy in mathematics lead to achievement, engagement and social empowerment? *Philosophy of Mathematics Education Journal 25*. Available at http://people.exeter.ac.uk/PErnest/pome25/index.html

Andersson, A. (in press, a). A "Curling Teacher" in Mathematics Education: Teacher Identities and Pedagogy Development. *Mathematics Education Research Journal*.

Andersson, A. (in press, b). Interplays between context and students' achievement of agency. Accepted for presentation and proceedings at Working group 10; Cerme 7, 9th-13th February 2011, in Rzeszow, Poland.

Andersson, A., Meaney, T., & Valero, P. (Submitted to Educational studies in mathematics)."I am [not always] a math-hater". Students' [dis]engagement in mathematics education.

Andersson, A., & Norén, E. (in press). Agency in mathematics education. Accepted for presentation and proceedings at Working group 10; Cerme 7, 9th-13th February 2011, in Rzeszow, Poland.

Andersson, A., & Ravn, O. (in press). A critical perspective on contextualisations in mathematics education. In O. Skovsmose & B. Greer (Eds), *Critique and politics of mathematics education*. Rotterdam: Sense Publishers.

Andersson, A., & Valero, P. (2009). Mathematics education giving meaning for social science students in Sweden. In C. Sandoval & Y. Ladino (Eds.), *Proceedings of the II International Congress of Research on Education, Pedagogy and Teacher Education*. Medellin: University of Antioquia – National Pedagogical University of Colombia.

Andersson, A., & Valero, P. (2011). Negotiating critical pedagogical discourses: Stories of contexts, mathematics and agency. In P. Ernest and B. Sriraman (Eds), *Critical Mathematics Education: Theory and Praxis*. USA: Information Age Publishing.

Ascher, M. (1998). *Ethnomatematics. A multicultural view of mathematical ideas*. Belmont: Wadsworth, Inc.

Atweh, B. (2004). Towards a model of social justice in mathematics education and its application to critique of international collaborations. In I. Putt, R. Faragher & M. McLean (Eds), *Mathematics Education Research Group of Australasia, July 2004.* Townsville: MERGA Inc.

Atweh, B., & Brady, K. (2009). Socially response-able mathematics education: Implications of an ethical approach. *Eurasia Journal of Mathematics, Science & Technology Education, 5*(3), 267-276.

Ball, D. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics, 8*(1), 40–48.

Bauchspies, W. K. (2009). Potentials, actuals and residues: entanglements of culture and subjectivity. *Subjectivity, 28*, 229-245.

Beswick, K., Watson, J., & Brown, N. (2006). Teachers' confidences and beliefs and their students' attitudes to mathematics. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Identities, cultures and learning spaces* (pp. 68-75). Sydney: Mathematics Education Research Group of Australasia.

Bibby, T. (2008). The experience of learning in classrooms: moving beyond Vygotsky. In T. Brown (Ed), *The Psychology of Mathematics Education: A Psychoanalytic Displacement* (pp. 37-60). Rotterdam: Sense Publishers.

Bibby, T. (2010). What does it mean to characterize mathematics as 'masculine'?: Bringing a psychoanalytic lens to bear on the teaching and learning of mathematics. In: M. Walshaw (ed), *Unpacking Pedagogy: New Perspectives for Mathematics Classrooms* (pp. 21-42). Charlotte: Information Age.

Biesta, G. J. J. (2005). What can critical pedagogy learn from postmodernism? Further reflections on the impossible future o critical pedagogy. In Ilan Gur Ze'eve (ed), *Critical theory and critical pedagogy today. Toward a new critical language in education* (pp.143-159). Haifa: Studies in Education (university of Haifa).

Biesta, G.J.J. (2006). *Beyond learning: Democratic education for a human future*. Boulder: Paradigm Publishers.

Biesta, G.J.J. (2009). Good education in an age of measurement: on the need to reconnect with the question of purpose in education. *Educational Assessment, Evaluation and Accountability, 21*(1), 33-46.

Biesta, G.J.J., & Tedder, M. (2006). How is agency possible? Towards an ecological understanding of agency-as-achievement. *Working paper 5*. Exeter: The Learning Lives project.

Bishop, A. (1998). Research, effectiveness and the practioners' world. In A. Sierpinska and J. Kilpatrick. *Mathematics education as a research domain: A search for identity* (pp. 33-45). Great Britain: Kluwer Academic Publishers.

Bishop, A. (1999). Mathematics teaching and values education – an intersection in need of research. Mathematics education and democratic education. Part 2. *ZDM, 31*(1-4), 1863-9690.

Bishop, A. (2001). What values do you teach when you teach mathematics? In P. Gates (ed.), *Issues in Mathematics Teaching* (pp. 93–104). London: Routledge.

Bishop, A. (2008). Mathematics teaching and values education – an intersection in need of research. In P. Clarkson, & N. Presmeg (Eds), *Critical issues in mathematics education. Major contributions of Alan Bishop* (pp. 231-238). New York: Springer.

Black, L., Mendick, H., Rodd, M., Solomon, Y. & Brown, T. (2009). Pain, pleasure and power: selecting and assessing defended subjects. In L. Black, H. Mendick, & Y. Solomon (Eds), *Mathematical relationships in education. Identities and Participation* (pp. 19-30). New York: Routledge.

Black, L., Mendick, H., & Solomon, Y. (2009). (Eds), *Mathematical relationships in education. Identities and participation*. New York: Routledge.

Black, L., Williams, J., Hernandez-Marritnez, P., Davis, P., Pampaka, M., & Wake, G. (2009). Developing a 'leading identity': the relationship between students' mathematical identities and their career and higher education aspirations. *Educational studies in mathematics, 73*, 55-72.

Boaler, J. (1993). The role of contexts in the mathematics classroom: do they make mathematics more real? *For the learning of mathematics, 13*(2), 12-17.

Boaler, J. (2002). *Experiencing school mathematics. Traditional and reform approaches to teaching and the impact on student learning. Revised and expanded edition*. London: Lawrence Erlbaum Associates, Publishers.

Boaler, J. (2003). Studying and capturing the complexity of practice. The case of "Dance of Agency". In Pateman , N. A., Dougherty, B. J., & Zilliox, J. T. (Eds), *Proceedings of the 27th International Group for the Psychology of Mathematics Education Conference Held Jointly with the 25th PME-NA Conference (Honolulu, Hawaii, July 13-18, 2003). Volume 1* (pp 3-16). Hawaiʻi: Center for Research and Development Group, University of Hawaiʻi.

Boaler, J. (2010). Discussion II. Can mathematics problems help with the inequities in the world? In L. Verschaffel, B. Greer, W. Van Dooren & S. Mukhopadhyay (Eds), *Words and worlds. Modelling verbal descriptions of situations* (pp. 131-140). Rotterdam: Sense Publishers.

Boaler, J., & Greeno, J. G. (2000). Identity, Agency, and Knowing in Mathematics Worlds. In J. Boaler (ed), *Multiple perspectives on mathematics teaching and learning*. London: Ablex Publishing.

Brown, R. (2009). Teaching for social justice: Exploring the development of student agency through participation in the literacy practices of a mathematics classroom. *Journal for Mathematics Teacher Education, 12*, 171-185.

Brown, M., Brown, P., & Bibby, T. (2008). 'I would rather die': reasons given by 16-year-olds for not continuing their study of mathematics. *Research in Mathematics Education, 10*(1), 3-18.

Bryman, A. (2001, 2004, 2008). *Social science methods. First, second and third editions*. Oxford: Oxford university press.

Burton, L. (2001). Research mathematicians as learners – and what mathematics education can learn from them. *British Educational Research Journal, 27*(5), 589-599.

Butler, J. (1990). *Gender trouble: feminism and the subversion of identity*. New York: Routledge.

Carspecken, P.F. (1996). *Critical ethnography in educational research: a theoretical and practical guide.* New York: Routledge.

César, M. (2007). Dialogic identities in students from cultural minorities or students categorized as presenting SEN: How do they shape learning, namely in mathematics. In ScTIGGroup (Eds.), *Second Socio-Cultural Theory in Educational Research and Practice Conference Proceedings.* Manchester: Manchester University.

Clarke, D. (2007). Ten key principles from research for the professional development of mathematics teachers. In G. C. Leder & H. J. Forgasz (Eds), *Stepping stones for the 21st century. Australasian mathematics education research* (pp. 27-37). Rotterdam: Sense Publishers.

Clarkson, P.C., Bishop, A. & Seah, W.T. (2010). Mathematics education and student values: The cultivation of mathematical wellbeing. In T. Lovat, R. Toomey & N. Clement (Eds) *International research handbook on values education and student wellbeing* (pp. 111-135). Dordrecht: Springer.

Clarkson, P., & Presmeg, N. (2008). *Critical issues in mathematics education. Major contributions of Alan Bishop* (pp. 231-238). New York: Springer

Clough, P. 2002. Narratives and fictions in educational research. London: Open University Press.

Cobb, P., Confrey, J., diSessa, A., Lehrer R., & Schauble, L. (2003). Design Experiments in Educational Research. *Educational Researcher, 32*(1) 9–13.

Cobb, P; Gresalfi, M., & Hodge, L.L. (2009). An interpretive scheme for analysing the identities that student develops in mathematics classrooms. *Journal for Research in Mathematics Education, 40*(1), 40-68.

Cobb, P., & Hodge, L. L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical Thinking and Learning, 4*(2&3), 249-284.

Cohen, A. (1994). *Self consciousness: An alternative anthropology of identity.* London: Routledge.

Cohen L., Manion, L., & Morrison, K. (2010). *Research methods in education. Sixth edition.* New York: Routledge.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44-48.

D'Ambrosio, U (2006). *Ethnomathematics. Link between Traditions and Modernity.* Rotterdam: Sense Publishers.

D'Ambrosio, U. (2007). Peace, social justice and ethnomathematics. In B. Sriraman (Ed), *International perspectives on social justice in mathematics education* (pp. 25-34). Charlotte: Information Age Publishing, Inc.

Davies, B. (2006). Subjectification: The Relevance of Butler's Analysis for Education. *British Journal of Sociology of Education, 27*(4), 425-438.

DeBellis, V. A., & Goldin, G.A. (2006). Affect and meta-affect in mathematical problem solving: A Representational Perspective. *Educational Studies in Mathematics, 63*(2), 131-147.

de Freitas, K. (2010). Regulating mathematics classroom discourse: text, context and intertextuality. In M. Walshaw (Ed). *Unpacking pedagogy: new perspectives for mathematics classrooms* (pp.129-152). USA: Information Age Publishing.

de Freitas, E. (2003). Contested positions: How fiction informs emphatic research. *International journal of education and the arts: A journal of scholarship, 4*(7).

de Freitas, E., & Nolan, C. (Eds). (2008). *Opening the research text. Critical insights and in(ter)ventions into mathematics education*. New York: Springer.

Dowling, P. (1998). *The sociology of mathematics education: mathematical myths, pedagogic texts*. London: Falmer Press.

Ekborg, M., Ideland, M., & Malmberg, C. (2009). Science for life: A conceptual framework for construction and analysis of socio-scientific cases. *Nordina, 5*(1), 35-46.

Emirbayer, M. & Mische, A. (1998). What is agency? *American Journal of Sociology, 103*(4), 962-1023.

Engestrom, Y. (1987). *Learning by Expanding. An Activity-theoretical approach to developmental research.* Helsinki: Orienta-Konsultit Oy

English, L.D. (Ed). (2002). *Handbook of international research in mathematics education. Second edition.* United States of America: Erlbaum.

English, L. D., Bartoloni Bussi, M., Jones, G.A., Lesh, R.A., & Sriraman, B. (Eds) (2008). *Handbook of international research in mathematics education, third edition.* New York: Routledge.

Englund, T. (2005). The discourse on equivalence in Swedish education policy. *Journal of Education Policy, 20*(1), 39–57.

Ernest, P. (1998). A postmodern perspective on research in mathematics education. In A.Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain. A search for identity* (pp. 71-85). Dordrecht: Kluwer Academic Publishers.

Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of mathematics education journal, 15*(1),1-16.

Ernest , P. (2009). Values and the social responsibility of mathematics education. In P.Ernest, B. Greer, & B. Sriraman (Eds), *Critical issues in mathematics education* (pp. 207-216). Charlotte: Information Age Publishing.

Fairclough, N. (2003). *Analysing discourse: textual analysis for social research*. New York: Routledge.

Foley, D., & Valenzuela, A. (2005). Critical ethnography: the politics of collaboration. In K. Denzin and Y.S. Lincoln (Eds), *The Sage handbook of qualitative research, third edition* (pp. 217-234*)*. Thousand Oaks: Sage.

Foucault, M. (1977). *Discipline and Punish*. New York: Vintage Books.

Foucault, M. (1980). *Power/knowledge: Selected Interviews and Other Writings 1972-1977*. London: Harvester.

Foucault, M. (1982). Afterword: The subject and power. In H.L. Dreyfus & P. Rabinow, *Michel Foucault: Beyond Structuralism and Hermeneutics (2nd ed., pp. 208-216)*. Chicago: University of Chicago Press.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Understanding teaching and classroom practice in mathematics. In F. K. J. Lester (Ed), *Second handbook of research on mathematics teaching and learning. Vol. 1*. (pp. 225–256). Charlotte, NC: NCTM.

Frankenstein, M. (1995). Equity in mathematics education: Class in the world outside the class. In W. G. Secada, E. Fennema & L. Byrd Adajian (Eds) *New directions for equity in mathematics education* (pp.165-190). Cambridge: Cambridge University Press.

Frankenstein, M. (2008). Quantitative form in arguments. In J. F. Matos, P. Valero, & K. Yasukawa (Eds). *Proceedings of the fifth international mathematics education and society conference.* Lisbon: Universidad de Lisboa and Department of Education, Learning and Philosophy, Aalborg University.

Frankenstein, M. (2009). Developing a critical mathematical numeracy through real real-life word problems. In L. Verschaffel, B. Greer, W. Van Dooren & S. Mukhopadhyay (Eds), *Words and worlds. Modelling verbal descriptions of situations* (pp. 111-130). Rotterdam: Sense Publishers.

Freire, P. (1992). *Pedagogy for the oppressed*. New York: Seabury Press.

Fuchs, S. (2007). Agency (and Intention). In G. Ritzer (ed), *Blackwell Encyclopedia of Sociology. Blackwell Publishing, 2007. Blackwell Reference Online*. Retrieved 27 August 2011 from http://www.blackwellreference.com

Fullan, A. (1991). *Leading in a culture of change*. San Francisco: Jossey-Bass.

García, G., Valero, P., Peñaloza, G., Mancera, G., Romero, J., & Camelo, F. (2009). *Reinventando el currículo y los escenarios de aprendizaje de las matemáticas. Un estudiodesde la educación matemática critica*. Bogotá: IDEP - UPN.

Gee, J. P. (2005). *An Introduction to Discourse Analysis: Theory and Method. Second edition*. New York: Routledge.

Gellert, U., & Jablonka, E. (2009). "I am not talking about reality". In L. Verschaffel, B. Greer, W. Van Dooren, & S. Mukhopadhyay (Eds), *Words and Worlds: Modelling verbal descriptions of situations* (pp. 39–53). Rotterdam: Sense Publishers.

Gerdes, P. (1996). Ethnomathematics and Mathematics Education. In A. J. Bishop (Ed), *International Handbook of Mathematics Education* (pp 909-944). Dordrecht: Kluwer Academic Publishers.

Goetz, T., Frenzel, A.C., Pekrun R., & Hall, N.C. (2006). The domain specificity of academic emotional experiences. *Journal of Experimental Education, 75*(1), 5–29.

Goodchield, S. (2009). Classroom research: Impact and long term effecr versus justice, liberation and empowerment? In P. Ernest, B. Greer, & B. Sriraman (Eds), *Critical issues in mathematics education* (pp. 217-235). Charlotte, NC: Information Age Publishing.

Gorard, S., Roberts, K., & Taylor, C. (2004). What kind of creature is a design experiment? *British Educational Research Journal, 30*(4), 577-590.

Gorard, S., & Taylor, C. (2004). *Combining methods in educational and social research*. Glasgow: Bell & Brain Ltd.

Gorgorió, N., & Planas, N. (2005). Social representations as mediators of mathematics learning in multiethnic classrooms. *European Journal of Psychology of Education, XX*(1), 91-104.

Greer, B.**,** Mukhopadhyay, S., Powell, A. B., & Nelson-Barber, S. (Eds), (2009). *Culturally responsive mathematics education.* New York: Routledge.

Greer, B., & Skovsmose, O. (in press). *Critique and politics of mathematics education.* Rotterdam: Sense Publishers.

Gresalfi, M. (2004). *Taking up opportunities to learn: Examining the construction of participatory mathematical identities in middle school students.* Stanford: Stanford University.

Gresalfi, M., Martin, T., Hand, V., & Greeno, J. (2009). Constructing competence: an analysis of student participation in the activity systems of mathematics classrooms. *Educational studies in Mathematics, 70*(1), 49–70.

Grootenboer, P., & Jorgensen, R. (2009). Towards a theory of identity and agency in coming to learn mathematics. *Eurasia Journal of Mathematics Science and Technology Education, 5*(3), 255-266.

Grootenboer, P., & Zevenbergen, R. (2007). Identity and mathematics: Towards a theory of agency in coming to learn mathematics. *Mathematics: Essential research, essential practice. Proceedings of the 30th annual conference of the mathematics mducation research group of Australasia Tasmania, Vol. 1* (pp. 335-344). Adelaide: MERGA.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*(1), 37-73.

Gutstein, E. (2006). *Reading and writing the world with mathematics. Toward a pedagogy for social justice.* Great Britain: Routledge.

Gutstein, E., & Peterson, B. (2006). *Rethinking mathematics. Teaching social justice by the numbers.* Wisconsin: Rethinking schools, Ltd.

Gutiérrez, R. (2007). Context matters: equity, success and the future of mathematics educiaotn. In T. Lamberg, & L. R Wiest, (Eds.), *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Stateline (Lake Tahoe).* Reno: University of Nevada.

Gutiérrez, R. (2010). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education,* (Equity Special Issue), 1-32.

Hammersley, M., & Atkinson, P. (2007). *Ethnography: principles in practice.* New York: Routledge.

Hargreaves, A. (2000). Four ages of professionalism and professional learning. *Teachers and teaching: History and practice, 6*(2), 151-182.

Hargreaves, A., Earl, L., Moore, S., & Manning, S. (2001). *Learning to change. Teaching beyond subjects and standards.* San Francisco: Jossey-Bass Inc.

Heller, J. (1961). *Catch 22.* USA: Simon & Schuster.

Hodgen, J., Küchemann, D., Brown, M., & Coe, R. (2010). Lower secondary school students' attitudes to mathematics: evidence from a large-scale survey in England. *Research in Mathematics Education, 12*(2), 155-156

Hodgen, J., & Marks, R. (2009). Mathematical 'ability' and identity: a socio-cultural perspective on assessment and selection. In L. Black, H. Mendick and Y. Solomon (Eds), *Mathematical Relationships in Education: Identities and Participation* (pp. 31-42). London: Routledge.

Holland, D., Lachicotte Jr, W., Skinner, D., & Cain, C. (1998, 2003). *Identity and Agency in Cultural Worlds*. London, Cambridge: Harvard University Press.

Hougaard, B. (2004). *Curlingföräldrar och servicebarn - en handbok i barnuppfostran*. Stockholm: Bokförlaget Prisma.

Hunter, R. (2010). Changing roles and identities in the construction of a community of mathematical inquiry. *Journal of Mathematics Teacher Education,* DOI 10.1007/s10857-010-9152-x.

Ingram, N. (2009) Engagement in the mathematics classroom. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In Search of Theories in Mathematics Education: Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol2, pp. 233-240). Thessaloniki: PME.

Jablonka, E., & Gellert, U. (2010). Ideological roots and uncontrolled flowering of alternative curriculum conceptions. In U. Gellert, E. Jablonka & C. Morgan (Eds.), *Proceedings of the Sixth International Mathematics and Society Conference* (pp. 31-49). Berlin: Freie Universität Berlin.

Jenkins, R. (2004). *Social identity. Second edition*. New York: Routledge.

Johansson, M. (2006). *Teaching mathematics with textbooks: a classroom and curricular perspective*. Doctoral thesis Nr.: 2006:23. Luleå: Luleå University of Technology.

Johnson, A.G. (2000). *The Blackwell dictionary of sociology. A user's guide to sociological language, second education*. Cornwall: TJ International.

Kapsap, A., & Silverman, F. L. (2008). A case study of the role of ethnomathematics among teacher education students of highly diverse cultural backgrounds. *Journal of Mathematics and Culture, 3*(1), 66-102.

Kelly, P. (2006). What is teacher learning? A socio-cultural perspective. *Oxford Review of Education, 32*(4), 505–519.

Kelly, P., & Lesh, R.A. (2000). *Handbook of research design in mathematics and science education*. New Jersey: Lawrence Erlbaum Associates, Inc.

Knijnik, G. (1998). Ethnomathematics and political struggles. *ZDM, 30*(6), 88-194.

Knijnik, G. (2007). Mathematics education and the Brazilian landless movement: three different mathematics in the context of the struggle for social justice. *Philosophy of Mathematics Education Journal, 21*(1).

Konsumentverket (2007). *SMS-lån. En kartläggning av unga vuxnas erfarenheter. Rapport 2007:17*. Karlstad: Konsumentverket.

Kram, K.E., & Ragins, B.R. (2007). The landscape of mentoring in the 21st century. In K.E Kram, & B.R. Ragins (Eds) *The Handbook of Mentoring at Work: Theory, Research, and Practice* (pp 659-689). Thousand Oaks: Sage Publications Inc.

Kvale, S., & Brinkmann, S. (2009). *Inter-Views*. USA: Sage Publication Inc.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational research journal, 27*(1), 29-63.

Lange, T. (2009). *Difficulties, Meaning and Marginalisation in Mathematics Learning as Seen through Children's Eyes*. Doctoral thesis. Aalborg: Aalborg University.

Lange, T. (2010). "Tell them that we like to decide for ourselves". Children's' agency in mathematics education. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (pp. 2587-2596). Lyon: Institut National De Recherche Pédagogique 2010.

Lasky, S. (2005). A socio-cultural approach to understanding teacher identity, agency and professional vulnerability in a context of secondary school reform. *Teaching and Teacher Education, 21*(8), 899-916.

Lave, J., & Wenger, E. (1991). *Situated learning. Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Leder, G., & Grootenboer, P. (2005). Affect and mathematics education. *Mathematics Education Research Journal, 17*(2), 1-8.

Leder, G., Pehkonen, E., & Törner, G. (Eds.) (2002). *Beliefs: A Hidden Variable in Mathematics Education?* Dordrecht: Kluwer Academic Publishers.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (ed.), *Multiple Perspectives on Mathematics Teaching and Learning* (pp.19–44). Westport: Ablex.

Lerman, S. (2006). Cultural psychology, anthropology and sociology: the developing 'strong' social turn. In J. Maasz & W. Schloeglmann (Eds.), *New mathematics education research and practice* (pp. 171-188). Rotterdam: Sense.

Lerman, S. (2009). Pedagogy, discourse and identity. In L. Black., H. Mendick & Y. Solomon (Eds), *Mathematical relationships in education. Identities and participation* (pp. 147-156). New York: Routledge.

Lerman, S., & Zevenbergen, R. (2004). The socio-political context of the meathemaics classroom. Using Bernstein's theoretical framework to understand classroom communications. In P. Valero, & R. Zevenbergen, (Eds), *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology* (pp. 27-42). Dordrecht: Kluwer Academic Publishers.

Lindqvist, U., Emanuelsson, L., Lindström, J.-O., & Rönnberg, I. (2003). *Lusten att lära - med fokus på matematik (No. 221)*. Stockholm: Statens skolverk.

Lovat, T., Toomey, R., & Clement, N. (Eds.) (2010). *International research handbook on values education and student wellbeing*. Dordrecht: Springer.

Lundin, S. (2008). *En kritisk analys av den svenska skolmatematikens förhistoria, uppkomst och utveckling*. Doctoral thesis. Uppsala : Acta Universitatis Upsaliensis.

Martin, D. (2000). *Mathematics success and failure among African-American youth*. New Jersey: Lawrence Erlbaum Associates, Inc.

Martin, D. B. (2009). Mapping a liberatory research and policy agenda. In D.B. Martin (Ed), *Mathematics teaching, learning, and liberation in the lives of black children* (pp. 3-38). New York: Routledge.

Martin, J.R., & Rose, D. (2007). *Working with discourse: meaning beyond the clause*. London: Continuum.

McLeod, D. (1992). Research on affect in mathematics education: a reconceptualization. In D.Grows (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 575-596). New York: McMillan Publishing Company.

Meaney, T. (2005). Mathematics as texts. In A. Chronaki & I.M. Christiansen (Eds), *Challenging perspectives on mathematics classroom communication* (pp. 109-142). USA: Information Age Publishing.

Meaney, T. (2007). Weighing up the Influence of Context on Judgements of Mathematical Literacy. International Journal of Science and Mathematics Education, *5*(4), 681-704.

Mellin-Olsen, S. (1987). *The politics of mathematics education*. Dordrecht: Kluwer Academic Publishers.

Mendick, H. (2006). *Masculinities in mathematics*. Maidenhead: Open University Press.

Mendick, H., Moreau, M.P., & Epstein, D. (2009). Special cases: Neoliberalism, choice, and mathematics. In L. Black, H. Mendick, & Y. Solomon (Eds), *Mathematical relationships in education* (pp. 71-81). New York: Routledge.

Mertens, D. M. (2005). *Research and Evaluation in Education and Psychology: Integrating Diversity with Quantitative, Qualitative and Mixed Methods, 2nd Edition*. Thousand Oaks: Sage Publications, Inc.

Ministry of Education (2000a). *Goals to aim for in mathematics education in upper secondary school*. Stockholm: Frizes.

Ministry of Education, (2000b). *Gy 2000:16. Social science programme: Goal, structure and syllabuses.* Borås: Fritzes.

Ministry of Education (2008, 2009). *Statistical information*. Available at www.skolverket.se

Ministry of Education, (2010a). *Gymnasieskolans kursprov vt 2010. En resultatredovisning.* Available at www.skolverket.se.

Ministry of Education, (2010b). *Svensk författningssamling SFS 2010:2039*. Västerås: Edita Västra Aros.

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational studies in mathematics, 61*, 219-245.

Mukhopadhyay, S., & Greer, B. (2001). Modelling with purpose: mathematics as a critical tool. In English, L.D. (Ed), *Handbook of international research in mathematics education. Second edition*. United States of America: Sheridan Books Inc.

Nieto, S. (2010). *Language, culture and teaching. Second edition*. New York: Routledge.

Nolan, C. (2010). Playing the field(s) of mathematics education: A teacher educator's journey into pedagogical and paradoxical possibilities. In M. Walshaw (Ed). *Unpacking pedagogy: new perspectives for mathematics classrooms* (pp.153-176). USA: Information Age Publishing.

Nolan, K., & de Freitas, E. (2008). Foreword to the research text: Mathematics education under cross-examination. In E. de Freitas & K. Nolan (Eds.), *Opening the research text: Critical insights and in(ter)ventions into mathematics education* (pp. 1-11). New York: Springer.

Norén, E. (2010). Discourses and Agency in a Multilingual Mathematics Classroom. *Nordic Educational Research Working-paper series.* Stockholm University: Stockholm.

Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings. Learning cultures and computers*. Dordrecht: Kluwer Academic Publishers.

OECD (2004). *Learning for tomorrow's world. First results from PISA*. Paris: OECD.

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Palm, T. (2009). Theory of authentic task situations. In L. Verschaffel, B. Greer, W. Van Dooren., & S. Mukhopadhyay (Eds), *Words and worlds. Modelling verbal descriptions of situation*s (pp. 3-20). Rotterdam: Sense Publishers.

Pickering, A. (1995). *The mangle of practice: Time, agency, and science*. Chicago: University of Chicago Press.

Popkewitz, T. (1999). A social epistemology of educational research. In T. Popkewitz, & L. Fendler (Eds.), *Critcal theories in education,* (pp. 17-42). London: Routledge.

Popkewitz, T. (2004). The alchemy of the mathematics curriculum: Inscriptions and the fabrication of the child. *American Education Research, 41*(1), 3-34.

Popkewitz, T. (2009). Curriculum study, curriculum history, and curriculum theory: the reason of reason. *Journal of curriculum studies, 41*(3), 301-319.

Powell, A. (2004). The Diversity backlash and the mathematical agency of students of colour. In M. Johnsen Høines, & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*. Bergen: Bergen University College.

Pring, R. (2000). *Philosophy of educational research*. London: Continuum.

Radford, L. (2008). The ethics of being and knowing: towards a cultural theory of learning. In L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in Mathematics Education: Epistemology, History, Classroom, and Culture* (pp. 215–234). Rotterdam: Sense Publishers.

Ratcliffe, M., & Grace, M. (2003). *Science education for citizenship*. Philadelphia: Open University Press.

Ravn, O., & Skovsmose, O. (2007). *Mathematics as measure. In revista Brasileira de historia da matematica*. Brazil: Publicação Oficial da Sociedade Brasileira de História da Matemática.

Robinson-Pant, A. P. (2005). *Cross-cultural perspectives on educational research.* UK: Open University Press.

Robson, C. (2002). *Real world research: a resource for social scientists and practitioner-researchers*. Oxford: Blackwell.

Ross, A. (2008). Organizing a curriculum for active citizenship education. In J. Arthur, I. Davies, & C. Hahn, (Eds). *The SAGE handbook of education for citizenship and democracy.* (pp 492-505). London: SAGE Publications Ltd.

Roth, W. M. (2007). Identity in science literacy. In W. M. Roth and K. Tobin (Eds), *Science, learning, identity: Sociocultural and Cultural-historical Perspectives* (pp. 147-185). Rotterdam: Sense Publishers.

Roth, W. M., & McRobbie, C. (1999). Lifeworlds and the "w/ri(gh)ting" of classroom research. *Journal of Curriculum Studies*, *31*, 501-522.

Roth W. M., & Tobin, K. (2007). (Eds), *Science, learning, identity: Sociocultural and Cultural-historical Perspectives*. Rotterdam: Sense Publishers.

Roth, W. M., Tobin,K., Zimmermann, A., Bryant, N., & Davis, C. (2002). Lessons on and from the dihybrid cross: An activity and theoretical study of learning in coteaching. *Journal of Research in Science Teaching, 39*(3), 253-282.

Ryan, E. (2006). *...regards, some girl with words. Genevieve's Journey*. Hartwell, Victoria: SidHarta Publishers.

School Inspectorate (2010). *Skolinspektionens Kvalitetsgranskning Rapport 2010:13*. Stockholm: Skolinspektionen.

Seah, W.T. (2008). Valuing values in mathematics education. In Clarkson, P. & Presmeg, N. (2008). *Critical issues in mathematics education. Major contributions of Alan Bishop* (pp. 239-252). New York: Springer.

Sfard, A. (2008). *Thinking as communicating*. New York: Cambridge University Press.

Sfard, A., Forman, E., & Kieran, K. (2001). Learning discourse: Sociocultural approaches to research in mathematics education. *Educational Studies in Mathematics, 46*(1/3), 1-12.

Sfard, A., & Prusak, A. (2005a). Telling identities: In a search of an analytical tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 2-24.

Sfard, A., & Prusak, A. (2005b). Identity that makes a difference: substantial learning as closing the gap between actual and designated identities. In H. L. Chick, & J. L. Vincent, (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, Vol. 1.* (pp. 37-52). Melbourne: PME.

Shanker, S. (1987). *Wittgenstein and the turning-point in the Philosophy of Mathematics*. London: Croom Helm Ltd.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer Academic Publishers.

Skovsmose, O. (1998). Linking mathematics education and democracy: citizenship, mathematical archeology, mathematicy and deliberative action. *Zentralblatt fur Didaktic der Mathmatik/International Reviews on Mathematics Education, 30*(6), 195-203.

Skovsmose, O. (2001). Landscapes of investigation. *ZDM, 33*(4) 123-132.

Skovsmose, O. (2005a). *Travelling through education: Uncertainty, mathematics, responsibility*. Rotterdam: Sense Publishers.

Skovsmose, O. (2005b). Meaning in Mathematics Education. In J. Kilpatrick, C. Hoyles, O. Skovsmose, & P. Valero (Eds), *Meaning in mathematics Education* (pp. 85-102). USA: Springer Science + Business Media, Inc.

Skosvmose, O. (2010). Critical mathematics education: in terms of concerns. In B. Sriraman, C. Bergsten, S. Goodchild, G. Palsdottir, B. D. Søndergaard & L. Haapasalo (Eds.), *The sourcebook on Nordic research in mathematics education* (pp. 671-682). Charlotte (USA): Information Age Publishing.

Skovsmose, O., & Borba, M. (2004). Research methodology and critical mathematics education. In Valero, P. & Zevenbergen, R. (Eds), *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology* (pp 207-226). Dordrecht: Kluwer Academic Publishers.

Skovsmose, O., & Nielsen, L. (1996). Critical Mathematics Education. In A. J. Bishop. *International Handbook of Mathematics Education* (pp 1257-1288). Dordrecht: Kluwer Academic Publishers.

Skovsmose, O., & Ravn, O. (2011). *Matematikfilosofi*. Aarhus: Systime.

Skovsmose, O., & Valero, P. (2002). Democratic Access to Powerful Mathematical Ideas. In L. D. English (Ed), *Handbook of International Research in Mathematics Education* (pp. 383-407). New Jersey: Lawrence Erlbaum Associates, Incorporated.

Solomon, Y. (2009). *Mathematical literacy. Developing identities of inclusion*. New York: Routledge.

Soyini Madison, D. (2005) *Critical ethnography: method, ethics, and performance*. Thousand Oaks: Sage.

Stentoft, D. (2009). *Challenging research conceptions in (mathematics) education: telling stories of multiplicity and complexity.* Aalborg: Uniprint.

Stentoft, D., & Valero, P. (2009). Identities-in-action: Exploring the fragility of discourse and identity in learning mathematics. *Nordic Studies in Mathematics Education*, *14*(2).

Stocker, D., & Wagner,D. (2007). Talking about teaching mathematics for social justice. *For the Learning of Mathematics*, 27(3), 17-21.

Sullivan, P., Tobias, S., & McDonough, A. (2006). Perhaps the decision of some students not to engage in learning mathematics is deliberate. *Educational Studies in Mathematics, 62,* 81–99.

Swanson, D.M. (2007). School mathematics: Discourse and the politics of context. In A. Chronaki & I.M. Christiansen (Eds), *Challenging perspectives on mathematics classroom communication* (pp. 261-294). USA: Information Age Publishing.

Swedish School Inspectorate (Skolinspektionen) (2010). *Skolinspektionens Kvalitetsgranskning Rapport 2010:13: Undervisningen i matematik i gymnasieskolan*. Stockholm: Skolinspektionen.

Szabo, A., Larson, N., Viklund, G., & Marklund. M. (2007). *Origo. Matematik kurs A för samhällsvetenskapliga och estetiska program*. Värnamo: Fälth & Hässler.

Thomas, J. (1993). *Doing critical ethnography*. USA: Sage.

Tsatsaroni, A., Evans, J., & Morgan, C. (2007). Pedagogic discourse, positioning and emotion: illustrations from school mathematics. *Reviews of science, mathematics and ICT education, 1*(1), 83-105.

Valero, P. (2002a). *Reform, democracy and mathematics education: Towards a socio-political frame for understanding change in the organization of secondary school mathematics.* Copenhagen: Danish University of Education.

Valero, P. (2002b). The myth of the active learner: From cognitive to socio-political interpretations of students in mathematics classrooms. In P. Valero & O. Skovsmose (Eds.), *Proceedings of the 3 International MES Conference* (pp. 1-13). Copenhagen: Centre for Research in Learning Mathematics.

Valero, P. (2004a). Socio-political perspectives on mathematics education. In P. Valero & R. Zevenbergen (Eds), *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology* (pp. 5-24). Dordrecht: Kluwer Academic Publishers.

Valero, P. (2004b). Postmodernism as critique to dominant mathematics education research. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 35–54). Greenwich, CT: Information Age Publishing.

Valero, P. (2007). A socio-political look at equity in the school organization of mathematics education. *ZDM, 39*(3) 225-233.

Valero, P. (2007). In between the global and the local: The politics of mathematics education reform in a globalized society. In B. Atweh, A. Calabrese Barton, M. Borba, N. Gough, C. Keitel, C. Vistro-Yu & R. Vithal (Eds.), *Internationalisation and Globalisation in Mathematics and Science Education* (pp. 421-439). New York: Springer.

Valero, P. (2010). Mathematics education as network of social practises. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello. *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education*. (pp. LIV-LXXX). Lyon: Institut National De Recherche Pédagogique 2010.

Valero, P. (2011). Re-interpreting students' interest in mathematics: Youth culture resisting modern subjectification. Paper presented at the 63 CIEAEM. Barcelona, July 24-29.

Valero, P., & Stentoft, D. (2010). The 'post' move of critical mathematics education. In H. Alrø, O. Ravn, & P. Valero (Eds.), *Critical mathematics education: Past, present and future* (pp. 183-195). Rotterdam: Sense Publishers.

Valero, P., & Zevenbergen, R. (eds). (2004). *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology*. Dordrecht: Kluwer Academic Publishers.

Verschaffel, L., Greer.B., & de Corte. E. (2000). *Making sense of word problems*. Lisse: Krips b.v. Meppel.

Vithal, R. (2003). *In search of a pedagogy of conflict and dialogue for mathematics education*. Dordrecht: Kluwer Academic Publishers.

Vithal, R. (2004). In P. Valero, & R. Zevenbergen (eds). *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology* (pp. 227-248). Dordrecht: Kluwer Academic Publishers.

Wackernagel, W., & Rees, E. (1996). *Our ecological footprint. Reducing human impact on the earth*. Gabriola Island: New Society Publishers.

Wagner, D. (2007). Students' critical awareness of voice and agency in mathematics classroom discourse. *Mathematical Thinking and Learning*, *9*(1), 31-50.

Wagner, D. (2010). The seductive queen - mathematics textbook protagonist. In U.Gellert,, E. Jablonka., & C. Morgan (Eds). *Proceedings of the Sixth International Mathematics Education and Society Conference* (pp. 467-477). Berlin: Freie Universität Berlin.

Walshaw, M. (2010). *Unpacking pedagogy: new perspectives for mathematics classrooms*. USA: Information Age Publishing.

Wedege, T. (1999). To know or not to know - mathematics, that is a question of context. *Educational Studies in Mathematics,* 39, 205-207.

Wedege, T. (2000). Technology, competences and mathematics. In D. Coben, G. FitzSimons & J. O'Donoghue (Eds.), *Perspectives on adults learning mathematics: Research and practise*. (pp. 191-207). Dordrecht: Kluwer Academic Publishers.

Wedege, T. (2010a). Connecting theories in mathematics education: from bricolage to professionalism. *Nordic studies in Mathematics education, 15*(1), 59-78.

Wedege, T. (2010b). Sociomathematics: A subject field and a research field. In U.Gellert, E. Jablonka & C. Morgan (Eds). *Proceedings of the Sixth International Mathematics Education and Society Conference* (pp. 478-487). Berlin: Freie Universität Berlin.

Wedege, T., & Evans, J. (2006). Adults' resistance to learn in school versus adults' competences in work: The case of mathematics. *Adults Learning Mathematics - an International Journal,* 1(2), 28-43.

Willis, P., & Trondman, M. (2000). Manifesto for Ethnography. *Ethnography, 1*(5), 5-16.

Winbourne, P. (2009). Choise: parents, teachers, children, and ability grouping in matehamtics. In L. Black., H. Mendick & Y. Solomon (Eds) *Mathematical relationships in education. Identities and participation* (pp. 58-70). New York: Routledge.

Wittgenstein, L. (1978). *Remarks on the Foundations of Mathematics. 3rd edition*. Oxford: Basil Blackwell.

Wittgenstein, L. (1983). *Philosophical Investigations. Second Edition* (First published 1953). Oxford: Basil Blackwell.

Wolcott, H.F. (2005). *The art of fieldwork*. USA: AltaMira press.

Wolcott, H.F. (2008). *Ethnography, A way of seeing. 2nd edition*. USA: AltaMira press.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation and autonomy in mathematics. *Journal for research in mathematics education, 27(*4), 458-477.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*(2), 113–121.

Addenda

Here there are the declarations of co-authorship for the papers that are part of the thesis.

ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.
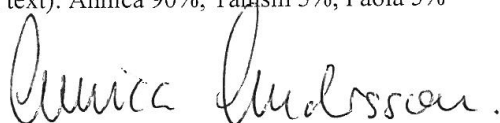
Paper title: Identity narratives of [dis]engagement in mathematics education contexts

Place of publication: Submitted to
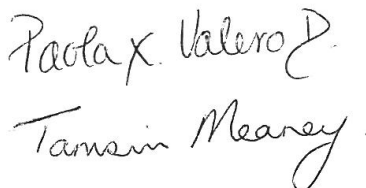
List of authors: Andersson, Annica
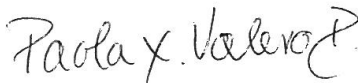
Valero, Paola

PhD student: Annica Andersson

Contribution: (%, text): Annica 90%, Paola, 10%

Signature, PhD student

Signature, co-author

ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.

Paper title: I am [not always] a math-hater": Students' (dis)engagement in mathematics education

Place of publication: Submitted to Educational studies in mathematics

List of authors: Andersson, Annica
           Meaney, Tamsin
           Valero, Paola

PhD student: Annica Andersson

Contribution: (%, text): Annica 90%, Tamsin 5%, Paola 5%

Signature, PhD student

Signature, co-author

ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.

Paper title: Examining a critical pedagogical discourse for agency and social empowerment.
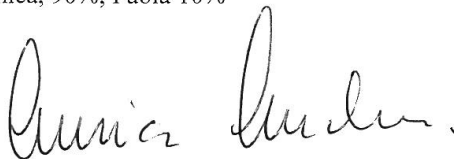
Place of publication: In P. Ernest and B. Sriraman (Eds), *Critical Mathematics Education: Theory and Praxis.* USA: Information Age Publishing.

List of authors: Andersson, Annica
            Valero, Paola

PhD student: Annica Andersson

Contribution: (%, text): Annica, 90%, Paola 10%

Signature, PhD student

Signature, co-author

ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.

Paper title: (2009). Mathematics education giving meaning to social science students.

Place of publication: Accepted for proceedings *at II Congreso Internacional de Investigación, Educación y Formación Docente.* Colombia: Medellin.

List of authors: Andersson, Annica
                 Valero, Paola

PhD student: Annica Andersson

Contribution: (%, text): Annica 90%, Paola 10%

Signature, PhD student

Signature, co-author

ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.
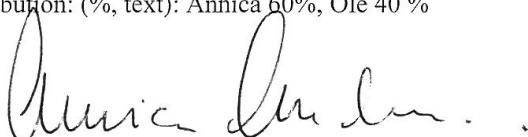
Paper title:. (In press). A critical perspective on contextualisations in mathematics education.

Place of publication: In O. Skovsmose & B. Greer (Eds), *Critique and politics of mathematics education*. Rotterdam: Sense Publishers.

List of authors: Andersson, Annica
                 Ravn, Ole

PhD student: Annica Andersson

Contribution: (%, text): Annica 60%, Ole 40 %

Signature, PhD student

Signature, co-author

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Co-author statement in connection with submission of PhD thesis

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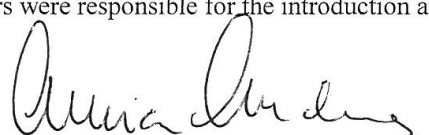
Paper title: AGENCY IN MATHEMATICS EDUCATION

Place of publication: proceedings CERME 7 (The Seventh Congress of the European Society for Research in Mathematics Education, held at the University of Rzeszów, Poland, between 9th and 13th February 2011)
http://www.cerme7.univ.rzeszow.pl/WG/10/CERME7_WG10_AnderssonNoren.pdf

List of authors: Annica Andersson, Eva Norén

PhD student: Annica Andersson

Contribution: 50 % of text. Annica was mainly responsible for the part concerning Biesta's Philosophy of (mathematics) education; Eva was mainly responsible for the part concerning Skovsmose's philosophy of critical mathematics education. Both authors were responsible for the introduction and conclusions.

Signature, PhD student

Signature, co-author

**Stockholms
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2011-08-17

Permission to publish article

Annica Andersson have my permission to publish our article AGENCY IN MATHEMATICS
EDUCATION presentetd at CERME 7 2011, in her thesis.

Eva Norén,
PhD mathematics education

Institutionen för matematikämnets och
naturvetenskapsämnenas didaktik

Stockholms universitet
Institutionen för matematikämnets och
naturvetenskapsämnenas didaktik   www.mnd.su.se
Stockholms universitet
106 91 Stockholm

Telefon: 08-1207 66 29

E-post: eva.noren@mnd.su.se