UNIVERSITY OF ICELAND
SCHOOL OF EDUCATION
Centre of Research in Mathematics Education
v/Stakkahlíð
105 Reykjavík
Iceland

On-going research in the history of mathematics education

A conference on research in the history of mathematics education
Date: June 20–24 2009
Location: Fjölbrautaskólinn í Garðabæ, 210 Garðabær, Iceland.

Programme overview (tentative):

June 20	June 21	June 22	June 23	June 24
Arrival and registration	8.30 Introduction	8.30 Announcements	8.30 Announcements	Departure
	8.45 – 10.25 Two presentations	8.40 – 10.20 Two presentations	8.40 – 10.20 Two presentations	
	10. 25 Coffee break	10. 20 Coffee break	10. 20 Coffee break	
	11.00 – 12.40 Two presentations	10.50 – 12.30 Two presentations	10.50 – 12.30 Two presentations	
	12.40 Lunch	12.30 Lunch	12.30 Lunch	
	13.40 – 15.20 Two presentations	13.30 – 15.10 Two presentations	13.30 – 15.10 Two presentations	
	15.20 Coffee break	15.10 Coffee break	15.10 Coffee break	
	15.50 – 16.40 Presentation	15.40 – 16.30 Presentation	15.40 – 16.30 Presentation	
	16.40 – 17.00 Announcements	16.30 Announcements	16.30 Closing session	
	17.00 – 21.00 Sightseeing and dinner by invitation of the Mayor of Gardabaer	17.00 – 21.30 Excursion to Thingvellir by invitation of the Univ. of Iceland, School of Education	17.00 Free afternoon 20.00 – 23.00 Conference dinner	

Presentations:

John Playfair in the Natural Philosophy Classroom

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While textbooks are deservedly considered valuable and interesting primary sources by mathematicians as well as by historians of mathematics education, these materials generally provide little insight into how classes were conducted each day or into what students actually learned. To develop a more complete picture of educational practice, textbooks must be combined with information gleaned from administrative records, student notebooks, student reminiscences, obituaries, and the like. Unearthing that sort of documentation, though, often depends as much on serendipity as on systematic research. John Playfair (1748-1819) served as professor of mathematics and then of natural philosophy at the University of Edinburgh. In addition to *Elements of Geometry* and *Illustrations of the Huttonian Theory of the Earth*, the books for which he is best known, he organized his lectures into *Outlines of Natural Philosophy* (2 vol., Edinburgh, 1812-1814). There are also at least 5 extant sets of notes taken by students who attended his natural philosophy course. This paper will analyze as many of

these notes as possible, focusing especially on the following questions: How closely do the notes conform to each other and to the textbook? Did the material Playfair covered change over time, such as before and after *Outlines* was published or when he revised the textbook in 1816 and 1819? Did the fact that he was primarily a mathematician early in his career inform his choice of topics and the manner in which he presented them? Were there aspects of the course that were uniquely Scottish?

Enlightenment vs. Philosophy in Icelandic Mathematics Education in the 19th Century

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Enlightenment brought interest in mathematics education to Iceland and efforts towards educating the colonized nation towards self-esteem against foreign merchants. Several handwritten and printed arithmetic textbooks were written and published in the 18th and early 19th century. The most prominent offspring of the movement was Björn Gunnlaugsson (1788–1876) who never went to school in Iceland but studied mathematics at the University of Copenhagen. He became a teacher at Iceland's only upper secondary school in 1822–1862 and made geodetic measurments to prepare the first correct map of Iceland's coast. In his teaching duties he taught Danish mathematics textbooks to fulfil the requirements of the University of Copenhagen.

When retired Gunnlaugsson wrote a book on mathematics in two volumes of which only one was ever printed and published in 1865. As a mathematician Gunnlaugsson was well at home in 18th century works on mathematics, such as by Euler and Gauss. However, after forty years of solitude as the only mathematician in Iceland, Gunnlaugsson had formed his own philosophical and religious attitude towards mathematical concepts and conventions. Gunnlaugsson's work, his background and influences on future mathematics education will be explored in the presentation.

Title to be announced

Eileen Donahue
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The Evolution of the Journal *L'Enseignement Mathématique* from the Initial Aims towards New Trends

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The journal *L'Enseignement Mathématique* was born as a journal oriented to mathematics teaching. Its goals, which included information and internationalization, were realized through the promotion of the birth of ICMI and the successive regular hosting of a section devoted to ICMI. After the First World War the crisis of ICMI slowly affected the policy of the journal. In the 1960s the journal was considered no more in line with the expectations of the world of mathematical instruction.

In my talk I outline some key moments in the evolution of the journal (editorial board, contributors, types of articles published,…) in the general frame of the changes in the world of instruction.

The International Commission on Mathematical Instruction (ICMI)
The Italian Contribution from the Founding to the 1950s

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The ICMI (acronym adopted in the 1950s) was created in 1908 in Rome during the IV International Congress of Mathematicians. Its first president was Felix Klein, eminent mathematician and promoter of significant reforms in the teaching of mathematics in Germany. Eighteen countries from all over the world joined, including Italy, whose delegates were Guido Castelnuovo, Federigo Enriques and Giovanni Vailati. The choice of delegates was not surprising: Castelnuovo was among the organizers of the Congress and a member (as was Enriques) of the renowned Italian school of algebraic geometry. Above all, both Castelnuovo and Enriques shared Klein's ways of conceiving research as well as of teaching mathematics. The choice of Vailati as a delegate was also a natural one, because at the time he was engaged in the project to reform Italian secondary school, and Klein was one of his points of reference.

The initial goal of the Commission was to "promote an inquiry and publish a general report on current trends in secondary teaching of mathematics in the various countries". Since its founding, the ICMI has gone through successive periods of more or less intense activity (connected with the dramatic events of the first half of the twentieth century) before arriving to the end of the 1960s, when it experienced a veritable renaissance based on new aims and work methodologies:

In my talk I will illustrate (partly on the basis of unpublished documents) the Italian contribution to ICMI activities from 1908 to the early 1950s, when the *Commissione Italiana per l'Insegnamento della Matematica*, was created, focusing on the following points: the most relevant figures (Castelnuovo, Enriques, Loria, Ascoli, etc.), with particular emphasis on their various initiatives in Education; the reports on the inquiries of the early periods and the influence on the school reforms in Italy; the political role of Salvatore Pincherle, president of the International Mathematical Union, in re-establishing International collaboration in 1928; the genesis of *Commissione Italiana per l'Insegnamento della Matematica.*

Essential Bibliography

Fondo Vailati, Library of Philosophy, University of Milan.
Fondo Ascoli, Private Family Archive, Torino.
ICMI Archives, Helsinki.

L. Giacardi, *Timeline*, *G. Ascoli, G. Castelnuovo, G. Scorza, F. Enriques, Questionnaires and Reports*, in F. Furinghetti, L. Giacardi (eds) http://www.icmihistory.unito.it/.

L. Giacardi, *La International Commission on Mathematical Instruction. Il contributo italiano dalla fondazione agli anni Cinquanta,* Ferrara, 21.11.2008 http://www.dm.unito.it/sism/ferrara/sunti.pdf

M. Menghini, F. Furinghetti, L. Giacardi & F. Arzarello (Eds.) 2008. *The first century of the International Commission on Mathematical Instruction (1908-2008). Reflecting and shaping the world of mathematics education*, Rome: Istituto della Enciclopedia Italiana.

On the Wings of Arithmetic and Geometry: Astronomical Knowledge in Iceland from the Reformation to the End of the Enlightenment

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After the Reformation the University of Copenhagen became *the* university of Iceland since the country was under the Danish throne. Young Icelanders would therefore go to Copenhagen for an education, at first in theology and later also in law, medicine and other fields. This was more or less the state of affairs until the second half of the nineteenth century.

During most of the period under consideration students at the University of Copenhagen were required to study introductory mathematics which included basic astronomy. The more philosophical aspects of astronomy such as the structure of the cosmos and the nature of heavenly bodies were usually discussed in the subject of natural philosophy which was at first based on Aristotelian ideas and in later centuries on Cartesian and finally Newtonian physics.

While in Copenhagen some of the Icelandic students became very interested in mathematical subjects, not only arithmetic, geometry and the mathematical parts of astronomy, but also closely related fields such as cartography and time-reckoning, and later surveying and triangulation.

We begin with **Guðbrandur Þorláksson** (1542-1627) and his work in the last quarter of the sixteenth century on cartography, the determination of latitude and the diurnal motion of the sun. Next we discuss **Gísli Einarsson** (1621-1688), the first appointed teacher of arithmetic, geometry and astronomy in Iceland. He calculated the almanac for Copenhagen for the year 1650 and made various astronomical observations at Skálholt. In the early eighteenth century **Magnús Arason** (1683-1728), a former student of the famous Danish scientist Ole Rømer (1644-1710), worked as a military engineer in the Danish army. As such he was sent to survey and map Iceland in 1721. Because of his early death the work was not completed. During his studies in Copenhagen he managed to publish several disputations, e.g. on lunar astronomy and on simple surveying methods. Later in the eighteenth century **Stefán Björnsson** (1721-1798), who was considered to be an excellent mathematician, worked on the triangulation of Denmark under the direction of Thomas Bugge (1744-1815). He published, among other works, disputations on Newtonian astronomy, a book on tetragons and papers in Icelandic on surveying and mechanics.

We end the overview by a short description of the work of **Rasmus Lievog** (1738-1811), a Norwegian who spent a quarter of a century in Iceland making astronomical and other observations at the Lambhus observatory under the direction of Thomas Bugge. His detailed observations of the eclipses of the Galilean satellites of Jupiter were later employed, together with other observations, in developing ephemerides in support of the Galileo space mission (1989-2003).

From Descriptive History to Interpretation and Explanation – the Case of Mathematics Education in Denmark in the 20th Century

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To celebrate the Unesco Mathematics year 2000 in Denmark it was decided to write a history of mathematics education in Denmark in the 20th century. It should include all formal

education up to BA at university. Ten authors worked on this for 7 years and the book on the subject has just appeared.
The coordinator of the author group will tell about the working process and the considerations about inventing constructs and grounded theory in an otherwise very descriptive historical account. Especially it will be discussed whether certain wave models can describe the development of math education, especially a model of oscillations between periods emphasizing understanding and periods emphasizing mechanical skills.

Title to be announced
Bernard Hodgson
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Choices of Routes in the History of Mathematics Education in Sweden

Bengt Johansson
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In my presentation I will give examples of critical choices of routes from the 400 year mathematics has been officially accepted as a subject for teaching in Sweden. The examples are taken mainly from studies of textbooks and instructional materials for teachers.

Toward a Cognitive Historiography of Mathematics Education

Jason Kastanis & Nikos Kastanis
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During the last years, a cognitive approach to mathematics education and history of mathematics is rapidly developing. This context favors the emergence of a new interest on the historiography of mathematics teaching and learning. Therefore, a reason is given to introduce to this approach to the historical consideration of mathematics education. We shall concentrate to the notions “practices of knowledge” and “cultural practice” as used in relevant mathematical and scientific studies, aiming to show their meaning and their role in the renewed historiography of mathematics teaching and learning.

Title to be announced
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Changing Representations and Practices in School Mathematics: the Case of Modern Math in Portugal

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The curricular movement known as Modern Math aimed at the transformation of representations and practices in school mathematics. Its study provides us with ways to understand how these changes came about. The purpose of this paper is to contribute to the

understanding of the ways in which representations of school mathematics gradually appropriated ideas from the Modern Math movement, how these new ideas merged into local educational traditions, and how were they transformed into meaningful practice. This work is centred on the Portuguese context from the middle 50s to the middle 60s of last century, and builds on Chervel's notion of school culture and Gruzinski' discussion of connected histories.

The Teaching of Intuitive Geometry in Early 1900s Italian Middle Schools: Programs, Mathematicians' Views and Praxis

Marta Menghini
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In 1881, intuitive geometry comes to life to be taught in Italy in the first three years of the Gymnasium (corresponding to the present middle school). It was explicitly considered as an introductory subject to let students better understand the rational geometry of the Lycée. The lack of a formal definition and of a detailed tasks' description of intuitive geometry caused continuous role changes in the Italian school programs. We discuss and analyze the reasons which led to different choices in the programs and in the books of intuitive geometry in the period between the 19th and the first half of the 20th century, which are also connected with the views of important mathematicians of the time.

The Changes of Attitudes to Teaching and Learning, and the Emergence of New Ideas about the Content and Methodology of the Mathematics Curriculum in England from 1950 to 1980

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This paper describes a period in the history of the development of the mathematics curriculum in England where a new teachers' organisation arose to challenge traditional views of mathematics teaching and its ideological basis, with an original vision of the relationship between teaching and learning and the creative nature of mathematics. The Association of Teachers of Mathematics was formed at a time when the 'New Mathematics' was about to emerge onto the educational scene, and the Organisation for European Economic Cooperation was proposing fundamental changes in the content of school mathematics curricula. The epistemological claims and the philosophical basis of the new paradigm are examined, and the extent of the influence of the changes in the teaching of mathematics in later years are considered.

Professional Debate and Social Structure in Swedish Mathematics Education 1920-1960
The Case of Geometry Instruction at Lower Secondary Level

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This paper treats a professional debate on how geometry instruction should be performed in schools. The reason to study this type of professional debate is that it can be considered an arena where the formal curriculum is interpreted and arguments about the teachers' daily matters are produced.
The main purpose is to present a prosopography where arguments about geometry instruction are related to a social structure within the group of persons that were engaged in the debate.
The main sources are periodicals on mathematics and science instruction, but also textbooks as I consider these a part of the argumentation about how geometry instruction should be carried out. The investigation of the social structure focus on the debaters' professional careers, which include educations, academic careers, political careers, bibliographies, sales figures regarding textbooks, positions in the school system and other positions linked to mathematics education. The investigation of the relation between the arguments and the social structure within the group of debaters is based on Pierre Bourdieu's theory about capital and field.
The advantage of using a prosopographic method together with Bourdieu's theory is that it can reveal how different types of arguments, but also competences, were linked to groups of debaters that held different positions in a community of mathematics teachers. Moreover, the analysis can reveal what ideas, arguments and competences that were given a value within the group of debaters. This might enhance our understanding of the motives behind the production of arguments, but also the influence of the arguments.

How to Relate Regional History to General Patterns of History?

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Over the last years, I did intense research about how mathematics teaching became implemented in Westphalia between 1800 and 1840. Since Westphalia was one of the provinces of the German state Prussia, this is evidently a quite regional history. By analyzing the concrete political, religious and cultural changes in this period, the case reveals to be an indicator for general changes in the educational systems in Europe after the French Revolution. Taking into account the contextual systems contributes to unravelling general patterns of the history of mathematics education.

Title to be announced

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Mathematics education in East Asia from antiquity to modern times

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Since the early 1990s the learning process of Asian students brought up in the tradition of the Confucian heritage culture (CHC) has become a much discussed issue. Likewise and as a consequence the teaching process of Asian teachers in CHC classrooms has attracted the attention of some educators. These two related issues are brought into focus in the so-called "The CHC Learner/Teacher Paradox". It is therefore natural to look at the history of mathematics education in some Asian countries like China, Japan and Korea. This talk attempts to give an account of this long episode from ancient to medieval to modern times with illustrative examples. By the nineteenth century Western influence became more and more dominating, making the revived attention to CHC classrooms all the more interesting from a pedagogical viewpoint.

Foreign Influences on Dutch Mathematics Education

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During the nineteenth century, mathematics was gradually established as a major topic within the Dutch secondary schoolsystem. In this period, French and German influences are easily established within the program and schoolbooks, and also with leading personalities of the time, like Jacob de Gelder (1765-1848) and Jan Versluys (1845-1920) But, once holding a strong position within the schoolsystem, Dutch schoolmathematics became more rigid, less open to influences from abroad. For instance, the Reform movement, inspired by Felix Klein, had only a limited and marginal effect. The prolific and infuential schoolbookwriter and didactician Pieter Wijdenes (1872-1972) openly declared he never read any textbook from abroad. Another key figure in the first half of the twentieth century, Edard Jan Dijksterhuis (1892-1965), was as historian of mathematics much more internationally orientated, but as a math didactician he also was rather conservative, even in the way he advocated some parts of the Meran Proposals. As fas as more modern ideas were propagated, this was mainly done by Tatiana Ehrenfest-Afanassjewa (1876-1964), born in Kiev, educated in St. Petersburg, who had studied math and physics in Göttingen with Klein and Hilbert. In 1912 she came with her husband, Paul Ehrenfest, a physicist, to Leiden where she lived until her death. When finally in the sixties of the last century Dutch schoolmathematics underwent a thorough reform, a decisive rol was played again by someone originally from another country: Hans Freudenthal, (1905-1990) from Luckenwalde, Germany. Freudenthal came in 1930 as an assistent to L.E.J. Brouwer to Amsterdam, and also remained in The Netherlands until his death. He attended for many years a didactical working group organised by Tatiana Ehrenfest, and there can be no doubt that this working group played an important role in arousing his interest in and developing his ideas about mathematics education. Maybe being a relative outsider was an advantage for looking openminded to a system that was in some ways the prisoner of its own succes.

History of mathematics education in training teachers of mathematics: considering its potentialities.

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The purpose of this paper is to analyze the potentialities of a history of mathematics education in training programmes of math teachers. A departing postulate, that teachers will tend to develop current pedagogical practice of a better quality by maintaining an historical relationship with past educational practices produced by their professional ancestors, will be put forward. Consequently, this study will answer the following questions: what history of mathematics education could be included in training of math teachers? To answer this question, the theoretical and methodological background of Cultural History will be used. Through history of mathematics education, teachers may become aware of processes and curricular models for the teaching of mathematics that historically had been circulating at a planetary scale acquiring sense at specific time and places.

An Example of Early Nordic Mathematics: On the Grey Zone between Education and Practical Experience

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It is well known that the Nordic people of the Viking Era sailed through most of the Northern Atlantic. When you navigate at open sea it is crucial to know your location. However, you will often lack terrestrial references for this. But, given certain circumstances, you may use solar altitude at noon to get your geographical latitude. The main problem for that is the variation of this altitude with time during the summer, being at its maximum at summer solstice. If you have some kind of a formula for an approximation to the variation it will clearly be quite valuable.
Since we have a function around its extremum we (modern people) know that it can be approximated by a parabola. Summing integers up to a given number will give a series which is simply related to the squares and hence to a parabola.
In the paper we will report on a formula for solar noon altitude where this is utilized to give a practical mnemonic device for navigators. The formula is attributed to a 12th century Icelandic farmers‘s hand named Star-Oddi Helgason, but may well have been used widely, especially in the later part of the Viking age. – Other items related to this will also be discussed.