The role of mathematical competencies in curriculum documents in different countries

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The inclusion of competencies in curriculum documents can be seen as an international reform movement in mathematics education. The purpose of this study is to understand which role mathematical competencies have in curriculum documents in different countries, with a focus on the relationship between competencies and content. Curriculum documents from 11 different countries were analysed. The results reveal three different themes of variation, concerning if the competencies are specific to mathematics, if competencies are described as learning goals, and if such learning goals are differentiated between grade levels.

Introduction

In this study, curriculum documents from different countries are analysed, focusing on the role of mathematical competencies. The inclusion of competencies in curriculum documents can be seen in the light of an international reform movement in mathematics education (cf. Boesen et al., 2014; Niss et al., 2016). This reform is visible in several countries (as is also evident through the analysis in this paper), where different notions are used for essentially the same thing, such as competencies, proficiencies, abilities, or practices. The notion of competencies is used here unless a direct reference is made to a document where another notion is used.

The reform highlights two aspects of the learning of mathematics: knowing mathematics and doing mathematics (Niss et al., 2016). The first aspect focuses on content, what to know, such as the mathematical notions, concepts, and methods. The second aspect focuses on competencies, how to act, describing the activities of doing mathematics (i.e., the practice of mathematics). Several questions could be asked concerning these two aspects, and some of these are addressed in this paper: What relationship exists between these two aspects (competencies and content)? Are they seen as two independent dimensions of mastering mathematics or is one of them seen as primary in some way?

Varying answers to these types of questions "will give rise to very different kinds of mathematics teaching and learning" (Niss et al., 2016, p. 612), making international comparisons concerning these questions very important in order to

understand the diversity of mathematics education in the world. This paper makes a contribution to this area of research by analysing curriculum documents from many different countries.

The use of competencies in curriculum documents

Two major strands of research can be located that focus on the use of competencies in curriculum documents. First, there is research that focuses on analyses of the message portrayed in curriculum documents. Second, there is research that focuses on different types of potential effects on teaching and learning from such documents.

Research on the content of curriculum documents

Analyses of curriculum documents from several countries highlight issues of clarity in these documents. An analysis of the former Swedish curriculum documents, valid between 1994 and 2011, shows that the reform message in these documents is vague and formulated with complex wording (Bergqvist & Bergqvist, 2016). The intended reform was a focus on competencies (or abilities as labelled in the Swedish context). Similarly, analyses of curriculum documents in USA have shown that "reformers wrote words to convey a new kind of mathematics teaching and learning, yet the meaning those words could convey was imprecise, at best" (Hill, 2001, p. 302).

Furthermore, an analysis of the Australian Curriculum focused on aspects of cohesiveness (Atweh, Miller, & Thornton, 2012). Through their analysis, the authors question "whether only lip service [...] is given to the General Capabilities, Cross-curricular Priorities, and the high order Proficiencies" (Atweh et al., 2012, p. 2). They notice a lack of cohesiveness concerning the role of proficiencies in the curriculum. In particular, they notice that proficiencies sometimes "are not presented as outcomes or aims to be developed and assessed" (Atweh et al., 2012, p. 7) but sometimes they are.

Research on potential effects of curriculum documents

There is also research about potential effects of competency-oriented curriculum documents on teaching. In general, it has been shown that educational reforms often do not give desired effects in schools (Cuban, 2013). Analyses of mathematical competencies, in the context of the NCTM Standards and in the Swedish 1994 curriculum context show similar results: The reform of including and focusing on mathematical competencies as a goal for mathematics education does not seem to have made a clear impact on school practice. For example, studies in relation to the NCTM Standards show that

- there are no clear differences in textbooks between before and after the introduction of the NCTM Standards (Jitendra et al., 2005),

- teaching is still "more like the kind of traditional teaching reported for most of the past century than the kind of teaching promoted in Principles and Standards" (Jacobs et al., 2006), and
- "reformers have not yet succeeded in getting district leaders to grasp […] the meaning of their reform proposals" (Spillane, 2000, p. 169).

In Sweden, a combination of national curriculum documents and national tests are used to convey reform messages. However, the 1994 reform, which for mathematics included a focus on competencies, was not successful, for example since

- textbooks focus on procedural competence (Boesen et al., 2014),
- teacher-made tests do not focus as much on competencies as the national tests (Boesen, 2006), and
- teachers have not modified their teaching in alignment with the reform (Boesen et al., 2014).

In line with these specific empirical results from USA and Sweden, in their survey of situations in several countries, Niss et al. (2016, p. 625) also note that "it has been found to be challenging for teachers to come to grips with notions of mathematical competence/competencies and their relatives and, not the least, with their implementation".

Connecting the content of curriculum documents with their implementation

As noted above, the implementation of the reform to focus on competencies as a central aspect of learning mathematics seems to be a difficult problem. One issue of implementation could be an unclear message in the curriculum documents, as also suggested in empirical studies of curriculum documents (see above). Perhaps the message is unclear because of the complexity of the message itself, about seeing mathematics knowledge also as competencies, and not only as knowledge about a certain content. But the message could also be unclear because of how the message is described in curriculum documents. Therefore, more analysis is needed of curriculum documents regarding this complex message of mathematical competencies. Such analysis is done in this paper.

Purpose

The purpose of this study is to understand the role of mathematical competencies in curriculum documents in different countries, with a focus on the relationship between competencies and content.

The focus on the relationship between competencies and content is chosen since this addresses the core question of how mathematics is characterized in a school context. Thus, this focus makes it possible to contribute with a deeper understanding of different ways to conceptualize the learning of mathematics, when including competencies in curriculum documents.

The purpose is not to compare the different documents to be able to draw conclusions about different countries, since the documents are of different types. For example, some documents are prescriptive, legal documents while other documents are guidelines or resources of some type. Instead, documents from different countries are here used to find different ways of describing the role of mathematical competencies, in particular in relation to content.

Method

The curriculum documents selected for analysis come from The ICMI Database Project[1]. Only documents available in English were included, due to limited proficiency in other languages. The criterion for inclusion of a document was that it should contain *an explicit presentation of a set of competencies*. This criterion is specified through a focus on two aspects of it: the notion of *competencies* and the notion of a *set*.

First, as discussed in the introduction of this paper, competencies refer to an answer to questions around what it takes "to become 'a doer' of mathematics" (Niss et al., 2016, p. 612). That is, competencies describe the activities of doing mathematics (i.e., the practice of mathematics). Thus, competencies are included in a curriculum document when different types of verbs are used, besides forms of "know", to describe what it means to master mathematics. However, it is not enough if a curriculum document uses notions that could be interpreted as competencies (such as "reasoning" or "solving problems"), but there needs to be an explicit presentation of a *set* of competencies. This part of the criterion comes from a result in the survey by Niss et al., (2016, p. 621): "It is further characteristic of any competency construct that it involves a number of distinctions between different instances of the construct or between different sub-constructs or strands".

This study focuses on how the relationship between competencies and content is described in curriculum documents. Therefore, the analysis focused on explicit descriptions about such relationships. In particular, two parts from documents are included in this analysis. First, direct statements about this relationship are included, for example, if competencies are described as the focus for teaching while knowledge of content is described as the goals for learning. Second, how competencies and content are presented structurally in the document is also included in the analysis, for example, if lists of competencies are given together with lists of content strands, but only the content strands are used when describing learning goals. Finally, these statements and structures from all curriculum documents are compared to characterize different ways of describing the role of mathematical competencies, in relation to content. That is, a bottom-up type of analysis is used, to find different themes in how the relationship between competencies and content is described.

Results

Curriculum documents from 11 different countries have been analysed. First, an overarching result is here presented, through three themes of variation that have been located in the analyses of the 11 documents. Thereafter, a short description of each document is given, focusing on explicit statements about the relationship between competencies and content, and on the structural presentation of this relationship. Each document is also characterized in relation to the three themes of variation.

Themes of variation

The analysis of the curriculum documents revealed different ways of describing the relationship between competencies and content. These results are here characterized through three themes of variation. The abbreviations given within each theme are used in the characterization of each document. The A category in each theme refers to a somewhat more focused or elaborated view of mathematical competencies.

First theme: Competencies are described as specific for mathematics (1A), or as more general for all subject areas, and thereby as more separated and independent from content (1B).

Second theme: Competencies are described as learning goals, that is, there is an aim for students to develop the competencies (2A), or competencies are described as teaching methods or classroom activities, that is, as an important way of learning mathematics, but where focus is on content when describing learning goals (2B).

Third theme, which is a sub theme of the 2A: When competencies are described as learning goals, this is done in different ways. In particular, competencies are described as specific learning goals for different grade levels, with variation over grade levels (3A), or they are described more generally as a focus of development over all grade levels, but with no variation when describing each grade level (3B). In the latter case, content is focused on when describing learning goals for each grade level.

Australia[2]

The Australian Curriculum for mathematics includes content strands and proficiency strands (1A). Proficiency strands describe "how content is explored or developed" and "the actions in which students can engage when learning and using the content". That is, proficiencies are described as means in order to learn the content (2B).

For each year level, the Australian Curriculum includes year level descriptions, content descriptions and achievement standards. The year level descriptions give an "overview of relationship between proficiencies and content" and relate explicitly to the proficiency strands but no explicit reference is made to the content strands nor to the content descriptions for the year level in

question. The content descriptions relate explicitly to the content strands but not to the proficiency strands. The achievement standards refer explicitly neither to the content strands nor to the proficiency strands (2B).

Canada – British Columbia[3]

The curriculum document from British Columbia includes curricular competencies, which describe "what students can do with mathematics" (1A). The content, on the other hand, "reflects what students should know". Learning standards are given for each grade level, structured in a table with two columns, one with competencies and one with content. Thus, the competencies are described as central learning goals (2A), on equal terms as content, and where all standards are specific for each grade level (3A).

Czech Republic[4]

The curriculum document in the Czech Republic describes objectives for mathematics, which focus on "the formation and development of key competencies by guiding pupils towards", which is followed by a list of math-specific competencies (1A).

Content is described through different thematic areas, and the subject matter is seen as "a means to master activity-oriented expected outcomes which are gradually combined and create preconditions for an effective and comprehensive use of acquired abilities and skills at the level of key competencies" (2A). The curriculum document describes expected outcomes at two different stages and structured around the thematic areas, where no explicit connections are made to the competencies (3B).

England[5]

The curriculum document in England describes aims for mathematics formulated through competencies (1A & 2A). However, there are no explicit descriptions of relationships between competencies and content. The detailed description of programme of study for each year level is structured around content, with specific goals that are not explicitly connected to the competencies (3B).

New Zealand[6]

There are no math-specific competencies in the curriculum document from New Zealand (1B). However, the document includes key competencies that are common for all subject areas, and they are described as "both end and means" (2A & 2B). The achievement objectives for mathematics are presented in different content strands, without any explicit connections to competencies (3B).

Northern Ireland[7]

For key stages 1 and 2 in the curriculum document in Northern Ireland, competencies are described as processes in mathematics (1A). This section is described on equal terms as other sections that describe content (e.g., number and

measures). In each section, learning goals are described (2A), which vary between stages (3A).

Norway[8]

The Norwegian curriculum document includes competencies (labelled as basic skills) that are for all subject areas, but these are also explicitly interpreted within each subject area (1A). There is a framework describing progression through year levels, using the general basic skills, which serves as "a basis and point of reference for developing subject and grade relevant competence aims" (2A). However, the competence aims in the curriculum are structured only through content areas, and there are no explicit connections to the general basic skills or to the math-specific competencies (3B).

Singapore[9]

The curriculum document from Singapore has mathematical problem solving as central, which is also placed in the centre of a pentagon, with labelled sides: attitudes, metacognition, processes, concepts, and skills. Processes include different mathematical competencies (1A), while concepts are described as "content categories".

Several of the processes are described as means in the learning process, for example, that communication "helps students develop their understanding of mathematics" and "through mathematical modelling, students learn to…" (2B). However, processes are also described as learning goals, for example: "The teaching of process skills should be deliberate and yet integrated with the learning of concepts and skills." (2A)

In one section of the curriculum, a table is presented with the processes and indicators of them. In another section, a much larger table is presented with content, connected to learning experiences, structured around many sub-strands and also around several levels, which is not the case for processes (3B).

South Africa[10]

The South African curriculum document includes "specific skills", which describe mathematical competencies (1A). There is one description of the relation between content and these skills: "Each content area contributes to the acquisition of specific skills", which emphasizes content as means towards learning the competencies (2A).

The competencies are only described through their names, while a table is given over "mathematics content knowledge", which is structured through content areas, general content focus and specific content focus. The curriculum document also includes grade overview tables, which are very extensive, describing aspects of teaching and assessment and showing a progression, which is structured around content (3B).

Sweden[11]

The Swedish curriculum document describes the development of mathematical competencies as the purpose of mathematics (1A & 2A), which is in a part of the curriculum separated from parts describing the content of mathematics. However, there are no explicit statements about the relation between content and competencies. The competencies are used for describing the knowledge requirements, that is, the grading criteria, which also vary for different grade levels (3A).

USA[12]

The Common Core Standards include competencies through standards for mathematical practice (1A). It is stated that students should develop these competencies at all educational levels, that is, they are described as learning goals (2A). The grade level standards, which "define what students should understand and be able to do", are structured around content areas. For each grade level, the mathematical practices are listed, in the same way for all grade levels (3B).

Conclusions and discussion

The analysis of the curriculum documents has revealed different ways of describing the relationship between competencies and content. These results have been characterized through three different themes of variation (see the first part of the results).

Concerning the first theme, most documents describe competencies specific for mathematics (1A), but one document describes only more general competencies (New Zealand). This issue, whether to derive mathematical competencies from more general competencies has been debated in research literature (Niss et al., 2016, p. 621), but the results here show that it is very common to describe competencies specific for mathematics. Concerning the second theme, most documents describe competencies as learning goals (2A), but one document primarily describes competencies as means (Australia). Some documents explicitly describe competencies both as means and ends in the learning process (New Zealand and Singapore). Concerning the third theme, most documents do not describe learning goals for competencies that are specific for different grade levels, but only as a more general focus of development (3B). Three documents specify different learning goals for competencies for different grade levels (Canada – British Columbia, Northern Ireland, and Sweden).

For all the themes of variation, there are also different levels of clarity in describing the role of competencies in relation to content. Issues of clarity have not been focused on specifically in this paper, but in many curriculum documents not much is said at all about the role of competencies in relation to content. Furthermore, even when competencies are explicitly described as learning goals,

which is common in these 11 documents, it is still not common to use competencies when describing specific learning goals, but then content is used as the main building blocks to describe progression in students' learning.

A Danish framework (Niss & Højgaard, 2011), not analysed here, gives a competence description of mathematics education, and describes competencies and content as two separate and independent dimensions of mathematics knowledge. That is, every formulation of a learning goal must include some competency and some content. No such clear description exists in the analysed documents, and the view presented in the Danish framework is not compatible with many of the documents from other countries. This is the case since the documents describe learning goals without the inclusion of competencies, or describe competencies and content at the same level, that is, learning goals are *either* formulated through competencies *or* formulated through content.

Based on these themes of variation, and the varying degree of clarity, it would be of interest to analyse the effects these variations might have on teaching, when relying on different types of descriptions of the relationship between competencies and content. Within the former Swedish national curriculum, there was no clear effects in the classrooms (Boesen et al., 2014) from curriculum documents that were unclear in their reform message (Bergqvist & Bergqvist, 2016). Aspects of unclarity in curriculum documents have been shown also in this study. In particular, unclarity concerns if and how competencies can and should be seen as learning goals, while at the same time most emphasis in the documents is on content when describing specific learning goals. Empirical studies are therefore needed to know how teachers interpret these different types of descriptions of competencies and content, and how teaching is influenced.

Furthermore, by analysing documents from 11 different countries, the results from this study can contribute with valuable contextual, and potentially explanatory, information in other international comparative studies. In particular, the differences seen in this study could relate to differences at a larger (political) context, or could help explain differences in classroom activities in different countries.

Notes

1. http://www.mathunion.org/icmi/activities/database-project/introduction/
2. http://www.australiancurriculum.edu.au
3. https://curriculum.gov.bc.ca/curriculum/mathematics/introduction
4. http://www.msmt.cz/areas-of-work/basic-education-1
5. https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study
6. http://nzcurriculum.tki.org.nz/The-New-Zealand-Curriculum
7. http://ccea.org.uk/curriculum

8. http://www.udir.no/in-english/
9. https://www.moe.gov.sg/education/syllabuses/sciences/
10. http://www.education.gov.za/Curriculum/NationalCurriculumStatementsGradesR-12.aspx
11. http://www.skolverket.se/publikationer?id=2687
12. http://www.corestandards.org/Math/

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