ABSTRACT

Title: THE MEETING WITH MATHEMATICAL PROBLEMS
An illumination of children's learning
Acta Universitatis Gothoburgensis 87:1992
ISBN 91-7346-250-0
Language: Swedish (English summary)
Keywords: Mathematical problem solving; Teaching arithmetic in primary school; Interaction in small groups; Writing, drawing and communicating in mathematics; Conceptions of arithmetic problems; The dynamics of problem solving; Phenomenography

The aim of the empirical study is to describe and analyse how primary school children in a school context experience arithmetic word problems and problem solving. The investigation has been carried out with a phenomenographic research approach and includes both an interview and a classroom study. In the interview study deep semi-structured interviews were held with 38 primary school pupils while they worked at solving problems from a sequence consisting of five different types of problem. In the classroom study three school classes were studied twice a week for a term while they worked with the same problems as the pupils in the interview study. In the classroom study problem solving was studied with the point of departure that pupils should draw, write, talk and calculate while solving problems. The classroom has a quasi-experimental design with pre- and post-tests in the participating and control classes, since one partial aim of the study is to see if such a change in working methods can contribute to pupils' ability to solve arithmetic problems.

The basic intentions underlying the design and content of the lessons in the classroom study was that the pupils should be able to speak their own language, carry out different actions, and vary their perspective on arithmetic problem solving and the problems posed. Sixty-eight lessons were observed and recorded on audio tape. The empirical material includes all the documents produced by pupils while solving the problems, comprising written stories, drawings and arithmetic calculations. The pupils who participated commented on the lessons in interviews and written reports. The class teachers have also given their comments on the lessons and the pupils' learning.

The results show that the pupils in the participating classes solved the problems in the post-test to a greater degree than did the pupils in the control classes. A covariance analysis shows that the difference between the two groups was statistically significant.

A pupil's understanding of a problem depends on their prior experiences, the problem situation and the specific problem. They have a preconception of a given problem which gives them a diffuse overall

understanding of it. Thereafter they differentiate within the problem and while solving it they refer to different parts of the content. When the pupils relate the problem's parts to one another and integrate them to a composite whole the problem takes shape such that they understand and conceptualise the problem. This understanding is seen in their orientation and approach to the problem and in their conception of the problem.

It is found that pupils can have two orientations, in which they have different goals or intentions for solving the problem.

Taken-for-granted orientation: Pupils have a *product*-intention; they want to *give* an answer to the problem.

Open orientation: Pupils have a *process*-intention; they want to *search for* an answer to the problem.

Pupils approach the problem in four qualitatively different ways.

Operand approach: Pupils focus on the numbers; they estimate a numerical answer to problems and do not carry out calculations.

Procedure approach: Pupils focus on numbers and operations; they perform a calculation to reach an answer.

Hypothesis approach: Pupils focus on all parts of the problem's content, and try to see how they are related. They do not, however, relate the numbers given to the relevant content of the problem.

Gestalt approach: Pupils focus on all the parts of the problem and on the relationships between them.

Pupils with a taken-for-granted orientation have an operand or a procedure approach. They see only the problem's surface and problem solving means that they have to give a numerical answer or perform an arithmetic operation. They apply well-known methods to the solution and solve the problem according to a fixed pattern. Pupils with an open orientation have a hypothesis or a gestalt approach. They go into the problem in depth and problem solving means that they relate the parts of the problem's content to one another. They are captivated by the problem solving process; they pose hypotheses, try different alternative solutions, and can vary their perspective.

A pupil's orientation, approach to and conceptions of the problem determine how the problem solving process takes shape. The process itself is characterised by the three components direction, reference and movement which in a dialectic relationship determine the outcome of the attempt to solve the problem.

Pupils' different ways of experiencing arithmetic problems and problem solving is formed in the meeting between their own ideas, the classroom situation, and the content of the lesson. The problems which pupils meet have a meaning for them and present themselves in the light of the pupils' earlier experience. Teaching of arithmetic problem solving should therefore to a greater extent give the pupils the opportunity to form arithmetic relationships and discover mathematical structures with a basis in their own experiences.