Mathematical knowledge for teaching (MKT)

Do we really know what it is?

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Why should MKT be of interest?

- "Every profession… has its own knowledge base: physicians, lawyers, priests, plumbers, motor mechanics, and so on.
- A moment's thought … exposes the fact that most of us could not do what lawyers, plumbers, and the rest, do".
- The relationship between knowledge of mathematics and effective teaching is not well understood and it can be oversimplified to the point of caricature.
- It is complicated and at times it is counter-intuitive.
- However, there is increasing recognition that effective teaching calls for distinctive forms of subject-related knowledge and thinking.

(Rowland, 2014)

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Issues underpinning MKT

- Interest in MKT draws on concerns about the quality of teachers' mathematical competence. It is often framed against tasks like
  - How do you explain that dividing by a fraction means invert and multiply?
  - Is 0.9 recurring equal to or different from 1?
- Interest in MKT is focused on the processes of teacher education
  - What mathematics should preservice teachers learn
  - What is the relationship between the mathematics they learn, the mathematics they teach and how they teach it?
- Interest in MKT draws on students' performance in international tests
  - What do teachers in country A do that will improve the performance of students in country B?

Do we know MKT when we see it?

- I am going to show a short clip from a Flemish grade 5 class at the start of their first lesson on percentages
- Children have been asked to bring from home households that refer to percentages.
- In this episode I see a teacher who is sensitive to a number of key issues.
  - She understands percentages as tools for managing the real world.
  - She understands that such topics only have meaning in relation to the real world.
  - She understands the need for appropriate contextualised motivation
- How do we capture such simple things?

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A short history of MKT

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Early perspectives on MKT

- Early mathematics teacher education programmes privileged mathematical content knowledge. For example
- The California State Board examination for elementary school teachers in March 1875 comprised ten items on mental arithmetic. One of these was:

  Divide 88 into two such parts that shall be to each other as 2/3 is to 4/5.

- Another question, one of ten written arithmetic problems, was:

  Find the cost of a draft on New York for $1,400 payable sixty days after sight, exchange being worth 102 1/2 percent and interest being reckoned at a rate of 7 percent per annum.

- There were pedagogical questions too…

  How do you interest lazy and careless pupils? Answer in full.

(shulman, 1986)

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Process-product approaches

- In the mid-20th century, 'performance-based' or 'competency based' teacher education models became popular.
- The idea was that concrete, observable behavioural criteria would underpin teacher training.
- Alongside, process-product studies were undertaken to identify those teaching behaviours that correlated with desirable learning outcomes.
- These became the competencies to be acquired by all teachers
- Detailed lists of skills were formulated, leading to a fragmentation in the teacher's role (Korthagen, 2004).

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Why process-product fails: A simple example

- Effective teachers maintain a brisk **lesson pace** and have higher student accuracy (Englert, 1984).
- Teachers with presence (nonverbal behaviour, **lesson pace,** and voice quality) are visually and auditorally dynamic (Ishler et al., 1988).
- In England, 'brisk **pace**' is accepted as an aspect of good teaching in official prescriptions and teacher discourse (Lefstein & Snell, 2013).
- However, "accelerating pupils' experience… necessitates slowing down the **pace** of teaching, and that government calls for urgency may, perversely, make lessons slower (Lefstein & Snell, 2013).
- Teaching associated with a faster **lesson pace** prompts superficial collaboration and participation at the expense of co−constructing, assessing and extending knowledge (Hennessy et al., 2007)

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A response to process-product

- In the 1970s, Humanistic Based Teacher Education (HBTE) became popular
- HBTE focused attention on the *person of the teacher*.
- It emphasised a *confluent education*, in which *thinking* and *feeling* flow together.
- It stressed the *unicity* and *dignity* of the individual.
- Thus, a central role was reserved for personal growth.
- Such goals remain incompatible with the imposition of standardised teaching competencies.
- So, where does this leave us today?

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The shaping of a new field

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Shulman's innovation

- Process-product and HBTE approaches to teacher education neglect the role of the subject, prompting a number of important questions
- What does teacher knowledge come from?
  - What do teachers know,
  - When did they come to know it
  - How did they come to know it?
- How do teachers decide what to teach?
- How do teachers decide how to represent what they teach?
- How do teachers manage students who do not understand?
- What is teacher explanation?
- Where do such explanations come from?

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Shulman's forms of teacher knowledge

- *Content knowledge*: The amount and organization of subject-related knowledge
- *General pedagogical knowledge*: Broad strategies of classroom management that transcend subject matter
- *Curriculum knowledge*: Awareness of the available materials and programmes, including
  - *lateral curriculum knowledge*: taught simultaneously in different subjects
  - *vertical curriculum knowledge*: taught in years before and after
- *Pedagogical content knowledge*: How to make a subject comprehensible to learners.
- *Knowledge of learners and their characteristics*
- *Knowledge of educational contexts*: Classroom, school, community…
- *Knowledge of educational ends, purposes and values*

(Shulman, 1986, 1987)

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The lost elements of Shulman 1

- Interest in Shulman's PCK (over 8000 citations) has masked three other forms of teacher knowledge that help organise the other.
- These are Propositional knowledge, Case knowledge and Strategic knowledge
- **Propositional knowledge** comprises statements about what is 'known' about teaching and learning. For example
- "When we examine the research on teaching and learning and explore its implications for practice, we are typically (and properly) examining propositions"
- He adds that there are three forms of propositional knowledge in teaching
  - Disciplined empirical or philosophical inquiry;
  - Practical experience;
  - Moral or ethical reasoning.

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The lost elements of Shulman 2

- **Case knowledge** draws on the use of case literature to "illuminate both the practical and the theoretical".
- It draws on the legal traditions in many countries whereby cases provide resources for teaching and theorising
- While cases may be "detailed descriptions of how an instructional event occurred", they should typically "be exemplars of principles, exemplifying in their detail a more abstract proposition or theoretical claim".
- He argues that there are three forms of case knowledge in teaching
  - Prototypes that exemplify theoretical principles,
  - Precedents that capture and communicate principles of practice
  - Parables that convey norms or values.

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The lost elements of Shulman 3

- Strategic knowledge refers to the exercise of propositional and case knowledge in principled action.
- It comes into play as the teacher confronts particular situations or problems, whether theoretical, practical, or moral, where principles collide and no simple solution is possible.
- Strategic knowledge is developed when the lessons of single principles contradict one another, or the precedents of particular cases are incompatible.
- For example, it is known from research on wait-time, that teachers who wait longer after posing a question encourage higher levels of cognitive processing

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Responses to Shulman

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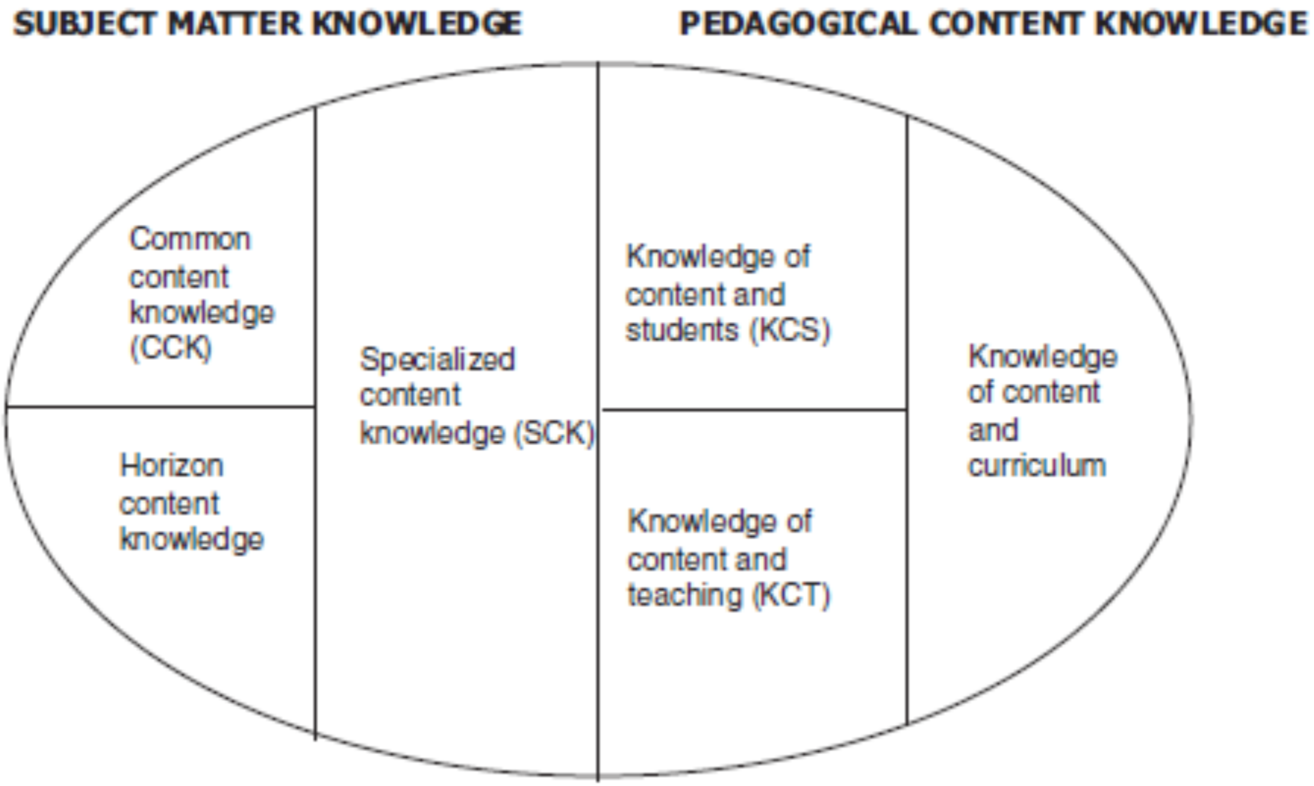
Ball, Hill and colleagues' adaptations



(Ball et al., 2008)

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In particular

- *Common content knowledge (CCK)* is the mathematical knowledge and skill used in settings other than teaching: recognising an error is CCK
- *Specialized content knowledge (SCK)* is the mathematical knowledge and skill unique to teaching: understanding an error is SCK
- *Horizon content knowledge (HCK)* looks forward to how mathematical topics are related to other topics taught later in the curriculum.
- *Knowledge of content and students (KCS)* combines knowing about students and knowing about mathematics. Teachers need to anticipate what students are likely to think and what they will find confusing.
- *Knowledge of content and teaching (KCT)* combines knowing about teaching and knowing about mathematics. Many of the mathematical tasks of teaching require a mathematical knowledge of the design of instruction
- *Knowledge of content and curriculum* is as construed by Shulman

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Operationalising the constructs

- Three constructs, *Common content knowledge (CCK), specialized content knowledge (SCK)* and *knowledge of content and students (KCS)* have been operationalised as part of a multiple choice instrument. For example

  Ms. Chambreaux's students are working on the following problem:

  Is 371 a prime number?

  As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

  a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

  b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

  c) Check to see whether 371 is divisible by any prime number less than 20.

  d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

- Their factor analysis yielded results largely commensurate with their hypothesised model, although many subscales had disappointing reliabilities.

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Other studies

- Hill et al. (2005) used a similar version of the same materials (CKT-M) to explore the relationship between primary teachers' CKT-M scores and children's achievement.
- They found that a one standard deviation increase in teacher performance yielded a 1/10 standard deviation increase in student scores over a year.

- Delaney (2007) adapted the MKT measures for use in Ireland.
- Items not appropriate to the Irish curriculum were removed
- Those that remained were educationally and linguistically similar and only minor adaptations were made – cookie became biscuit, for example
- He showed that the difficulty of the US items correlated closely with that of the adapted items in Ireland.

Mathematical quality of instruction (MQI)

- Drawing on a process-product and teacher deficit perspective, six dimensions were operationalised for an observational framework
- *Mathematics errors*: The presence of computational, linguistic, representational, or other mathematical errors in instruction
- *Responding to students inappropriately*: Teachers either misinterpret or, in the case of student misunderstanding, fail to respond to student utterances
- *Connecting classroom practice to mathematics*: Teachers connect classroom practice to worthwhile mathematics rather than activities that do not require mathematical thinking
- *Richness of the mathematics*: Teachers use and connect multiple representations, encourage mathematical explanation, justification, proof and reasoning
- *Responding to students appropriately*: Teachers correctly interpret students' mathematical utterances and address student misunderstandings
- *Mathematical language*: Teaches exploit accurate mathematical language in instruction.

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Coding for MQI

- Lessons were split into 5-minute segments.
- Each segment was coded for each 33 codes designed to represent MQI
- Instructional format and content: 3 codes
- Teacher's mathematical knowledge: 12 codes
- Teacher's use of mathematics with students 8 codes
- Teacher equity: 10 codes
- Each lesson was then allocated a score of *low* (1), *medium* (2), or *high* (3).
- These were then summed to give an overall score for each teacher across nine lessons each.

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Results: Correlating MQI with MKT

Scale	*Correlation to Measure Scores (Spearman's rho)*
Connecting classroom activities to mathematics	–0.49
Responding to students appropriately	0.65 (*)
Responding to students inappropriately	–0.41
Mathematical language	0.30
Errors Total	–0.83 (**)
Error—language subscale	–0.80 (**)
Richness of the mathematics	0.53

*Significant at the 0.05 level.
**Significant at the 0.01 level.

During the analysis, mathematical errors split to create seven dimensions

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Further perspectives on Ball and Hill

- It is well-known that mathematics learning draws on two forms of knowledge: procedural knowledge and connected conceptual knowledge
- Fauskanger (2015) exploited *available* MKT measures to address procedural knowledge and several open tasks to assess conceptual knowledge.
- He found that the multiple-choice MKT responses gave no indication of teachers' open response – the MKT items were not predictive.
- Schoenfeld (2007) has criticised the format of the MKT items and argued the need for further research to determine the extent to which they reflect the desired *competencies*.

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Looking at MKT through other lense: Some confounding factors

Ball, Hill and teacher expertise

- Berliner (2001) has synthesised the characteristics of an expert teacher.
- Teacher expertise is specific to a domain and developed over hundreds and thousands of hours;
- It continues to develop but is not necessarily linear;
- Experts have a better structured knowledge than novices;
- Experts represent problems in qualitatively different ways from novices, their representations are deeper and richer;
- Experts recognize meaningful patterns faster than novices; see 1, 2, 4, 8, 16
- Experts are more flexible and more opportunistic planners;
- Experts can change representations quickly when it is necessary;

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More notes on expertise

- Experts impose meaning on ambiguous stimuli. They are much more "top down processors." Novices are misled by ambiguity and are more likely to be "bottom up" processors;
- Experts may start to solve a problem slower than a novice, but overall they are faster problem solvers;
- Experts are usually more constrained by task requirements and the social constraints of a situation than are novices;
- Experts develop automaticity in their behaviour to allow conscious processing of more complex information;
- Experts have developed self-regulatory processes as they engage in their activities.
- Against Berliner's perspective on expertise, and it seems fairly compelling, I struggle to see how we can operationalise measures of teacher knowledge

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Mathematics teaching cross-culturally

- Mathematics teaching varies considerably from one cultural context to another.
- If you compare culturally western (Socratic) teaching with culturally eastern (Confucian), as has Jinfa Cai for example, these difference are vast
- But even within world regions, there are major differences in, for example:
  - Curricular emphases
  - Textbooks
  - Teaching strategies
  - Lesson structures
  - Learning objectives
  - Time wasted – sanctioned and unsanctioned

Looking at curricula: linear equations

- Finland.

  By the end of grade 8, students in grades grades 6-9, *will know how to… solve a first degree equation*.

- Flanders.

  Students in the first grade of secondary education will *solve equations of the first degree with one unknown and simple problems which can be converted to such an equations*.

  Students in the second grade of secondary education will *solve equations of the first and second degree in one unknown and problems which can be converted into such equations*.

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- Hungary

In year 5 students should *solve simple equations of the first degree by deduction, breaking down, checking by substitution along with simple problems expressed verbally.*

In year 6 they should *solve simple equations of the first degree and one variable with freely selected method.*

In year 7 they should *solve simple equations of the first degree by deduction and the balance principle. Interpret texts and solve verbally expressed problems. Solve equations of the first degree and one variable by the graphical method.*

In year 8 students should *solve deductively equations of the first degree in relation to the base set and solution set. Analyse texts and translate them into the language of mathematics. Solve verbally expressed mathematical problems.*

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Learning outcomes

	Flanders	England	Hungary	Spain	All
Conceptual knowledge	71.2	78.6	64.1	77.3	73
Derived knowledge	4.5	1.0	6.4	2.7	4
Structural knowledge	17.1	1.0	39.7	14.7	17
Procedural knowledge	56.8	53.4	51.3	68.0	57
Mathematical efficiency	12.6	9.7	35.9	14.7	17
Problem-solving	7.2	20.4	30.8	38.7	22
Reasoning	35.1	30.1	44.9	25.3	34
Total episodes	111	103	78	75	367
Total lessons	20	15	18	16	69
Total didactic codes	299	237	320	264	1120
Codes per episode	2.7	2.3	4.1	3.5	3.05

Percentage of episodes per country in which each outcome was observed (Andrews, 2009a)

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Didactical strategies

	Flanders	England	Hungary	Spain	All
	%	%	%	%	%
Activating	23.4	11.7	34.6	13.3	20.4
Exercising	2.7	7.8	5.1	.0	4.1
Explaining	52.3	52.4	59.0	64.0	56.1
Sharing	61.3	60.2	97.4	61.3	68.7
Exploring	6.3	3.9	.0	5.3	4.1
Coaching	38.7	54.4	44.9	76.0	52.0
Assessing	19.8	13.6	35.9	1.3	17.7
Motivating	9.9	12.6	46.2	56.0	27.8
Questioning	48.6	5.8	87.2	70.7	49.3
Differentiating	6.3	7.8	.0	4.0	4.9

Percentage of episodes per country in which each strategy was observed (Andrews, 2009b)

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Even things as simple as lesson structures

	Flanders	England	Hungary	Spain
Total lessons	20	15	18	16
Mean lesson length	50.1	53.1	45.9	58.2
Total episodes	111	103	78	75
Total didactic codes	299	237	320	264
Codes per episode	2.7[1]	2.3[2]	4.1[3]	3.5[4]
Episodes per lesson	5.6	6.9[5]	4.3[6]	4.7
Mean episode duration	8.9	7.7[7]	10.4	12.5[8]
Standard deviation	6.5	5.0	6.3	8.0

Such matters lead us to a new section

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Relating MKT to what students are expected to learn

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The strands of mathematical proficiency

- **Conceptual understanding**: students need to develop an interconnected understanding of mathematical concepts, operations, and relations;
- **Procedural fluency**: students need to develop an automated, flexible and efficient set of mathematical procedures;
- **Strategic competence**: students need to be able to formulate, represent and solve mathematical problems;
- **Adaptive reasoning**: students need to develop the capacity for logical thought and mathematical argumentation;
- **Productive disposition**: students need to see mathematics as a sensible, useful, and worthwhile subject to be learned, coupled with a belief in the value of diligent work and in one's own efficacy as a solver of problems.

(Kilpatrick et al., 2001)

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What is required of a problem solver?

- **Appropriate knowledge**: You cannot solve problems if you don't know mathematics and its syntax.
- **A set of problem-solving strategies or heuristics**: You cannot solve problems if you have no strategies for doing so.
- **A meta-cognitive competence**: You cannot solve problems if you cannot monitor what you do and why you do it
- **A belief that the problem is worth solving**: You cannot solve problems if you do not believe they are worth solving

(Andrews and Xenofontos, 2015)

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What is required of teachers

- **Teachers encourage problematising**: In problem solving classrooms students actively engage with intellectually challenging problems
- **Teachers grant students the authority to work on such problems**: In problem solving classrooms students develop agency and authority, and reflect upon their solution strategies.
- **Teachers devolve accountability**: In problem solving classrooms students expose their work to the scrutiny of others and the disciplinary norms of mathematics
- **Teachers provide appropriate resources**: In problem solving classrooms teachers provide flexible instructional support to support the above and the development of positive beliefs.

(Andrews and Xenofontos, 2015)

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Alternatives to MKT?

- In the light of such complex learning objectives – objectives internationally recognised – it is difficult to understand how one might construct a measure of MKT.
- This leads me pose a different question
- Is a focus on measuring MKT misplaced?
- Should our attention be focused on identifying the best people to become teachers and then supporting them in their becoming teachers with an MKT appropriate for their professional work?
- We have been trialling a short questionnaire with students in Stockholm. It is completed during the first week of their course
- Factor analyses have yielded seven constructs several of which combine in ways that indicate which students should probably not be on a teacher education programme

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Summarising so far

- Shulman’s very useful introductory work has limitations with respect to uncovering the nature of MKT.
- Attempts to quantify MKT, in its various forms, represent a return to the process-product studies of the mid 1900s.
- However, framing MKT in, say, the manner of Ball and Hill, may be helpful: but horizon curriculum knowledge, for example, is not absolute but relative.
- Quantitative MKT studies may have had limited success in particular cultural contexts, but they fail to understand that teachers teach according to deep-seated traditions largely hidden from them – genomgång is a fine example.
- Teachers may experience changes in curricula, they may have their views influenced by particular personal events, but by and large they change little. In sum,
- Teachers work within intended , received and idealised curricula (Andrews, 2011)

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Looking afresh at the problem

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The knowledge quartet (484 hits)

- Foundation: Teachers' mathematics-related knowledge, beliefs and understanding, incorporating Shulman's classic taxonomy of kinds of knowledge without undue concern for the boundaries between them.
- Transformation: Knowledge-in-action as demonstrated both in planning to teach and in the act of teaching itself.
- Connection: Ways that teachers achieve coherence within and between lessons: it includes the sequencing of material for instruction and an awareness of the relative cognitive demands of different topics and tasks.
- Contingency: Witnessed in classroom events that were not planned for. In commonplace language, it is the ability to 'think on one's feet'.
- The whole framework comprises 21 codes

(Rowland, 2012; Rowland et al., 2005)

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More on the KQ

- The framework has been used to
  - analyse qualitatively how primary pre-service teachers (Rowland et al., 2005) and secondary pre-service teachers (Thwaites et al., 2011) enact mathematical knowledge for teaching;
  - compare its efficacy with both primary and secondary teacher education students (Rowland, 2012);
  - examine how secondary teachers enact contingency in their lessons (Rowland and Zazkis, 2013);
  - observe primary teacher education students (Rowland & Turner).
- Since 2011, responding to criticisms that the KQ framework is interpretively vague, an international team has been working on the production of a coding schedule appropriate for cross-cultural work (Weston et al., 2012).
- This development takes me full circle and back, possibly, to process-product

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Conclusion

- However we define, operationalise and improve teachers' MKT, and it remains a hugely complex task, two facts remain:
- "Consciously, we teach what we know; unconsciously, we teach who we are" (Hamachek (1999, p. 209).
- "students' learning of mathematics is not independent of the cultures, and therefore the curricula, in which they are raised; the nature and ambitions of the schools they attend; the experience, competence and expectations of their teachers; the aspirations espoused at home; and their own and their friends' goals and inclinations" (Andrews et al., 2014, p.8)
- Understanding MKT is only a part of the bigger picture.

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Tack för att ni lyssnade

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