To set an example: Shifts in awareness when working with Venn diagrams

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Working with sets challenges pre-service elementary teachers mathematical thinking. Even though they find Venn diagrams relevant for K-3 students, they struggle with their own mathematical thinking when it comes to the concept of sets. This might have consequences on their future teaching in grades K-3. We have identified what pre-service teachers struggle with Venn diagrams during a workshop on statistics. We discovered that working with properties of objects is not enough to understand set intersection. Particularly, numerical tasks can be helpful in bringing about shifts in levels of awareness. As a result of this insight, a series of tasks are now proposed for a short intervention, in order to further pre-service teachers' understanding of Venn diagrams.

Introduction

As a teacher educator, it is important to support and challenge future teachers' conceptual understanding. Mathematical concepts form a complex network of properties and connections, where some objects (e.g. squares) make up subcategories of several other categories (e.g. rhombi and rectangles). Venn diagrams and basic set theory can serve as a tool for pre-service teachers to grasp this complexity (Bagni, 2006; Moyer & Bolyard, 2003).

Basic set theoretic concepts carry intuitive meanings (Bagni, 2006), and Venn diagrams where the interior of closed plane curves represent sets, are simple and non-controversial, even for children (Freudenthal, 1969). However, previous research is inconclusive regarding the benefit of Venn diagrams. Moyer and Bolyard (2003) present classifications activities that support the development of constructing and validating sound mathematical arguments, while Hodgson (1996) found that Venn diagrams might actually hinder problem solving.

Identification of a problem

As a reoccurring part of the first mathematics course in the primary teacher education at Gothenburg University, two lectures and associated workshops have introduced basic set theoretical concepts and Venn diagrams to the pre-service teacher. The first lecture and workshop focused on primary classroom activities such as classification of objects according to properties in Venn diagrams. In the second lecture and workshop the pre-service teachers' own mathematical thinking was challenged with word problems and numerical examples.

While the pre-service teachers see the possibility of enhancing engagement and constructing rich didactical situations for young children, they also struggle when understanding tasks involving numerical examples beyond properties of objects. While the discovery of objects belonging to several sets seemed unproblematic in the first workshop, the understanding of intersection became an obstacle when working with numerical problems.

We discovered that working with properties of objects is not enough to understand set intersection. In order to create meaningful activities and support their future students' mathematical development, pre-service teachers need to be aware of their awareness when working with problems (awareness-in-action), but also of this awareness in turn (awareness-in-discipline) (Mason, 2011). To facilitate this process, teacher educators need to raise awareness yet another level (awareness-in-counsel). Particularly, numerical tasks can raise the level of awareness to awareness-in-discipline and awareness-in-counsel.

Developing tasks to raise level of awareness

In a forthcoming project, we will develop a series of related word problems and collect data on pre-service teachers' work on these problems. We will address the following research questions:

- 1. How do pre-service teachers shift between levels of awareness when working with a series of tasks using Venn diagrams?
- 2. How do teacher educators shift between levels of awareness when designing, utilizing and evaluating a series of tasks using Venn diagram?

The purpose of this presentation is to put forward this forthcoming project and highlight pre-service teachers' shifts between levels of awareness when working with Venn diagrams.

References

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