Ordinary Senior Secondary

# Mathematics Curriculum Standards

(Experimental Version)

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# Section One – Forward

Mathematics is a science that seeks to study space formats and quantitative relationships. It is a scientific language and an effective tool to depict natural and societal regularities. Mathematical science forms the foundation of natural sciences, technological sciences, etc. It is increasingly exerting its functions in the development of economical sciences, social sciences and human sciences. Mathematical applications are increasingly widespread, and begin to diffuse continuously into every aspects of societal living. When mathematics is integrated with computer sciences, worth is created for the well-being of the many aspects of the society, and development is propelled for the productivity of the society. Mathematics is unique and performs an irreplaceable role in the formation of human rational reasoning, and fosters development of human intelligences. Mathematics is an important constituent of human culture, and mathematical literacy is a basic quality that must be possessed by all citizens.

As mathematics education is part of education in general, it plays a vital role in the activities of developing and perfecting the education of mankind, in the many facets of forming attitudes and thinking methods for inquiring the human world, and in the processes of propelling society's growth and development. In a modern society, mathematics education is an important part of life-long education. It builds up the foundation for the citizens to further themselves, and is needed for one's life-long development. Mathematics education occupies a special position in school education. It enables students to grasp fundamental knowledge, basic skills, and basic ideas in mathematics, as well as develops students to express clearly and think systematically, so as to enable students to possess realistic and practical attitudes, and spirits of perseverance. Students learn how to deploy mathematics thinking methods to solve problems and know the world they live in.

## 1. Properties of the Curriculum

Senior secondary mathematics curriculum is one main curriculum in the post-obligatory education stage of common senior secondary schooling. It comprises the most basic contents in mathematics, and is a fundamental curriculum for the development of citizen's qualities.

Senior secondary mathematics curriculum allows students to know the relationships of mathematics and nature, as well as mathematics and human society, know the scientific and cultural values of mathematics, and enhance abilities of posing problems, analyzing and solving problems. It is instrumental as it lays the foundation for forming scientific reasoning, developing intelligences and innovation consciousness.

Senior secondary mathematics curriculum can help students know the worth of mathematical applications, enhance their application consciousness, form abilities in solving simple practical problems.

Senior secondary mathematics curriculum is fundamental to the studies of senior secondary physics, chemistry, technology, etc., and their further studies. At the same time, because of students' life-long developments, it cultivates world-views and values of science and as such has important meanings in raising the qualities of all citizens.

## 2. Basic Rationale of the Curriculum

**1. Construct a common foundation, provide a platform for development.**

Senior secondary education belongs to basic education. Senior secondary mathematics curriculum should exhibit fundamental characteristics, comprising two aspects of meanings: First, after the stage of obligatory education, there is a need to provide students a higher level of mathematical foundation to adapt to modern everyday living and future development, so as to allow them to acquire higher levels of mathematical literacy. Second, there is a need to prepare students to further their studies in mathematics. The senior mathematics curriculum consists of the compulsory series and the optional series. Compulsory series are meant to satisfy the common mathematical needs of all students, whereas the optional series are for the individual needs. It is noteworthy that the optional series are still fundamental to the needs of students' development.

**2. Provide multifarious curricula, and adapt choices to the needs of individuals.**

Senior mathematics curriculum should demonstrate multifarious and selective characteristics so that different students can have different developments in mathematics.

Senior mathematics curriculum should provide students room for selection and development. There should be a number of levels and type of choices so as to foster students' individual development and thinking pertaining to their future career planning. Under the guidance of teachers, students can engage in autonomous selection of courses, and if necessary can undertake appropriate conversion and modification. At the same time, senior mathematics curriculum should reserve some room for the teachers and schools to devise curriculum development plans to enrich and perfect continuously the options provided to the students in accordance with the basic needs and individual conditions of the students.

**3. Propose learning formats that are constructive, autonomous, and eager to explore.**

Students' mathematical learning activities should not be constrained to reception, memorization, imitation and exercise. Instead, senior mathematics curriculum should promote ways of mathematics learning such as autonomous exploration, hands-on practical, cooperative exchange, reading and self-learning. These modes of learning can promote students' initiatives in learning so that under the guidance of the teachers the learning processes become one that are "re-create" in nature. At the same time, senior mathematics curriculum should set up learning activities involving mathematical explorations and mathematical modeling. This would not only create facilitative conditions for students to engage in constructive, self-initiated, and multifarious modes of learning, but also stimulate students' interests in mathematics learning, and encourage students during the learning processes to develop habits of independent thinking and constructive explorations. Through multiple formats of autonomous learning and exploratory activities, senior mathematics curriculum should strive to enable students to experience the processes of mathematical discoveries and creation, and to develop their innovation consciousness.

**4. Pay attention to elevate students' abilities in mathematical thinking.**

Senior mathematics curriculum should pay attention to elevate students' mathematical cognitive abilities, and this is a basic objective of mathematics learning. When we learn mathematics and use mathematics to solve problems, we are continuously involving in intuition and perception, observation and discovery, induction and analogy, spatial imagination, abstraction and generalization, symbol representation, computation and evaluation, data processing, deductive reasoning and proving, reflection and processes involving cognitive constructions. These processes are concrete demonstrations of one's mathematical cognitive abilities, and are instrumental to help students engage in thinking and making judgment of mathematical patterns implicit in objective objects and events. Mathematical cognitive abilities demonstrate unique functions in the formation of objective reasoning.

**5. Develop students' mathematical application awareness.**

Since the commencement of the second half of the 20th century, immense development in mathematical applications is one prominent characteristic of development in mathematics. During the knowledge society era, mathematics moves from the backstage to the forefront. Integration of mathematics and information technology enables mathematics generating worth for the society in many aspects. At the same time, the prospect of mathematical development is widely extended. For a long period of time mathematics education in China has not been sufficiently emphasized, particularly in relating mathematics with practical reality and in connecting mathematics with other subject areas. Therefore, there is a need to enhance greatly mathematical applications and relate mathematics to reality in senior mathematics. Recently, practices of mathematical modeling at the secondary and university levels in China reveal that promotion of instructional activities of mathematical applications fulfils the needs of the society, facilitates stimulation of learning interests in mathematics, enhances students' application awareness, and extends perspectives of the students.

Senior mathematics curriculum should provide practical backgrounds of basic contents to reflect the worth of mathematical applications, conduct learning activities of mathematical modeling, and set up some courses of special topics that exhibit some important mathematical applications. Senior mathematics curriculum should strive to enable students to experience the functions of mathematics in solving practical problems, and the connections of mathematics with everyday living and other subject areas, as well as to enhance students to form and develop application awareness progressively in mathematics, and practical abilities as well.

**6. Keep in pace with society's progress to know the "Double Basics".**

In China, there is a tradition of mathematics education paying particular attention to the teaching of fundamental knowledge, training of basic skills, and development of abilities. Senior mathematics curriculum in the new century should propagate this tradition. At the same time, due to advances of the era, in particular due to the widespread applications of mathematics and development of computer technology and modern information technology, there is a need to form new "Double Basics" that fulfils the needs of the era. For example, due to the needs to adapt to the development of the information era, senior mathematics curriculum should add contents of algorithms, so that the most basic information processing and statistical knowledge can be treated as new fundamental knowledge and basic skills in mathematics. At the same time, there is a need to cut and diminish complicated computations, artificial tactics-oriented difficult problems, and contents excessively emphasizing intricate details so as to conquer tendencies of phenomenon of "disorganization and disorientation of the double basics".

**7. Emphasize nature of mathematics and pay appropriate attention to formalization.**

Formalization is a basic characteristic of mathematics. During teaching of mathematics, learning how to express formalization is a basic requirement. However, expression of formalization should not be solely emphasized, and there is a need to pay attention to the nature of mathematics as well. Otherwise, animating and lively mathematical cognitive activities would be drowned in the sea of formalization. Modern development in mathematics makes clear that entire formalization is impossible. Therefore, senior mathematics curriculum should revert back to simplicity and truth, striving hard to reveal developmental processes and nature of concepts, rules, and conclusions in mathematics. Mathematics curriculum should include logical deduction, and emphasize reasoning as well. Through analyses of typical examples and students' autonomous exploratory activities, students are enabled to comprehend mathematical concepts and processes so that conclusions are drawn progressively, to realize the methods of thinking implicit in these concepts and processes, to quest for developmental trajectory of history of mathematics, as well as to transform mathematics from the academic to the more acceptable educational form.

**8. Exhibit the cultural values of mathematics.**

Mathematics is an important constituent of human culture. Mathematics curriculum should reflect appropriately the history, application and developmental tendency of mathematics. In addition, it should reflect the function of mathematics in propelling social development and the social needs of mathematics. Apart from these, propelling function of social developments on mathematics developments, thinking system of the social sciences, the aesthetic value of mathematics, and the innovation spirits of the mathematicians are included as well. Mathematics curriculum should help students familiarize with the function of mathematics in the development of human civilization and the progressive formation of correct mathematical perspective. Because of these, senior mathematics curriculum should promote exhibition of cultural values of mathematics, propose appropriate contents like "mathematical culture" as learning requirements. There are needs to set up courses such as "selected topics in the history of mathematics”.

**9. Pay attention to the integration of information technology and mathematics curriculum.**

Widespread applications of modern information technology are now exerting deep influences on contents of the mathematics curriculum, mathematics teaching, and mathematics learning. Senior mathematics curriculum should promote realization of organization of information technology and curriculum contents (e.g. algorithms are integrated with related parts in the mathematics curriculum). The basic rule of integration is to facilitate students to know the nature of mathematics. Senior mathematics curriculum should promote the use of information technology to display those contents that in the past may not be presented easily. After guaranteeing the prerequisite of training of hand calculations, students should strive to use scientific calculators, all sorts of mathematics education educational platforms, so as to enhance integration of mathematics teaching and information technology, encourage students to deploy computers and calculators to carry out explorations and discoveries.

**10. Establish reasonable, scientific evaluation system.**

Demand of human developments by the modern society has caused deep changes to the evaluation system. Senior mathematics curriculum should establish an appropriate and scientific evaluation system, including aspects such as concepts, contents, formats and systems of evaluation. Evaluation should not only pay attention to students’ mathematics learning outcomes, but also to mathematics learning processes; not only pay attention to the levels of mathematics learning of the students, but also to the changes of affection and attitudes exhibited in mathematical activities. In mathematics education, evaluation should establish a variety of objectives, and pay attention to the development of students’ personality and potentials. For example, process evaluation should pay attention to the evaluation of students’ understanding of mathematics concepts and mathematics thinking processes. Teachers should pay attention to the evaluation of processes involving how students pose, analyze and solve problems mathematically, as well as to the processes exhibiting attitudes of cooperation with others, consciousness of expression and exchanges, and spirits of exploration. Regarding learning activities such as mathematical exploration and mathematical modeling, teachers should develop corresponding contents and methods of evaluation.

## 3. Design Considerations of the Curriculum

Senior mathematics curriculum seeks to integrate organically basic rationale of educational reform, design of the curriculum framework, determination of contents, as well as curriculum implementation.

### (1) Senior Secondary Mathematics Curriculum Framework

**i. Curriculum Framework**

Senior mathematics curriculum comprises of compulsory and optional studies. Compulsory curriculum consists of 5 modules, whereas there are 4 series in the optional curriculum. Series 1 and 2 are composed of several modules, and series 3 and 4 several special topics. Each module carries 2 credits (36 hours of instruction), whereas each special topic carries 1 credit (18 hours of instruction). As such, two special topics are equivalent to one module. Curriculum framework is shown in the diagram below:

<insert diagram here>

Optional Study Series; Compulsory Modules; Optional Study; Mathematics

Note: the in the diagram above represent a module (36 hours of instruction, represents special topics (18 hours of instruction)

**ii. Compulsory Curriculum**

Every student is required to study the mathematics contents of the compulsory curriculum, which comprises of 5 modules:

Mathematics 1: Set, concept of function, and basic elementary function I (exponential function, logarithmic function, power function).

Mathematics 2: Preliminary solid geometry, preliminary plane analytic geometry.

Mathematics 3: Preliminary algorithms, statistics, probability.

Mathematics 4: Basic elementary function II (trigonometric function), vectors on a plane, trigonometric identity transformation.

Mathematics 5: Solution of a triangle, sequence, inequality.

**iii. Optional Curriculum**

Concerning the optional curriculum, students can select in accordance with their interests and aspirations for future development. The optional curriculum comprises of series 1, series 2, series 3 and series 4.

* Series 1: Consists of 2 modules.

Optional Study 1-1: Common logic terminology, conic section and equation, derivative and its application.

Optional Study 1-2: Case studies of statistics, inference and proof, extension of number system and introduction of complex number, block diagram.

*Series 2: Consists of 3 modules.

Optional Study 2-1: Common logic terminology, conic section and equation, vectors in space and solid geometry.

Optional Study 2-2: Derivative and its application, inference and proof, extension of number system and introduction of complex number.

Optional Study 2-3: Principle of enumeration, case studies of statistics, probability.

*Series 3: Consists of 6 special topics.

Optional Study 3-1: Selected topics of history of mathematics.

Optional Study 3-2: Information security and cryptogram.

Optional Study 3-3: The geometry of the sphere.

Optional Study 3-4: Symmetry and group.

Optional Study 3-5: Euler's formula and classification of closed surfaces.

Optional Study 3-6: Trisection of an angle and extension of a number field.

*Series 4: Consists of 10 special topics.

Optional Study 4-1: Selected topics of geometrical proofs.

Optional Study 4-2: Matrix and transformation.

Optional Study 4-3: Sequence and difference.

Optional Study 4-4: Coordinates system and parametric equations.

Optional Study 4-5: Selected topics of inequalities.

Optional Study 4-6: Elementary number theory.

Optional Study 4-7: Optimum seeking method and preliminary experimental design.

Optional Study 4-8: Overall planning (critical path method) and preliminary graph theory.

Optional Study 4-9: Risk and decision making.

Optional Study 4-10: Switching circuits and Boolean algebra.

**iv. Remarks concerning setting up of the curriculum**

*** Principle and intention of setting up the curriculum.**

Principle for ascertaining contents of the compulsory curriculum: Satisfy basic mathematical needs of future citizens, as well as equip and prepare students with mathematics that are necessary to further their studies.

Principle for ascertaining contents of the optional curriculum: Satisfy students' interests and needs for their future development, as well as build up the foundation for students' further studies and acquisition of higher mathematical literacy.

In particular, series 1 is meant for those students who wish to further themselves in the humanities and social sciences, and series 2 is set up for those who wish to develop themselves in science and technology, and economics. Contents of both series 1 and series 2 are fundamental contents of the optional curriculum.

Series 3 and series 4 are meant for those students who are interested in and wish to elevate their levels of mathematical literacy. Contents involved reflect some important mathematical thinking that are helpful to build up further students' foundation, increase their application awareness, are beneficial to students' life-long development and extension of mathematical perspectives, as well as good for increasing students' recognition of scientific values, application values and cultural values of mathematics. In particular, scope of the special topics will be broadened as the curriculum develops. Students can select topics in accordance with their own interests and aspirations. According to the characteristics of the contents of series 3, it will not become contents of selective tertiary entrance examinations. Evaluation of learning of this part of contents should adopt an integration of quantitative and qualitative methods. It should be undertaken by the schools and the results accrued may be used as a reference for admission into institutes of higher learning.

*** Setting up contents of mathematical exploration, mathematical modeling, and mathematics culture.**

Senior mathematics curriculum requires the diffusion of different forms of ideas of mathematical exploration and mathematical modeling into each of the modules and special topics. At the senior secondary stage of schooling, teachers should arrange at least one relatively comprehensive mathematical exploration and mathematical modeling activities. Senior mathematics curriculum demands an organic integration of contents of mathematics culture with contents of each of the modules. Specific requirements may refer to those of mathematical exploration and mathematical modeling (see p.???).

*** Logical sequence of the modules.**

Compulsory curriculum is the foundation of the series 1 and series 2 of the optional curriculum. Basically, series 3 and series 4 of the optional curriculum do not rely on other curriculum series, and therefore can be implemented together with them. Implementation of these special topics may not consider any order. In the compulsory curriculum, Mathematics 1 is the foundation of mathematics 2, mathematics 3, mathematics 4 and mathematics 5.

*** Implementation of Series 3 and Series 4 curriculum.**

After securing the foundation in the implementation of the compulsory curriculum, and the optional study series 1 and series 2, schools may implement some special topics of the series 3 and series 4 in accordance with individual school's situations so as to satisfy the basic selection needs of the students. According to one's situations, schools should seek to enrich and perfect progressively, as well as exploit and deploy constructively curriculum resources from outside (including distance learning resources). Concerning implementation of the curriculum, teachers ought to devise developmental plans according to their individual conditions.

### (2) Course Selection Recommendations for Students

Students' interests, aspirations and their respective conditions are all different. Different institutes of higher learning and different areas of specialization have different mathematics requirements on the students. In some cases, specializations of the same area can have different mathematics requirements on the students. Due to the advances of the era, whether in the natural sciences and areas in the science and technology, or in human sciences and areas in the social sciences, we need some students who are equipped with higher mathematics literacy. This serves important functions for the development of the society, as well as science and technology. Based on this, students can select different curriculum combinations. After selection, they are allowed to undertake appropriate adjustments according to their individual conditions. Listed below are several basic recommendations of curriculum combination provided to students as references:

1. Students complete 10 credits of the compulsory curriculum to meet the mathematics requirements for senior secondary school graduation.

2. After the completion of 10 compulsory credits, those students who would like to specialize in the human and social sciences can have two choices. The first choice is to gain 4 credits by selecting optional study 1-1 and optional study 1-2 of series 1, gain 2 credits by selecting 2 special topics from series 3, accumulating a total of 16 credits. The second choice is for those students who are interested in mathematics and aspire to acquire a higher level of mathematics literacy. Apart from the 16 credits mentioned in the first choice, students can gain 4 more credits from series 4, accumulating a total of 20 credits.

3. Those students who would like to specialize in science and technology (including some economic areas), after completing the 10 credits of the compulsory curriculum, gain 2 credits by selecting 2 special topics from series 3, gain 2 credits by selecting 2 special topics from series 4, accumulating a total of 20 credits. The second choice is for those students who are interested in mathematics and aspire to acquire a higher level of mathematics literacy. Apart from the 20 credits mentioned in the first choice, students can gain 4 more credits from series 4, accumulating a total of 24 credits.

As far as curriculum combinations are concerned, there is a certain degree of flexibility so that students can convert from one combination into another. After students have made the selection, they can apply to their schools to make modifications in accordance with their aspirations and conditions. After testing and assessment, the credits earned can be interchanged accordingly.

### (3) Major Behavioral Verbs Used in *Standards*

Objectives used in *Standards* consist of three aspects: knowledge and skills, processes and methods, affection, attitudes and values. Levels of the behavioral verbs used may be classified as follows:

| Objective Domains | Levels | Behavioral Verbs |
|---|---|---|
| Knowledge and Skills | Know /Familiarize/ Imitate | familiarize, realize, know, identify, perceive, recognize, become familiar with, begin to realize, begin to acquire, begin to understand, find |

| | | |
|---|---|---|
| | Understand/Operate Independently | describe, state, express, manifest, announce, depict, explain, predict, imagine, understand, induce, summarize, abstract, extract, compare, contrast, decide, judge, able to find, able, deploy, begin to apply, begin to discuss |
| | Master/Apply/Transfer | master, derive, analyze, deduce, prove, research (study), discuss, select, decide, solve problems |
| Processes and Methods | Involve/Imitate | involve, observe, perceive, experience, operate(manipulate), locate, borrow, imitate, collect, recall, revise, participate, try |
| | Discover/Explore | design, organize, tidy up, analyze, discover, exchange(communicate), research (study), explore(investigate), inquire, find out, solve, find |
| Affect, Attitudes and Values | Respond/Concur | feel, recognize, familiarize, begin to realize, realize |
| | Realize/Interiorize | acquire, elevate, enhance, form, nourish, establish, engender, develop |

## Section Two – Curriculum Objectives

Overall objectives of senior mathematics curriculum are: Based on the foundation of the nine-year obligatory education mathematics curriculum, students move a step further to elevate their mathematics literacy necessary for future citizenship, so as to satisfy individual development and societal progress. Concrete objectives are listed as follow:

1. Acquire necessary fundamental knowledge and basic skills in mathematics; understand basic mathematical concepts and nature of mathematical conclusions; familiarize with the background of genesis and application of concepts and conclusions; realize mathematical thinking and methods inherent in mathematics, and the function they play in the ensuing studies. Through different forms of autonomous learning, exploratory activities, experience the processes of mathematical discoveries and creation.
2. Elevate abilities of spatial imagination; abstract generalization, deductive reasoning, computation and evaluation, and data processing.
3. Elevate abilities in posing, analyzing and solving problems (including simple and practical problems), abilities in mathematical expression and exchanges, as well as abilities in acquiring mathematical knowledge independently.
4. Develop mathematics application and innovation awareness; strive to engage in thinking and making judgments for some mathematical models implicit in the realistic world.
5. Elevate interests in mathematics learning; establish confidence in learning mathematics well; form spirits of perseverance and dedication, as well as scientific attitudes.
6. Possess a certain degree of mathematical perspective; progressively recognize the scientific value, application value and cultural values of mathematics; form habits of critical thinking; pay high regard to the rational spirits of mathematics; realize the aesthetic meaning of mathematics, so as to move a step further to establish dialectical and historical world view of objectivism.

# Section Three – Content Standards

## 1. Compulsory Curriculum

The compulsory curriculum is the foundation of the whole senior mathematics curriculum. It comprises of 5 modules carrying a total of 10 credits, the contents of which are studied all students. The contents are chosen in accordance with two principles: (1) satisfy the basic mathematics needs of future citizens; (2) provide mathematics essential for preparing students to further their studies.

Contents of the five modules are:

Mathematics 1: set, concept of function and basic elementary function I (exponential function, logarithmic function, power function).

Mathematics 2: preliminary solid geometry, preliminary plane analytical geometry.

Mathematics 3: preliminary algorithm, statistics, probability.

Mathematics 4: basic elementary function II (trigonometric function), plane vector, trigonometric identical transformation.

Mathematics 5: solution of triangle, sequence, inequality.

The above contents covered the major part of fundamental knowledge and basic skills in the traditional mathematics curriculum at the senior secondary stage of schooling. Contents included are: set, function, sequence, inequality, solution of triangle, preliminary solid geometry, and preliminary plane analytical geometry. What are different from the traditional curriculum are that, under the condition that the mathematics foundation is guaranteed, the compulsory curriculum emphasizes the formation, development and practical application of mathematics. It is noteworthy that skills and difficulty levels should not be set beyond the abilities of the students.

Apart from these, contents like vector, algorithm, probability and statistics are added as part of fundamental knowledge.

Vector is one of the most important and most basic concepts in modern mathematics, and forms a bridge communicating amongst contents like geometry, algebra and trigonometry. It possesses rich practical background and potentials for widespread applications.

Modern society is information-driven. People often need to extract information from the collected data for informed decision making. In the compulsory curriculum students shall learn basic idea and fundamental knowledge of statistics and probability, which are essential knowledge needed by all citizens.

Algorithm is a new topic in the compulsory curriculum. It has become an important foundation of computational science, and is increasingly playing a vital role in advancing science and technology, as well as social development. Ideas of algorithm and its preliminary knowledge is becoming general knowledge of the common citizens. In the compulsory curriculum, students shall learn basic ideas of algorithm and its preliminary knowledge. Algorithmic thinking shall tread through related parts of the senior mathematics curriculum.

Presentation of the compulsory curriculum should strive to demonstrate the process going from the concrete to the abstract, exhibit basic thinking methods and internal connection implicit in mathematical knowledge, as well as make visible the formation, development and practical application of mathematical knowledge. Teachers and editors of teaching materials should arrange some practice assignments taking into account concrete contents in appropriate places of the curriculum (e.g. statistics, linear programming).

## Mathematics 1

In this module, students shall learn concepts of sets and functions, as well as basic elementary functions I (i.e. exponential function, logarithmic function, power function).

Set theory was created by Cantor, a German mathematician, in the late $19^{th}$ century. Its language is a basic language of modern mathematics. Using the language one can represent some mathematical contents precisely and concretely. The senior secondary mathematics curriculum treats set theory simply as one language for students to learn, and students will learn how to deploy the most basic language of set theory to represent related mathematical objects, as well as develop abilities for exchanges using mathematical language.

Functions are important mathematical models used for describing change patterns and regularities of the objective world. At the senior secondary stage, functions are not merely envisaged as dependent relationships amongst variables. At the same time language of sets and correspondence is used to depict functions. Thinking methods involving functions shall tread its way through the senior mathematics curriculum from the beginning right up to the end. Students shall learn concrete basic elementary functions such as exponential function and logarithmic function. Coupled with practical problems, students feel the processes and methods of using functions to construct models, realize the importance of functions to mathematics and other subjects, begin to deploy ideas of functions to comprehend and process simple problems encountered in realistic everyday living. Students shall learn how to use the properties of function to evaluate the approximate solutions of an equation, and realize the organic relationships between functions and equations.

**Contents and Requirements**

**1. Set Theory (about 4 class hours)**

(1) Meanings of a set and its representation

(i)Through real examples, familiarize with the meanings of a set, realize the "belong to" relationship between an element and a set.

(ii) Able to select natural language, pictorial language, set theory language (enumeration method, or description method) to describe different concrete problems, feel the meanings and function of the set theory language.

(2) Basic relationships between sets

(i) Understand the meanings of inclusion and equality between sets, able to identify subsets of a given set.

(ii) In concrete situations, familiarize with the meanings of universe and empty set.

(3) Basic operations of sets

(i) Understand the meanings of union and intersection of two sets, able to find the union and intersection of two simple sets.

(ii) Understand the meanings of complementary set of a subset of a given set, able to find the complementary set of a given subset.

(iii) Able to use Venn diagram to represent set relationships and operations, realize the function of intuitionistic diagrams to the understanding of abstract concepts.

**2. Concepts of Functions and Basic Elementary Functions I (about 32 class hours)**

(1) Functions

(i) Through rich real examples, students move a step forward to realize that functions are important mathematical models used to describe the dependent relationships amongst variables. Based on this foundation, students learn to use the set and correspondence languages to depict functions, realize the function of correspondence relationships to depict concepts of functions, familiarize with the constituents of functions, able to find out the domain and range of some simple functions, and familiarize with concepts of mappings.

(ii) In practical situations, able to choose appropriate methods to represent functions (graphical method, tabulation method, analytic method) in accordance with the needs.

(iii) Through concrete real examples, familiarize with simple segmentation functions, and able to apply them.

(iv) Using functions already taught, particularly quadratic functions, comprehend the monotone characteristics of functions, maximum (minimum) values and their geometrical meanings; using concrete functions familiarize with the meanings of odd/even characteristics.

(v) Learn how to use graphs of functions to comprehend and study properties of functions (see Example 1).

(2) Exponential functions

(i) Through concrete examples (e.g. fission of cells, decay of $^{14}C$ in archaeology, quantitative changes of residues of medicine left behind in human bodies), familiarize with the practical background of exponential function model.

(ii) Comprehend the meanings of power of rational exponent; through concrete real examples familiarize with the meanings of power of real exponent, and master the operations of power.

(iii) Understand the concepts and meanings of exponential functions, able to borrow calculators and computers to draw the graph of concrete exponential functions, explore and understand the monotone characteristics and special points of exponential functions.

(iv) Realize that exponential function is an important kind of mathematical models during the process of solving simple practical problems.

(3) Logarithmic functions

(i) Comprehend concepts of logarithms and the properties of their operations, know how to use the "change of base" formulae to convert general logarithms into natural logarithms or common logarithms; through reading familiarize with the history of the discovery of logarithm, as well as its contribution to the simplification of calculations.

(ii) Through concrete real examples, familiarize with the intuitive quantitative relationships depicted by models of logarithmic functions, begin to understand concepts of logarithmic functions, realize that logarithmic function is an important kind of mathematical model, able to borrow calculators and computers to draw the graph of concrete logarithmic functions, explore and understand the monotone characteristics and special points of exponential functions.

(iii) Know that exponential function $y = a^x$ and logarithmic function $y = log_a x$ are inverse functions of each other ($a > 0$, $a \neq 1$).

(4) Power functions

Through concrete examples, familiar with concepts of power functions; using graphs of $y = x$, $y = x^2$, $y = 1/x$, $y = x^{1/2}$, familiarize with the behavior of changes of these graphs.

(5) Functions and equations

(i) Taking the graph of a quadratic function into account, decide whether the roots of a quadratic equation exists, and if exists the number of the roots, so as to familiarize with the connection of zeros of a function with roots of an equation.

(ii) In accordance with the graph of a concrete function, able to borrow a calculator to use bisection method to evaluate the approximate solutions of the corresponding graph, familiar with the use of this common method to arrive at the approximate solutions of an equation.

(6) Modeling using functions and their applications

(i) Using computational tools to compare the differences in terms of growth and increments amongst exponential functions, logarithmic functions, and power functions. Using real examples realize the meanings of different kinds of growth and increments in mathematical modeling using functions, e.g. rise in a linear manner, exponential explosion, and logarithmic growth.

(ii) Collect real examples of some commonly used models involving functions in everyday living (exponential functions, logarithmic functions, power functions, and segmentation functions), familiarize with the widespread application of modeling using functions.

(7) Practical assignment

According to some special topics, collect related information of historical events and figures (Kepler, Galileo, Descartes, Newton, Leibniz, Euler, etc.) that have immense influences on developments in mathematics during the period before and after the 17$^{th}$ century, as well as real examples of functions in realistic everyday living. Adopt group learning format to write an article on the formation, development or application of related concepts of functions, and to engage in exchanges after the lesson. For specific and concrete requirements, please refer to the requirements on mathematical cultures (see page ???).

**Remarks and Recommendations**

1. Set is an undefined concept. During teaching, teachers should make use of students' everyday experiences and their mathematical knowledge acquired. Through rich real examples, students are initiated to understand the meanings of sets. The best way to learn language of sets is through practice. During teaching, teachers should create environments and opportunities to allow students to use language of the sets in expression and communication. Students would then gradually familiarize with the characteristics of the natural language, set language and pictorial language during their practical uses. Students are able to convert from one language to another, and finally master the language of the sets. Regarding the relationships amongst sets and the teaching of the associated operations, usage of Venn diagram is important. This is

because it can help students learn, master, and use the language of the sets and other mathematical languages.

2. Teaching of concepts of functions should start from both practical backgrounds and definitions so as to help students understand the basic properties of functions. Generally there are two methods to introduce concepts of functions. The first method is to introduce mappings first, and then learn functions. The second method is through concrete real examples to allow students realize the special correspondence relationships between two sets of numbers (i.e. functions). Considering the cognitive characteristics of most senior secondary students and the need to help students understand the basic properties of concepts of functions, the second method is to be recommended. Teachers can make use of what students have mastered, such as concrete functions and descriptive descriptions of these functions so as to guide students to connect their everyday experiences with the practical problems, attempt to list various types of functions, and finally construct general concepts of functions. Then, teachers can ask students to study concrete functions such as exponential and logarithmic functions so as to deepen their understanding of concepts of functions. It is noteworthy that functions are important concepts needed repeated realization, upward spiraling, and progressive in-depth understanding before one can really master and apply them flexibly.

3. In teaching, teachers should emphasize understanding of basic properties of concepts of functions, avoid excessive training of complicated skills when the domain, range are evaluated and basic properties of functions are discussed. Teachers should try their best to avoid setting atypical questions pertaining to the evaluation of domain and range of functions.

4. Regarding teaching of powers and exponents, teachers should help students recall what they have acquired on concepts of integer exponent of powers and the associated properties of operations. Taking concrete real examples into account, teachers can introduce rational exponent of powers and the associated operational properties, as well as meanings of real exponents of power and the associated operational properties. Students advance to realize the idea of using rational numbers to approach as close as possible toward an irrational number. Teachers should allow students to use calculators and computers to engage in practical operations so as to get a feeling of the "approaching" process.

5. Regarding treatment of inverse functions, students are required only to use concrete functions as examples to engage in discussion and intuitive comprehension. For example, teachers can use exponential functions and logarithmic functions of different bases for comparison to illustrate that exponential functions $y = a^x$ and logarithmic functions $y = log_a x$ ($a>0$, $a \neq 1$) are inverse functions. There is no need to

discuss the formal definition of inverse functions, and there is no requirement to find out the inverse function of a given function.

6. During teaching of application of functions, students should guide students continuously to experience that function is a basic mathematical model used for describing the changing patterns and regularities of the objective world. Students should experience the intimate relationships between functions (such as exponential and logarithmic functions) and the realistic world, as well as their function in depicting realistic problems.

7. Teachers should pay attention to encourage students to deploy modern educational technology to explore and solve problems. For example, students can use calculators and computers to draw the graphs of exponential and logarithmic functions. They can explore and compare the changing patterns and regularities of these graphs, study the properties of these functions, and evaluate the approximate solutions of equations.

**Exemplary Cases**

**Example 1** Gordon, a member of the track and field team, is under supervised training by his coach preparing for a 3000 meters running race. The training requirements are:

(1) After starting the race with uniform acceleration, velocity reached 5*m* per second after 10 seconds, then run at uniform velocity until at 2 minutes.

(2) Starts to decrease velocity at a uniform rate, at 5 minutes decreased to 4*m* per second.

(3) Within 1 minute, gradually accelerate to reach a velocity at 5*m* per second, maintain running at uniform velocity.

(4) At the last 200 meters, accelerate uniformly and dash for the finishing line with a velocity reaching 8*m* per second.

Based on these training requirements, solve the following problems:

(1) Draw the graph of the function relating Gordon’s velocity and the time he runs.

(2) Write down the function relating running velocity and time when Gordon is engaged in the long race training.

(3) According to the stipulated requirements, calculate the time taken to complete the 3000*m* running race.

**Solution**: (1)

<insert graph here: start race, during the race, accelerate, dash for the finishing line>

(2)

<insert equations here>

(3) omitted.

**Example 2** The release of fluorides used in electrical appliances (e.g. refrigerators) destroys the ozone layer of the upper atmosphere. The quantity of ozone $Q$ in the atmosphere changes can be represented by the exponential function, satisfying equation $Q = Q_0e^{-0.0025t}$, $Q_0$ is the initial quantity of ozone.

(1) As time increases, will the quantity of ozone in the atmosphere increase or decrease?

(2) How many years will it take for half of the ozone to disappear?

## Mathematics 2

In this module, students shall learn elementary solid geometry and plane analytical geometry.

Geometry is a mathematical science that seeks to study the relationships of shapes, magnitude and positions of objects in the realistic world. We generally adopt methods such as intuitive perception, confirmatory operation, reasoning and argumentation, measurement and calculation to know and explore the geometrical figures and their properties. Three-dimensional space is the realistic space people live in. Therefore, knowledge of spatial figures, inculcation and development of students' spatial imagination and reasoning argumentation abilities, deployment of abilities so as to engage in exchanges with others using the language of the figures, as well as the use of intuitive geometric ability, are all basic requirements of the compulsory series curriculum in the senior secondary curriculum. In the section on elementary solid geometry, students shall start from observing spatial geometric objects in a holistic manner, recognize spatial figures, and then use cuboids as embodiments to know intuitively and comprehend the positional relationships of points, lines and planes in space. Students are able to use language of mathematics to express properties related to parallelism and perpendicularity, judge and determine these properties, and to carry out argumentation of some of these conclusions. Students shall become familiarize with the calculation methods of surface area and volume of some simple geometric objects.

**Contents and Requirements**

**1. Elementary Solid Geometry (about 18 class hours)**

(1) Geometrical objects in space

(i) Use models of real objects and computer software to observe a large quantity of spatial figures; know structure characteristics of cylinders, cones, frustum and solid spheres and their simple constellation, as well as to use these characteristics to describe the structure of simple objects in realistic everyday living.

(ii) Able to draw three-view drawings of simple figures and objects in space (cuboids, solid spheres, cylinders, cones, prisms and their combinations); able to identify the solid models represented by the above-mentioned three-view drawings; able to use materials (e.g. cardboards) to make models; able to use "oblique method" to draw their intuitionistic diagrams.

(iii) Through observation, use two types of methods (parallel projection, central projection) to draw the three views and the intuitionistic diagrams; familiarize with the different ways of representation of figures in space.

(iv) Complete a practical assignment, such as draw the three views and the intuitionistic diagrams of some architecture (there is no rigorous requirement on size and type of lines subject to the condition that characteristics of the figures are not affected).

(v) Familiarize with the formulae for calculating the surface area and volume of solid sphere, prism, pyramid, and frustum (there is no need to memorize the formulae).

(2) Positional relationships amongst points, lines and planes.

(i) Borrowing models of cuboids, abstract definitions of the positional relationships of lines and planes in space based on the foundation of knowing intuitively and comprehending the positional relationships of points, lines and planes. Also, familiarize with the following axioms and theorems that can be used as bases for inferences.

* Axiom 1: If two points of a straight line lie on a plane, then the whole straight line lies on this plane.

* Axiom 2: There is one and only one plane passing through three points not lying on a straight line.

* Axiom 3: If there is one common point of two planes which do not coincide, then there is one and only one common straight line passing through this point.

* Axiom 4: Two lines parallel to the same straight line are parallel.

* Theorem: If the corresponding two sides of two angles in space are respectively parallel, then the two angles are either equal or complementary.

(ii) Through intuitive perception, confirmatory operation, reasoning and argumentation, and use the above-mentioned definitions, axioms and theorems in solid geometry as starting point, know and comprehend related properties and conditions for judging and determining parallel and perpendicular lines and planes in space.

Through intuitive perception and confirmatory operation, induce the following theorems used for judgment and determination:

* If one straight line outside is parallel to a straight line inside a plane, then this straight line is parallel to the plane.

* If two intersecting straight lines of one plane are parallel to another plane, then the two planes are parallel.

* If a straight line is perpendicular to two intersecting straight lines on a plane, then this straight line is perpendicular to the plane.

* If a plane passes through a perpendicular line of another plane, then the two planes are perpendicular.

Through intuitive perception and confirmatory operation, induce the following theorems used for judgment and determination of properties, and prove these theorems as well:

* If a straight line is parallel to a plane, then the line of intersection of any plane passing through this straight line and the plane are parallel to the straight line.

*If two planes are parallel, then the lines of intersection of any plane that intersects these two planes are mutually parallel.

* The two straight lines perpendicular to the same plane are parallel.

* If two planes are perpendicular, then a straight line of one plane perpendicular to the line of intersection of the two planes is perpendicular to another plane.

(iii) Able to deploy conclusions already acquired to prove some simple propositions of positional relationships in space.

**2. Elementary Plane Analytical Geometry (about 18 class hours)**

(1) Straight line and equation

(i) In a plane rectangular coordinates system, coupled with concrete figures, explore the geometric essentials ascertaining the position of straight lines.

(ii) Comprehend concept of inclination angle and slope of straight line; involve in the process of using algebraic method to depict the slope of a straight line; master the formula of calculating the slope of a straight line passing through two points.

(iii) Able to judge and determine that two straight lines are parallel or perpendicular in accordance with the slopes.

(iv) According to the geometric essentials ascertaining the position of straight line, explore and master the different forms of equations of straight line (point slope form, two point form, and general form); realize the relationship between slope intercept form and linear equation.

(v) Able to use method of solving system of equations to find the coordinates of the point of intersection of two straight lines.

(vi) Explore and master the formula of distance between two points, formula of distance of a point to a straight line; able to find the distance between two parallel straight lines.

(2) Circle and equation

(i) Recall the geometric essentials ascertaining circle; explore and master the standard equation and general equation of a circle in a plane rectangular coordinates system.

(ii) Able to determine the positional relationships between a straight line and a circle, as well as between two circles based on the given equations of straight line and circle.

(iii) Able to use equations of straight line and circle to solve some simple problems.

(3) During the initial process of learning plane analytical geometry, realize the idea of using algebraic method to handle geometric problems.

(4) Rectangular coordinates system in space

(i) Through concrete situations and contexts, feel the necessity of establishing rectangular coordinates system in space; familiarize with rectangular coordinates in space; able to use rectangular coordinates system in space to depict position of points.

(ii) Through representation of coordinates of vertex of special cuboids (all edges are parallel to the coordinate axes respectively), explore and find the distance formula between two points in space.

**Remarks and Recommendations**

1. Teaching of elementary solid geometry emphasizes helping students to form spatial imagination ability progressively. Design of contents in this section follows the principle of progression, i.e. from parts to the whole, and from concrete to the abstract. Teacher should provide rich models of real objects, or spatial geometric objects displayed through the use computer software, to help students recognize the structural characteristics of geometric objects in space. Furthermore, teacher should use these characteristics to describe structure of simple objects in realistic everyday living, consolidate and enhance learning and comprehension of related topics of three-view drawings at the obligatory stage of schooling, help students apply parallel projection and central projection, and proceed to master methods and skills of representing spatial figures on a plane (see Example 1).

2. Geometry teaching, through knowledge of practical models, should pay attention to guide students to learn how to convert natural languages into languages expressed in the form of figures and symbols. Teachers can make use of concrete cuboids as embodiment of relationships of points, lines and planes. Based upon the foundation of intuitive perception, teacher enables students to know positional relationships of some general points, lines and planes in space. Through observation, experimentation and elucidation of figures, students proceed to familiarize with basic

properties of parallel and perpendicular relationships, and the associated methods of judgment and determination. Students can use mathematical language accurately to express positional relationships of geometrical targets, and are able to solve some simple deductive reasoning and application problems (see Example 2).

3. During teaching of elementary solid geometry, there are requirements on proofs of theorems on properties related to parallel and perpendicular relationships of lines and planes. Corresponding theorems that are used for judging and determining properties demand merely intuitive perception and confirmation by operation. In series 2 of the optional curriculum these theorems shall be proved using the vector method.

4. During teaching those schools having better resources and conditions should deploy modern information technology appropriately to display figures in space. This would provide iconic support for understanding and mastering geometric properties (including proofs) of figures, and elevate students' geometric intuition ability. Teacher can guide and help students deploy knowledge of solid geometry to select topics for exploratory studies.

5. In the teaching of elementary plane analytical geometry, teacher should help students involve in the following processes: geometric problems should first be treated algebraically; use algebraic language to describe essentials of geometry and their relationships; proceed to transform geometric problem into algebra problem; process algebra problem; analyze the geometric meanings of algebraic structures; and finally solve the problem. This idea should be threaded through teaching of plane analytical geometry from the beginning right to the end. Students are helped continuously to realize method of thinking pertaining to the integration of numbers and shapes.

**Exemplary Cases**

**Example 1** Shown below is the three-view drawing of a trophy. Please draw the intuitive configuration of the trophy and find its volume.

<insert figure here; front view, side view, plan view>

**Example 2** Observe your classroom, state the positional relationships of points, lines and planes observed. Please give your reasons.

## Mathematics 3

In this module, students shall learn elementary algorithm, statistics and probability.

Algorithm is an important part of mathematics and its applications. It is an important foundation of computational science. Due to galloping development in modern information technology, algorithm is increasingly playing a vital role in science and technology, as well as social development. Algorithm is increasingly amalgamated with the many aspects of social living, and its thinking method has become one of the mathematical qualities needed by people at modern times. It is noteworthy that ancient Chinese mathematics is replete with rich ideas of algorithm. In this module, based on the foundation of having an initial feeling of algorithm at the obligatory stage of schooling, students experience the function of block diagrams in problem solving. Through imitation, operation and exploration, students learn how to design block diagrams to express the problem solving processes. Students realize basic ideas of algorithms, as well as the importance and validity of algorithms. They develop abilities to think and express systematically, and their logical thinking ability is elevated accordingly.

Modern society is information-driven. People often need to collect data. Based on the data collected people extract valuable information and make appropriate decisions. Statistics is a subject which seeks to study how to collect, organize and analyze data appropriately. It can provide evidences for informed decision making. Random phenomena are abundant in daily lives, and probability is a subject which seeks to study random phenomena. As such, it furnishes important models of thinking and methods of problem solving in order to know the objective world. At the same time, it provides the theory foundation for the developments in statistics. Because of these, basic knowledge of statistics and probability are necessary for citizenship in the future. In this module, based on the foundation of learning statistics and probability at the obligatory stage of schooling, students learn how to do random sampling through practical problem contexts, use the sample to estimate the population and the basic methods of carrying out regression. Students realize ideas of using samples to estimate population and its characteristics. Through solving practical problems, students involve systematically in the whole process of data collection and processing, and realize differences between statistical thinking and certain thinking. Students shall integrate concrete examples to learn some basic properties of probability and some simple models of probability. They deepen their understanding of random phenomena, and through experimentation and modeling use computers or calculators to estimate the probability of a simple event happened.

**Contents and Requirements**

**1. Elementary Algorithm (about 12 class hours)**

(1) Meanings of algorithm, procedural block diagram

(i) Through analyses of processes and steps of solving practical problems (e.g. solving of problems involving system of linear equations in two unknowns), realize the idea of algorithm; familiarize with the meanings of algorithm.

(ii) Through imitation, operation, and exploration, involve in expressing the processes of problem solving while designing block diagrams. During the processes of solving practical problems (e.g. solving of problems involving system of linear equations in three unknowns), comprehend the three basic logical structures of block diagrams: sequence, conditional branch, and loop.

(2) Basic algorithmic statements

Involve in the process of transforming procedural block diagrams of concrete problems into program statements; understand a few basic algorithmic statements: input statement, output statement, assignment statement, conditional statement, and loop statement; proceed to realize basic idea of algorithm.

(3) Through reading cases of algorithms in ancient Chinese mathematics, realize the contribution of ancient Chinese mathematics to mathematics development in the world.

**2. Statistics (about 16 class hours)**

(1) Random sampling

(i) Able to raise some valuable statistics problems in realistic everyday living and other subjects.

(ii) Combined with concrete practical problem contexts, comprehend the necessity and importance of random sampling.

(iii) During processes of participating in statistical problem solving, students are able to use simple random sampling method to draw samples from a population; through analyses of real cases, familiarize with methods of stratified sampling and systematic sampling.

(iv) Able to collect data through methods such as experimentation, information search, and design of questionnaire.

(2) Using sample to estimate population

(i) Realize through real examples the meanings and functions of distribution; during the processes of sample data representation, students learn how to tabulate frequency distribution table, draw frequency distribution histogram, frequency line graph, stem-and-leaf diagram, and realize their respective characteristics.

(ii) Through real examples comprehend the meanings and function of standard deviation of sample data; learn how to calculate the standard deviation of data.

(iii) Able to select appropriate sample in accordance with the requirements of practical problems; able to extract basic characteristics of numbers (e.g. mean, standard deviation) from sample data, and to put forward an appropriate explanation.

(iv) During the process of solving statistical problem, move a step further to realize the idea of using sample to estimate population; able to use sample frequency distribution to estimate population distribution; able to use basic sample characteristics of numbers to estimate basic population characteristics of numbers;

begin to realize the randomness of sample frequency distribution and characteristics of numbers.

(v) Able to use basic method of random sampling and idea of using sample to estimate population and solve some simple practical problems; able to provide some evidences for informed decision by means of data analysis.

(vi) Form consciousness of preliminary evaluation of processes related to data processing.

(3) Correlation of variables

(i) Through data collection of two variables related to each other encountered in realistic problems, construct and use a scatter diagram to know intuitively the correlation relationship of the two variables.

(ii) Involve in the use of different estimation method to describe the process of describing the linear relationship of two variables; know the idea of least squares method; able to establish linear regression equation in accordance with given coefficient formula of linear regression equation (see Example 2).

**3. Probability (about 8 class hours)**

(1) In concrete situations and contexts, familiarize with the uncertainty of random events and stability of frequencies; proceed to familiarize with the meaning of probability, as well as the difference between frequency and probability.

(2) Through concrete examples, familiarize with the probability addition formula of two mutually exclusive events.

(3) Through concrete examples, comprehend classical probabilistic model and the associated probability computational formulae; able to use enumeration method to calculate the number of basic events in some random events, and the probability that these events happened.

(4) Familiarize with the meanings of random numbers; able to deploy modeling methods (including generation of random numbers to carry out modeling using calculators) to estimate probability; begin to realize the meanings of geometric probabilistic models (see Example 3).

(5) Through reading materials, familiarize with the processes of knowing random phenomena by human being.

**Remarks and Recommendations**

1. Algorithm is new content in the senior secondary mathematics curriculum. Its ideas are very important, but not mysterious. The use of elimination method to solve system of linear equations in two unknowns, and processes of evaluation of the greatest common factor are examples of algorithms. Contents of algorithm in this module pertain to the establishment of connection of algorithm in mathematics and computer technology so as to express algorithm in a formal manner. For those schools

that are better resourced, teachers should strive to use computers for its realization. Because of the need to express algorithm systematically and clearly, teachers generally are required to organize problem solving processes as procedural block diagrams. In order to operate the algorithms on computers, teachers need to translate natural languages or procedural block diagrams into computer languages. The primary objective of this module is to enable students to realize the ideas of algorithm, and elevate logical thinking ability. Teacher should not treat this part of content simply as learning of programming language and design of programs.

2. Algorithm should be taught with real examples so as to enable students to learn some basic logical structures and statements during the problem solving processes. For those schools that are better resourced, teachers should encourage students to try their best to practice algorithms using computers.

3. Apart from confining algorithms as contents of this module, the ideas involved should diffuse into related contents in the senior secondary mathematics curriculum. Students should be encouraged to use algorithm to solve related problems.

4. Teacher should guide students to realize the function and basic idea of statistics. One characteristic of statistics is to use a small part of data to infer properties of the whole. Students should realize the differences between statistical thinking and certainty thinking, and are aware of the randomness of statistical results. There are chances that statistical inferences are wrong.

5. Statistics seek to extract information from the data. During teaching, teachers should guide students to choose different methods so as to obtain appropriate samples in accordance with the practical problem requirements, and to extract number characteristics from the sample data. Teachers should not treat statistics as number crunching and graph plotting. Regarding concepts in statistics (e.g. population, sample), there is a need to integrate with concrete problems for descriptive explication. Teacher should not quest for rigorous formal definitions of these concepts.

6. Teaching of statistics should be undertaken using concrete examples. During teaching, through processing of some typical examples, students involve relatively systematically in the whole process of data processing. In this process, students learn some data processing methods, and deploy the knowledge and methods acquired to solve practical problems. For example, during learning of contents of linear relationships, teacher can encourage students to explore a multitude of methods to ascertain the linear regression straight line. Based on this foundation, teacher can guide students to realize the idea of method of least squares and to find the linear regression equation according to given formulae. For those students who are interested, teacher can encourage them to attempt to derive linear regression equations.

7. The core problem of probability teaching is to enable students to become familiar with random phenomena and meanings of probability. Through the large quantity of real examples in everyday living, teacher should encourage students to do hands-on experiments. Students comprehend correctly the uncertainty of random events happened, and the stability of its frequency. They attempt to clarify some common mistakes encountered in everyday living (e.g. the probability of winning a lottery is 1/1000, one is sure to win if one buys 1000 lotteries).

8. Teaching of classical probability model should allow students to comprehend the characteristics of the model using real examples, i.e. the limitation of experimentation results and that each of the experimental results happened with equal probability. Students begin to learn to convert some practical problems into classical probability models. The emphasis of teaching should not be placed on "how to compute".

9. Students are encouraged to try their best to use calculators and computers to process data and to carry out modeling so as to realize better statistical thinking and the meanings of probability. For example, students can use calculators to generate random numbers to model the experiment of tossing a coin.

**Exemplary Cases**

**Example 1** The scores gained by two basketball players for each match of the seasonal tournament are shown below:

*A*'s score: 12, 15, 24, 25, 31, 31, 36, 36, 37, 39, 44, 49, 50.

*B*'s score: 8, 13, 14, 16, 23, 26, 28, 33, 38, 39, 51.

The above-mentioned data can be represented using the following diagram. The numbers in the middle represent the tenth digit of the scores, whereas the numbers on the two sides represent the unit digit of the scores gained by the two players respectively.

<insert stem-and-leave diagram here> indicates that the score gained by player A in this match is 25.

The diagram shown is generally known as stem-and-leaf diagram. Please compare the achievements of the two players in accordance with the above diagram.

As seen in the stem-and-leaf diagram, the scores gained by player *A* are roughly symmetric, the median is 36. Except for one special score the scores gained by player *B* are also roughly symmetrical, the median is 26. Therefore the performance of player *A* is relatively more stable, and the overall situation is better than that of player *B*.

There are two advantages of using stem-and-leaf diagram for data representation. First, there is no loss of information in the statistical diagram since all information can be obtained from the stem-and-leave diagram. Second, we can always use stem-and-leaf diagram for recording during tournament, facilitating documentation and representation. However, stem-and-leaf diagram can only be used to represent two-digit whole numbers. Although it can be used to represent the tournament results of more than two players (or two or more documentation), it is not so intuitive and clear when two records are displayed together.

**Example 2** The table shown below tabulates daily temperature and number of cups of hot tea sold by a tuck shop for a 6-day period.

| Temperature/□ | Number of Cups |
|---|---|
| 26 | 20 |
| 18 | 24 |
| 13 | 34 |
| 10 | 38 |
| 4 | 50 |
| -1 | 64 |

(i) Use the data in the above table to draw a scatter diagram.

(ii) Can you discover the approximate relationships between temperature and number of cups in the scatter diagram?

(iii) If the approximate relationship is linear, please draw a straight line to represent this linear relationship approximately.

(iv) If the temperature of a certain day is -5□, forecast the number of hot tea sold by this tuck shop.

When using a straight line to approximate the linear relationship between temperature and number of cups, students may choose two points to ascertain a straight line that reflects the changes, e.g. (4, 50) and (18, 24). Students can select one straight line so that the number of data points lying on one side of the straight line is basically equal to that of the other side. Another method is to choose several groups of points, ascertain several straight line equations, calculate the mean of the slopes and intercepts of each of these straight lines, and finally use the means obtained as the slope and intercept of the regression line.

**Example 3** A large quantity of beans is randomly scattered on the diagram shown (may use calculators or computers to model this process), calculate the ratio of beans fallen inside a circle and that fallen inside a square. Based on these estimate the value of $\pi$, and begin to realize the meanings of geometric probability model.

<insert diagram here or on the right side of the paper>

**Mathematics 4**

In this module, students shall learn trigonometric functions, vectors on a plane (i.e. plane vectors), and trigonometric identical transformations.

Trigonometric functions are basic elementary functions. They are important mathematical models to describe periodic phenomena, and have important function in mathematics and other disciplinary areas. In this module, through real examples, students learn trigonometric functions and their basic properties, realize the function of trigonometric functions in solving problems with periodic changes and regularities.

Vectors are amongst the most important and basic concepts in modern mathematics. The background of vectors is very rich and practical. They are tools to communicate with algebra, geometry and trigonometric functions. In this module, students shall become familiar with the rich practical background of vectors, understand plane vectors and the meanings of their operations, able to use the language of the vectors and methods to express and solve some problems in mathematics and physics, develop operational abilities, as well as abilities in solving practical problems.

There are certain types of application of trigonometric identical transformations in mathematics. Students' abilities of inference and operations are developed as well. In this module, students shall deploy methods of vectors to derive basic formulae of trigonometric identical transformations, followed by deploying these formulae to carry out simple identical transformations.

**Contents and Requirements**

**1. Trigonometric Functions (about 16 class hours)**

(1) Arbitrary degree, radian measures

Familiarize with concepts of arbitrary angles and the radian measure. Able to convert from radian measures into degree measures, and vice versa.

(2) Trigonometric functions

(i) Make use of the unit circle to understand definition of trigonometric functions (*sine*, *cosine*, *tangent*) of an arbitrary angle.

(ii) Make use of the directed line segments of trigonometric functions in the unit circle to derive the induction formulae (*sine*, *cosine*, and *tangent* of $\pi/2 \pm \alpha$, $\pi \pm \alpha$), able to draw the graphs of $y = \sin x$, $y = \cos x$, $y = \tan x$, familiarize with the periodicity of trigonometric functions.

(iii) Make use of graphs to understand the properties (such as monotonicity, maximum and minimum, points of intersection with the $x$-axis) of *sine* function, *cosine* function on $[0, 2\pi]$, and *tangent* function on $(-\pi/2, \pi/2)$.

(iv) Understand basic formulae relating trigonometric functions of the same angle:

$$\sin^2 x + \cos^2 x = 1, \quad \sin x / \cos x = \tan x.$$

(v) Using concrete real examples, familiarize with the practical meanings of $y = Asin(\omega x + \varphi)$; able to make use of a calculator or computer to draw the graph of $y = Asin(\omega x + \varphi)$, and observe the influence of parameters $A$, $\omega$, and $\varphi$ on changes of the graphs of functions.

(vi) Able to use trigonometric functions to solve some simple practical problems; realize that trigonometric functions are important function models to describe phenomena of periodic changes.

**2. Plane Vectors (about 12 class hours)**

(1) Practical background of plane vectors and basic concepts

Through examples of force and its analyses, students are familiar with the practical background of vectors; understand plane vectors and the meanings of equality of two vectors, as well as understand geometric representation of vectors.

(2) Linear operation of vectors

(i) Through real examples, master operations of addition and subtraction of vectors, and to understand their geometric meanings.

(ii) Through real examples, master operation of scalar multiplication of vectors and understand their geometric meanings; understand meanings of collinearity of two vectors.

(iii) Familiarize with properties of linear operation of vectors and their geometric meanings.

(3) Basic theorems of plane vectors and their coordinate representation.

(i) Familiarize with basic theorems of plane vectors and their meanings.

(ii) Master orthogonal decomposition of plane vectors and their coordinate representation.

(iii) Able to use coordinates to represent operations of addition, subtraction and scalar multiplication of plane vectors.

(iv) Understand the conditions under which two plane vectors, when represented using coordinates, are collinear

(4) Inner product of plane vectors

(i) Through real examples of "power" in physics, understand the meanings of inner product of plane vectors, as well as the associated physical meanings.

(ii) Realize relationships of inner product of plane vectors with the projection of vectors.

(iii) Master coordinates representation of inner product; able to carry out operations of inner product of plane vectors.

(iv) Able to use inner product to represent the included angle of two vectors; able to use inner product to determine the perpendicular relationship of two plane vectors.

(5) Application of vectors

Involve in using method of vectors to solve some simple plane geometric problems, as well as problems related to forces and processes pertaining to some practical problems. Students realize that vector is a tool that may be used to solve geometric problems, physics problems, etc. Students develop abilities of operations, as well as abilities of solving practical problems.

**3. Trigonometric Identical Transformation (about 8 class hours)**

(1) Involve in the process of deriving the *cosine* formula of difference of two angles using inner product of vectors, and proceed further to realize the function of vector method.

(2) Using the *cosine* formula of difference of two angles, able to derive the *sine*, *cosine*, and *tangent* formulae of sum and difference of two angles, as well as the *sine*, *cosine*, and *tangent* formulae of an angle that has been doubled; familiarize with the interrelationships amongst them.

(3) Able to apply the above-mentioned formulae to carry out identical transformation (including guiding students to derive transformation formulae involving product of trigonometric functions into sums and differences, sums and differences of trigonometric functions into products, and trigonometric functions of angle measures that have been halved. There is no need to memorize these formulae.)

**Remarks and Recommendations**

1. During teaching of trigonometric functions, teachers should create rich contexts in accordance with students' life experiences so as to enable students to realize the meanings of models of trigonometric functions. For example, using simple pendulum, spring oscillator, circular motion of a point, as well as real examples of music, waves, tides, and seasonal changes, students feel the abundant presence of periodic phenomena, know the change patterns and regularities of periodic phenomena, and realize that trigonometric functions are important models that can be used to depict periodic phenomena.

2. During teaching of trigonometric functions, teachers should make full use of the function of unit circle. Unit circle can help students know intuitively arbitrary angles and trigonometric functions of these angles, understand the periodicity, induction formulae, and relational expressions of trigonometric functions of same angle, as well as graphs and basic properties of trigonometric functions. Borrowing the intuition of unit circle, teachers can guide students to explore autonomously trigonometric functions and related properties, develop their abilities of analyzing and

solving problems.

3. Remind students to pay attention to the connection and synthesis of different subjects. During learning of related contents in other subjects (e.g. motion of simple pendulum, propagation of waves, alternating currents), students should pay attention to use trigonometric functions to analyze and comprehend.

4. Radian measure is a concept students find difficult to accept. During teaching, teachers should enable students to realize that radian measure is also a unit of angular measure (angle subtended at the centre of a circle by an arc equal in length to $1/2\pi$ of the circumference, or $1/2\pi$ of round angle). Due to the ensuing studies of the curriculum, students shall comprehend this concept progressively. There is no need to treat this topic in depth.

5. Teaching of concepts of vectors should start from the background of physics and geometry. Physics background comprises of concepts of force, velocity and acceleration, whereas geometry background is the directed line segment. Familiarization of these physics and geometric background are very important for students to comprehend concepts of vectors, and applications of vectors to solve practical problems. Teachers can guide students to deploy vectors to solve some physics and geometry problems. Examples are: the use of vectors to calculate the work done by a force which acts on a body to cause it to move along given directions; and the use of vectors to solve the positional relationships (i.e. parallel or perpendicular) of two straight lines on a plane. Regarding non-orthogonal decomposition of vectors, students are only required to know them in general terms and there is no need to expand the topic further.

6. During teaching of trigonometric identical transformation, teachers can guide students to use the inner product of vectors to derive the *cosine* formula of difference of two angles, and through this formula derive *sine*, *cosine* and *tangent* formulae of sum and difference of two angles, and *sine*, *cosine* and *tangent* formulae of angles that have been doubled. Teachers should encourage students to explore independently, and to engage in discussion and communication. Teachers guide students to derive formulae involving product of trigonometric functions into sum and difference, sum and difference of trigonometric functions into product, and trigonometric functions of an angle that has been halved. These may be treated as basic training of trigonometric identical transformation.

7. During teaching of this module, teachers should encourage students to use calculators and computers to explore and solve problems. Examples are: finding the values of trigonometric functions, solving measurement problems, analyzing the influence of the variation of parameters on function $y = A\ sin(\omega x + \varphi)$. Teachers can insert mathematical exploration and mathematical modeling activities in related

contents of trigonometric functions, plane vectors, and trigonometric identical transformations.

**Exemplary Case**

**Example 1**

Tides, caused by the gravity of the sun and moon, are rise and fall of the sea level at regular times of the day. Generally, ships enter their courses at high tide to park at the dock, and leave for the ocean at low tide after unloading the cargo. The table below shows the daily relationship of clock time and water depth for certain season of a harbor.

<insert table here: clock time, water depth/meter)

(1) Choose one trigonometric function to describe approximately the functional relationships of water depth and time of this harbor, and produce the approximate value of water depth at every hour of the day.

(2) The distance between the bottom of the ship and the sea level surface (depth of immersion) is 4 *m*. Safety regulations stipulate that the minimum distance between the bottom of the ship and the bottom of the sea (safety gap) is 1.5 *m*. At what time can the cargo ship enter the harbor? How long can it stay?

(3) If the depth of immersion of a ship is 4 *m* and the safety gap is 1.5 *m*, the ship starts to unload the cargo at 2:00, depth of immersion decreases at the rate of 0.3 *m* per hour, at what time of the day the ship is required to stop unloading its cargo and drive to deeper water districts.

## Mathematics 5

In this module, students shall learn solution of a triangle, sequence of numbers, and inequalities.

Based on the knowledge foundation students already acquired, students explore relationships amongst sides and angles of a given triangle. Students discover and master the quantitative relationships between the length of the sides and the measure of the angles. At the same time, students know how to deploy these relationships to solve some practical problems related to measurement and geometrical computations.

As number sequence is a special kind of function, it is a basic mathematical model that reflects patterns and regularities in nature. In this module, students shall establish mathematical models of arithmetic progression and geometrical progression through analyzing a large number of practical problems in everyday living. Students explore and master some basic quantitative relationships of these models, feel the widespread applications of these two types of number sequence models, and use them to solve some practical problems.

Equal and unequal relationships are basic quantitative relationships of objective events and objects, and they are important contents of research in mathematics. Establishing concepts of inequality, processing of unequal relationships, and processing of problems dealing with equality relationships are equally important. In this module, through concrete contexts, students shall feel the existence of a large quantity of unequal relationships in realistic world and everyday living, comprehend the meanings and values of system of inequalities in depicting unequal relationships, master basic methods in solving quadratic inequalities in one unknown, and able to solve some practical problems. Students are able to use plane regions to represent a system of linear inequalities in two unknowns, and attempt to solve some simple problems of linear programming in two unknowns. Students shall know basic inequalities and their simple applications, and realize the interrelationships amongst inequalities, equations, and functions.

**Contents and Requirements**

**1. Solution of a Triangle (about 8 class hours)**

(1) Through exploring the relationships of the length of the sides and the measure of the angles of any given triangle, students master *sine* theorem, *cosine* theorem, and are able to solve some simple metric problems of a triangle.

**2. Number Sequence (about 12 class hours)**

(1) Concepts of number sequence and its simple method of representation

Through real examples in everyday living, familiarize with concepts of number sequence and several simple methods of representation (tabulation, graph, formula of general term); know that number sequence is a special kind of function.

(2) Arithmetic progression, geometric progression

(i) Through real examples, understand concepts of arithmetic progression and geometric progression.

(ii) Explore and master the formula of the general term of arithmetic progression and geometric progression, as well as the formula of the sum of the first $n$ terms.

(iii) Able to discover common difference and common ratio relationships in number sequences within concrete problem contexts.

(iv) Realize the inter-relationships between arithmetic/geometric progressions with linear/exponential functions.

**2. Inequalities (about 16 class hours)**

(1) Unequal relationships

Through concrete situations and contexts, feel the existence of a large quantity of unequal relationships in the realistic world and everyday living.

(2) Quadratic inequalities in one unknown.

(i) Involve in the process of abstracting model of quadratic inequalities in one unknown from practical situations and contexts.

(ii) Through graphs of functions familiarize with the relationships of quadratic inequalities in one unknown with the corresponding functions and equations.

(iii) Able to solve quadratic inequalities in one unknown; able to design the procedural block diagram of the solution process given a quadratic inequality in one unknown.

(3) System of linear inequalities in two unknowns and simple linear programming problems.

(i) Abstract a system of linear inequalities in two unknowns from practical situations and contexts.

(ii) Familiarize with the geometrical meanings of linear inequality in two unknowns; able to use plane regions to represent system of linear inequalities in two unknowns (see example 2).

(iii) Able to abstract some simple linear programming problems in two unknowns, and solve them accordingly (see example 3).

(4) Basic Inequality: $\sqrt{(ab)} \leqq (a+b)/2 \quad (a, b \geqq 0)$.

(i) Explore and familiarize with the proving process of the Basic Inequality.

(ii) Able to use Basic Inequality to solve simple maximum (or minimum) problems.

**Remarks and Recommendations**

1. Teaching of solution of a triangle should emphasize the function of *sine* theorem and *cosine* theorem in the exploration of the relationships of the sides and angles of a triangle. Teachers guide students to learn that knowledge of these relationships is one means to solve measurement problems. There is no need to train students excessively on complicated identical transformations.

2. There are widespread applications of arithmetic progression and geometric progression. During teaching, teachers should pay attention to use examples (e.g. education loan, housing loan, decay of radioactive materials, population growth) to enable students to understand the function of these number sequence models. Students acquire abilities of abstracting number sequence models from practical problems.

3. In the teaching of number sequence, teachers should guarantee training of basic skills and, by means of necessary practicing, guide students to master basic relationships amongst quantities in the number sequence. Attention needed to be paid to control the difficulty and complexity levels of the training.

4. In the teaching of quadratic inequality in one unknown, teachers should pay

attention to enable students to familiarize with the practical background of quadratic inequality in one unknown. When solving quadratic inequality in one unknown, students can start by finding the roots of corresponding equations, and then find out the solutions of inequality in accordance with the graph of the corresponding function. Also, students can use algebraic method to find the solutions. Teachers should encourage students to design the procedural block diagram to solve quadratic inequality in one unknown.

5. There are rich practical backgrounds in inequalities. They are important tools in depicting regions. Depicting region is a basic step of solving linear programming problems. During teaching, teachers can introduce system of linear inequalities in two unknowns from the practical background.

6. Linear programming is a concrete model of optimization. During teaching of this module, teachers should guide students to realize basic ideas of linear programming, and borrow geometric intuition to solve some simple linear programming problems. There is no need to introduce a lot of terminologies.

**Exemplary Cases**

**Example 1** Income and Comparison of Education Savings Schemes

Request students to collect local information on education savings, and think about the following problems:

(1) According to the education savings scheme, deposit 50 dollars each month, deposit continuously for a 3-year period, how much income can one get if the principal and interest are drawn as a lump sum at maturity (3 years). How much income can one get for a 6 years scheme?

(2) According to the education savings scheme, deposit *a* dollars each month, deposit continuously for a 3-year period, how much income can one get if the principal and interest are drawn as a lump sum at maturity (3 years). How much can one get for a 6-year scheme?

(3) According to the education savings scheme, deposit 50 dollars each month, deposit continuously for a 3-year period. Compared with equivalent conditions in "Small Deposit and Lump Withdrawal", how much more income can one get if the principal and interest are drawn as a lump sum at maturity (3 years).

(4) If one would like to accumulate 10 thousand dollars of education savings' principal and interest drawn as a lump sum in 3 years time, how many dollars should one deposit each month?

(5) If one would like to accumulate *a* thousand dollars of education savings' principal and interest drawn as a lump sum in 3 years time, how many dollars should one deposit each month?

(6) According to the education savings scheme, if one originally plan to deposit 100 dollars each month, deposit continuously for a 6-year period, but after 4 years would like to draw in advance the total savings of principal and interest as a lump sum, how much income can one get?

(7) According to the education savings scheme, if one originally plan to deposit $a$ dollars each month, deposit continuously for a 6-year period, but after $b$ years would like to draw in advance the total savings of principal and interest as a lump sum, how much income can one get?

(8) Open-ended question: If one does not use the education savings scheme and opts for other savings format, explore the maximum income one may get if one deposits 100 dollars each month, deposits for a 6-year period, and interest is based on current standards. Compare this income with what may be obtained using the education savings scheme.

**Example 2**

A chemical fertilizer factory manufactures two mixtures of fertilizer $A$ and $B$. The primary ingredients are phosphates and nitrates. One carton of fertilizer $A$ needs 4 tonnes of phosphate and 18 tonnes of nitrate, resulting in 10000 dollars as profits, whereas one carton of fertilizer $B$ needs 1 tonne of phosphate and 15 tonnes of nitrate, resulting in 5000 as profits. Now 10 tonnes of phosphate and 66 tonnes of nitrates are available for production of the two types of fertilizers. Please list the expressions of mathematical relationships of the given conditions, and draw the graphs accordingly.

**Solution:** Let $x$, $y$ be the number of cartons of mixtures of fertilizers $A$ and $B$ manufactured. Then,

<insert system of inequalities here>

<insert graph of example 2 here>

**Example 3**

One factory manufactures two kinds of commodities I and II to tailor to the needs of the market. The income incurred are 2000 and 3000 dollars respectively for the two commodities. Both I and II require equipments $A$ and $B$ for their processing. Processing of one unit of I requires one hour processing time for $A$ and two hours for $B$. Processing of one unit of II requires two hours processing time for $A$ and one hour for $B$. For $A$ and $B$, the number of hours that may be used for manufacturing are 400 and 500 respectively. Devise a production plan that may maximize the income.

**Solution**: The mathematical model of this model is linear programming in two unknowns.

Let $x$ and $y$ units of I and II are manufactured. The constraint conditions are:

<insert system of linear inequalities here>

<insert graph of example 3 here>

The objective function is $f = 3x + 2y$.

It is required to find out appropriate values of $x$ and $y$ such that $f = 3x + 2y$ is maximized.

Draw the feasible region first as shown in the diagram. Consider $3x + 2y = a$, where $a$ is a parameter. Transform this equation into $y = - 3/2\ x + a/2$. This is a family of straight lines of slope equals to -3/2 and varied according to the value of $a$. $a/2$ is the intercept of the straight line cutting the $y$ axis. When $a/2$ is maximized, $a$ is likewise maximized. Here the objective function attains its maximum value while satisfying the constraint conditions. Of course, this family of straight lines should intersect with the feasible region.

In this problem, the $(x, y)$ that makes $3x + 2y$ attains its maximum value is the point of intersection (200, 100) of the two straight lines $2x + y = 500$ and $x + 2y = 400$. Therefore, the factory should manufacture 200 and 100 units of commodities I and II respectively to secure a maximum income of 800 thousand dollars.

**Example 4** A factory manufactures a pool in the form of a cuboid without a cover. The capacity of this pool is 4800 $m^3$ and the depth is 3 $m$. If the manufacturing cost of the bottom of the pool is 150 dollars per $m^2$ and that of the sides of the pool is 120 dollars per $m^2$, design a pool so that the manufacturing cost is minimized. What is the minimum manufacturing cost?

## 2. Optional Curriculum

### Remarks of Series 1 and Series 2

Based on the foundation of completing the compulsory curriculum, students who would like to learn mathematics further can select series 1 and series 2 for study in accordance with their needs and interests.

Series 1 is meant for students who would like to develop themselves further in the social sciences and the humanities. It comprises of two modules totaling 4 credits. Series 2 is meant for those who would like to develop themselves further in economics, science and technology. It comprises of 3 modules totaling 6 credits.

Contents of series 1 are listed below:

Option 1-1 Common terminologies of logic, conic section and equation, derivative and its application;

Option 1-2 Statistical cases, inference and proof, extension of number system and introduction of complex number, block diagram;

Contents of series 2 are listed below:

Option 2-1 Common terminologies of logic, conic section and equation, vectors in space and solid geometry;

Option 2-2 Derivative and its application, inference and proof, extension of number system and introduction of complex number;

Option 2-3 Enumeration principle, statistical cases, probability

In series 1 and series 2 of the curriculum, some contents and their requirements are the same, e.g. common terminologies of logic, statistical cases, extension of number system and complex number; some contents are basically the same, but the requirements are different, e.g. derivative and its application, conic section and equation, inference and proof; some contents are rather different, e.g. series 1 has included some contents on block diagram, and series 2 has included contents such as vectors in space and solid geometry, enumeration principle, discrete random variables and their distributions.

## Series 1

### Option 1-1

In this module, students shall study common terminologies of logic, conic section and equation, derivative and its application.

Using terminologies of logic correctly is a basic quality possessed by all modern citizens of a society. Irrespective of engaging in thinking, communication and various kind of work, we need to use terminologies of logic correctly to represent our own thinking. In this module, based on the foundation of the stage of obligatory education, students learn common terminologies of logic, realize the function of terminologies of logic in expression and argumentation, deploy these terminologies of logic to express mathematical contents correctly, and to engage better in communication.

In this module, based on the foundation of studying preliminary plane analytical geometry in the compulsory curriculum, students shall learn conic section and equation, familiarize with the relationships between conic sections and quadratic equations, master the basic geometric properties of conic sections, feel the function of conic sections in depicting the realistic world and solving practical problems, and proceed to realize the idea of integration of numbers and shapes.

Creation of calculus is a milestone in mathematics development. Its development and widespread application marked a new era of transition towards mathematics of the recent times. It furnishes important methods and strategies to study variables and functions. Concept of derivative is a core concept in calculus and it has very rich practical background and widespread application. In this module, through a large quantity of real examples, students shall involve in processes depicting mean rate of change to instantaneous rate of change, understand the meanings of derivative, and realize ideas of derivative and its implicit meanings. Students apply derivatives to explore properties such as the monotone and extreme values of functions, as well as their practical applications. Students feel the function of derivatives in solving mathematics problems and practical problems, and realize that the formation of calculus is of value to human civilization.

**Contents and Requirements**

**1. Common Terminologies of Logic (about 8 class hours)**

(1) Propositions and their relationships

(i) Familiarize with converse proposition, negative proposition and converse-negative proposition of a proposition.

(ii) Understand the meanings of necessary condition, sufficient condition, and sufficient and necessary condition; able to analyze the inter-relationships of the four kinds of proposition.

(2) Simple logical connectives

Through mathematical examples, familiarize with the meanings of logical connectives: “or”, “and”, “not”.

(3) Universal quantifier and existential quantifier

(i) Through rich real examples in mathematics and everyday living, understand the meanings of universal quantifier and existential quantifier.

(ii) Able to negate a proposition containing one quantifier correctly.

**2. Conic Section and Equation (about 12 class hours)**

(1) Familiarize with conic section and its practical background, feel the function of conic section in depicting the realistic world and solving practical problems.

(2) Involve in the process of abstracting an ellipse model from concrete situations (see Example 1); master definition of an ellipse, standard equation and its simple geometric properties.

(3) Familiarize with definition, geometric shape and standard equation of parabola and hyperbola, and know their simple geometric properties.

(4) Through learning conic section and equation, students proceed to realize the idea of integration of numbers and shapes.

(5) Familiarize with simple application of conic sections.

**3. Derivative and its Application (about 16 class hours)**

(1) Concept of derivative and its geometric meanings

(i) Through analyses of a large quantity of real examples, students involve in processes transiting from mean rate of change to instantaneous rate of change; familiarize with concept of derivative and its practical background; know that instantaneous rate of change is derivative, and realize the ideas and inner meanings of derivative (see Example 2 and Example 3).

(ii) Through graphs of functions understand intuitively the geometric meanings of derivative.

(2) Operation of derivatives

(i) In accordance with the definition of derivative find the derivatives of the functions $y = c$, $y = x$, $y = x^2$, $y = x^3$, $y = 1/x$, $y = \sqrt{x}$.

(ii) Able to use the formulae of derivatives of given basic elementary functions and arithmetic operations to find the derivatives of simple functions; able to find the derivative of composite function (limited to functions of the form $f(ax + b)$ ).

(iii) Able to use table of derivative formulae.

(3) Application of derivatives in studying functions

(i) Combined with real examples, borrow geometric intuition to explore and familiarize with the relationship of monotonicity of a function with its derivative (see Example 4 ); able to use derivative to study the monotonicity of functions, and able to find the monotone intervals of polynomial functions with degrees not exceeding 3.

(ii) Combined with graph of a function, familiarize with the necessary condition and sufficient condition that a function takes on extreme values at some points; able to use derivative to find the maximal value and minimal value of a polynomial function with degree not exceeding 3, as well as find the maximum values and minimum values of a polynomial function with degree not exceeding 3 within a closed interval; realize the generality and effectiveness of method of derivatives in studying properties of functions.

(4) Examples of optimization problem in everyday life

For example, through maximization problems such as maximization of profits, minimization of material use, and highest efficiency, students realize the function of derivative in solving practical problems (see Example 5).

(5) Mathematical culture

Collect background information of the era when calculus was created, as well as information of related leading figures, and engage in exchanges; realize the meanings and value of calculus on development of human culture. For concrete requirements, please refer to the section “mathematical culture” in *Standards* (see page ???).

**Remarks and Recommendations**

1. During teaching of common terminologies of logic, teachers should pay particular attention to the following problems:

(i) Propositions considered here refer to those with clearly specified conditions and conclusions. Students need only to become familiar with converse proposition, negative proposition and converse-negative proposition of a proposition. The emphasis is on the interrelationships amongst the four types of propositions, as well as the necessary condition, sufficient condition, and sufficient and necessary condition of a given proposition.

(ii) Regarding the meanings of logical connectives “or”, “and”, and “not”, students are only required to become familiar with them through real examples in mathematics. Teachers should help students to express correctly related mathematics contents.

(iii) Regarding the use of quantifiers, the emphasis is on the understanding of their meanings. There is no need to chase after formal definitions of quantifiers.

(iv) During the process of guiding students to use common terminologies of logic, attention should be paid to enable students to master usage of common terminologies of logic, correct logical mistakes appeared, and realize the accuracy and clarity in the use of common terminologies of logic to express mathematical contents. Mechanical memorization of terminologies of logic and abstract explanations should be avoided. The use of truth table is not required.

2. When introducing conic section, teachers should deploy rich examples (e.g. orbit of a planet, locus of a projectile, mirror surface of a torchlight) to enable students to become familiar with the background and application of conic section.

3. Teachers should demonstrate the process of obtaining an ellipse through cutting a cone by a plane so as to deepen students' understanding of conic section. Whenever resources and conditions permit, schools should exhibit sufficiently the function of modern educational technology. They can use computers to demonstrate how a conic section may be obtained as a result of cutting a cone by a plane (see Example 1).

4. Teachers should demonstrate practical applications of conic section to students, e.g. trajectory of a lead ball thrown, orbit of a satellite.

5. In this module, concept of derivative is introduced through practical background and examples of concrete applications. In teaching, teachers can guide students to involve in processes transiting from mean rate of change to instantaneous rate of change, and subsequently know that derivative is instantaneous rate of change. Students can study real examples of application of derivative such as growth rate, expansion rate, efficiency, density and velocity. Through feeling the function of derivative in studying function and solving practical problems, students realize ideas of derivative and its inner meanings. Treatment of special topic in this way aims to help students understand the background, thinking and function of derivative intuitively.

6. In teaching, teachers should prevent treating derivative merely as studying some rules and steps and ignore its ideas and values. Teachers should enable students to recognize that any rate of change of events can be described using derivative.

**Exemplary Cases**

**Example 1** In Figure 1, a plane intersects with a cone. The line of intersection of this plane and the cone is an ellipse. Two spheres, one is big and one is small, are placed inside the cone so that they are tangent with the plane and the cone respectively. The points where the cutting plane is tangent with the spheres are exactly the two foci of the ellipse.

<Insert Figure 1 and 2 here> Quantity of sewage discharge, standard, time

**Example 2** National Environmental Protection Bureau stipulates the standards for sewage discharge of two incorporations. Before enforcement of new standards, inspection is made and the results are shown in Figure 2. Which incorporation is better in handling sewage disposal? (*W* represents the quantity of sewage discharge).

At $t_0$, although $W_1(t_0) = W_2(t_0)$, since <insert formula here>, it can be said that within unit time the rate of sewage disposal is greater for incorporation *A* than incorporation *B*. Therefore, incorporation *A* is a little bit better than incorporation *B*.

**Example 3** We know that when an athlete dives from a high platform of 10 *m* high, the velocities at different times are different during the process of moving in the air and subsequently diving into water. Suppose that after *t* seconds, the height of the athlete relative to the ground level is: $H(t) = -4.9t^2 + 6.5t + 10$, what is the velocity of the athlete after 2 seconds (instantaneous velocity) ?

The average velocity of that athlete from 2 seconds to 2.1 seconds (denoted as [2, 2.1] is:

<insert equation here>

Similarly, we can calculate the average velocities at [2, 2.01], [2, 2.001], etc. We can also calculate the average velocities at [1.99, 2], [1.999, 2], etc.

| Time/s | Interval/s | Average Velocity/ ($m/s$) | | Time/s | Interval/s | Average Velocity/ ($m/s$) |
|---|---|---|---|---|---|---|
| [2, 2.1] | 0.1 | -13.59 | | [1.9,2] | 0.1 | -12.61 |
| [2, 2.01] | 0.01 | -13.149 | | [1.99,2] | 0.01 | -13.051 |
| [2, 2.001] | 0.001 | -13.104 9 | | [1.999,2] | 0.001 | -13.095 1 |
| [2, 2.000 1] | 0.000 1 | -13.100 49 | | [1.999 9,2] | 0.000 1 | -13.099 51 |
| [2, 2.000 01] | 0.000 01 | -13.100 049 | | [1.999 99,2] | 0.000 01 | -13.099 951 |
| … | … | … | | … | … | … |

From this table we can see that when the time interval becomes smaller and smaller, the average velocity tends to become a constant. This constant (13.1) can be treated as the velocity of the athlete at 2 seconds.

As shown in the diagram are straight line *l* and circle *c*, when *l* starts to rotate (angle of rotation is less than 90°) from $l_0$ around *O* uniformly, the area of the shaded area swept by it is *S* and this is a function of *t*. Its graph is something like:

<insert diagrams here>

**Example 5** The length of the side of a square iron plate is *a*. Four small squares,

the length of the side of which is $x$, are truncated at the four corners of the plate and then a box without a lid is made.

(1) Express volume of the box $V$ as a function of $x$.

(2) Find the value of $x$ such that the volume of the box $V$ is greatest.

**Option 1-2**

In this module, students shall learn statistical cases, inference and proof, extension of number system and introduction of complex number, and block diagram.

Based on the foundation of statistics learning in the compulsory curriculum, and through discussion of typical exemplary cases, students are familiar with some common statistical methods and their applications, proceed to realize the basic ideas of using statistical methods to solve practical problems, as well as know the function of statistical method in decision making.

"Inference and Proof" is a basic thinking process in mathematics. It is the most commonly used thinking method for study and everyday living. Inferences generally include plausible reasoning and deductive reasoning. Plausible reasoning is a deduction process involving prediction of some results based on known facts and correct conclusions (including definitions, axioms and theorems), results of experiments and practical work, as well as personal experiences and intuition. Induction and analogy are common plausible reasoning thinking method. During the problem solving process, plausible reasoning is useful for guessing and discovering conclusions, as well as exploring and providing lines of thoughts. This facilitates development of innovative consciousness. Deductive reasoning is a process involving drawing of new conclusions based on known facts and correct conclusions (including definitions, axioms and theorems), as well as those obtained through strict logical rules. Development and elevation of deductive reasoning or logical proof abilities are important objectives in the mathematics curriculum. Relationships between plausible reasoning and deductive reasoning are intimate and they reinforce each other. Proof generally comprises of logical proof and proof by experiments and practices. Correctness of mathematical conclusion has to be guaranteed through logical proof, i.e. through correct usage of inference rules to arrive at a conclusion based on the foundation of correct premises. In this module, through recall of knowledge already learned, students proceed to realize plausible reasoning, deductive reasoning, as well as the connection and differences between these two types of reasoning. Students realize characteristics of proof, familiarize with basic methods of mathematical proof, including methods of direct proof (e.g. analytic method, synthetic method) and methods of indirect proof (e.g. prove by contradiction). Students should feel the function of logical proof in mathematics and everyday living, and develop habits such that proposals are reasonable and arguments are supported by evidences.

Process of extension of number system exhibits the discovery and creation processes in mathematics. At the same time it exhibits the objective demand and background of the formation and development of mathematics. Introduction of complex number at the stage of senior secondary education is another extension of number system. In this module, students shall familiarize with processes of extension of number system in problem contexts, as well as the necessity of introducing complex numbers. Students learn some basic knowledge of complex number and realize the function of rational thinking in the extension of the number system.

Block diagram, in the form of diagrams, represents interrelationships of parts and components of a system. It is used to express clearly relationships of the various parts of relatively complicated systems. Block diagram has been used extensively in areas such as algorithm, design of computer programming, expression of procedural flowcharts, and comparison of project designs. It shall become an important way of representation in everyday living and exchanges in different disciplinary areas. In this module, students shall learn how to use flowchart and structural diagram to depict mathematics problems, as well as problem solving processes of other problems. At the same time, students experience the use of block diagram to represent the mathematical problem solving process during the learning process, as well as the superiority of the developmental process. Abilities of abstract generalization and logical thinking are elevated. Students can express and communicate clearly their thoughts to others.

**Contents and Requirements**

**1. Statistical exemplary cases (about 14 class hours)**

Through typical exemplary cases, students learn some of the common statistical methods. They are able to apply these methods preliminarily to solve some practical problems.

(i) Through exploration of typical exemplary cases (e.g. "Is there any relationship between lung cancer and smoking?"), familiarize with the basic ideas, methods and preliminary applications of test of independence (only 2 x 2 cross-tabulations are required).

(ii) Through exploration of typical exemplary cases (e.g. "Quality control", and "Is the new medicine effective?"), familiarize with basic ideas, methods and preliminary applications of inference principle and hypothesis testing (see Example 1).

(iii) Through exploration of typical exemplary cases (e.g. "Classification of insects"), familiarize with the basic ideas, methods and preliminary applications of cluster analysis.

(iv) Through exploration of typical exemplary cases (e.g. "Relationships between human weight and height"), familiarize with the basic ideas, methods and preliminary applications of regression.

**2. Inference and Proof (about 10 class hours)**

(1) Plausible reasoning and deductive reasoning

(i) Combined with real mathematics examples learned and real examples in everyday life, students familiarize with the meanings of plausible reasoning, able to use induction and analogy to carry out simple inferences, realize and know the function of plausible reasoning in mathematical discoveries (see Example 2 and Example 3).

(ii) Combined with real mathematics examples learned and real examples in everyday life, students realize the importance of deductive reasoning, master the basic format of deductive reasoning, and are able to use deductive reasoning to carry out some simple inferences.

(iii) Through concrete examples, students familiarize with the connection and differences between plausible reasoning and deductive reasoning.

(2) Direct proof and indirect proof

(i) Combined with real mathematics examples learned, students familiarize with the two basic methods of direct proof – analytic method and synthetic method; familiarize with the thinking processes and characteristics of analytic method and synthetic method.

(ii) Combined with real mathematics examples learned, students familiarize with one basic method of indirect proof – method of proof by contradiction; familiarize with the thinking processes and characteristics of method of proof by contradiction.

(3) Mathematical culture

(i) Through real examples introduce the followings: Euclid's Elements, Marx's Capital, Jefferson's Declaration of Independence, and Newton's Three Laws. Realize the thinking of axiomatization.

(ii) Introduce function of computer in areas of automatic deduction and mathematical proof.

**3. Extension of number system and introduction of complex number (about 4 class hours)**

(i) Familiarize with process of extension of number system in problem contexts; realize the contribution of practical needs and the inner contradiction within mathematics (operation rules of numbers, theory of equation) to the process of extension of number system; feel the function of human rational thinking and the connection of mathematics and realistic world.

(ii) Understand basic concepts of complex number, as well as the sufficient and necessary conditions of the equality of complex numbers.

(iii) Familiarize with algebraic representation method of complex number, and its geometric meanings.

(iv) Able to carry out arithmetic operations of complex numbers in algebraic representation; familiarize with the geometric meanings of the addition and subtraction operations of complex numbers in algebraic representation.

**4. Block Diagrams (about 6 class hours)**

(1) Flow charts

(i) Through concrete real examples, students know procedural block diagram further.

(ii) Through concrete real examples, students familiarize with procedural block diagram (i.e. overall planning diagram or critical path diagram; see Example 4, Example 5).

(iii) Able to draw the flowcharts of simple practical problems, and realize the function of flowchart in solving practical problems.

(2) Structural Diagrams

(i) Through real examples, familiarize with structural diagram; able to use structural diagram to organize knowledge already learned and information previously collected.

(ii) Combined with the structural diagram constructed engage in exchanges with others; realize the function of structural diagram to reveal interrelationships of events and objects.

**Remarks and Recommendations**

1. During teaching of exemplary statistical cases, teachers should encourage students to involve in the processes of data processing, develop students' intuitive feeling of data, recognize the characteristics of statistical methods (e.g. mistakes may be made in statistical inference; the randomness of outcomes of estimation), and realize the immense applications of statistical methods. Teachers should try their best to provide students opportunities for practical activities. They can integrate with mathematical modeling activities, select one exemplary case and require students to practice it personally. Regarding contents of exemplary statistical cases, teachers require students to familiarize with basic ideas and preliminary applications of a few statistical methods. There is no demand on the theoretical foundation so as to avoid students to engage in sheer memory and mechanical application of formulae in calculations.

2. During teaching, teachers should encourage students to use calculators, computers and other modern technologies to process data. For those schools that are better resourced, teachers can deploy some common statistical software to solve

practical problems.

3. During teaching, teachers should guide students to use plausible reasoning to explore and guess some mathematical conclusions through the use of real examples. Teachers should also use deductive reasoning to ascertain the correctness of conclusions obtained, or to use counter-examples to overthrow wrong conjectures. The emphasis is to understand plausible reasoning and deductive reasoning through the use of concrete examples, and not to quest for abstract representation of concepts.

4. Content included in this module comprises of a summary of basic methods of proof. During teaching, teachers should guide students to recognize the characteristics of different methods of proof through the use of real examples so that students realize the necessity of proof. Regarding skills of proof, there is no need to demand too much from the students.

5. Teaching of block diagrams should start from analyses of real examples. Teachers should guide students to use block diagrams to represent the main considerations and procedures of mathematical computation and proving process, as well as the flow of tasks of practical problems, and structural relationships of some system of mathematical knowledge. Through the use of block diagrams, students are enabled to understand characteristics of flowcharts and structural diagrams, master the usage of block diagrams, and experience the superiority of using block diagrams to handle processes of problem solving.

6. During teaching of concept of complex number and its operations, teachers should pay attention to avoid complicated computation and skills training. For those students who are interested, teachers can arrange some contents that allow for extension, e.g. find the roots of $x^3 = 1$, and introduce basic theorems of algebra.

**Exemplary Cases**

**Example 1** The probability that sheep in some regions are infected by some disease is 0.4. A sheep is ill or not is mutually independent of each other. Now, one new medicine for the prevention of this disease is under experimentation. Five sheep are randomly chosen. The result is that all five sheep were not infected. The question to ask is whether this medicine is effective or not.

*At first sight, one may draw the conclusion that the medicine is definitively effective. This is because all the sheep were not infected after taking the medicine. But, if we ponder further, we find that there are problems. This is because the majority of sheep will not be infected even they do not take medicine, and the proportion of sheep that are infected is only 0.4. That all five sheep are not inflected is not due to the effects of the medicine. One natural thought in the analysis of this problem is: if the medicine is not effective, the probability that five sheep randomly drawn are infected is not great. If the probability that this event happened is very small and that it may be* *considered as almost not likely to happen at all, then the fact that all five sheep were not infected allow us to draw the conclusion that the medicine ought to be effective, i.e. the medicine is effective.*

*Assume that the medicine is not effective, probability that the five sheep are not infected is:*

<insert formula here>

*This probability is very small and the probability that the event happened is negligible. But the event did happen, indicating that our assumption is not correct and that the medicine is effective.*

*The thought behind the analyses resembles that of proof by contradiction. But this is not so. After we make the assumption, we discover that an event happened even though the probability of happening is so negligible to the extent that it almost does not happen. In this way, we have falsified our assumption.*

*What needs to point out is that, when we draw the conclusion that "the medicine is effective", we have to be aware that we may make mistakes. The probability that this mistake happened is 0.078. That is to say, we have around 92% confidence that the medicine is effective.*

**Example 2** Explore and find the quantitative relationships between the number of faces, vertices and sides of a convex polyhedron (discovery of the Euler's formula).

**Example 3** An analogy between circles on a plane and spheres in space.

| Concepts in plane geometry | Similar concepts in solid geometry |
|---|---|
| Circle | Sphere |
| Tangent of circle | Tangent plane of sphere |
| Chord of circle | Circular cross-section of sphere |
| Circumference of circle | Surface area of sphere |
| Area of circle | Volume of sphere |

| Properties of circle | Properties of sphere |
|---|---|
| Line joining the center of circle and mid-point of chord (not diameter) is perpendicular to the chord. | Line joining the center of sphere and center of circular cross-section (those small circular cross-sections that do not pass through center of sphere) is perpendicular to the circular cross-section. |
| Chords equidistant to the center of circle are equal; chords non-equidistant to the center of circle are not equal; chord is | Circular cross-sections equidistant to the center of sphere are equal; circular cross-sections non-equidistant to the |

| | |
|---|---|
| longer when its distance to the center of circle is nearer. | center of sphere are not equal; circular cross-section is larger when its distance to the center of sphere is nearer. |
| …… | …… |

**Example 4**

There are altogether three procedures in the processing of some parts in a factory: rough-processing, re-processing and fine-processing. After completion of each procedure, there is a need to inspect the product. Product that passes through inspection proceeds to the stage of fine-processing, whereas those product which fails to pass through needs re-processing. Re-processed product that passes through inspection proceeds to the stage of fine-processing, whereas those product which fails to pass through is regarded as rejected product. Product that passes through inspection is rated as finished product, and those product which fails to pass through is regarded as rejected product. Please use a flowchart to represent processing of parts.

<insert flowchart here> arrival of parts, rough-processing, inspection, pass, re-processing, re-inspection, fail, rejected product, fine-processing, final inspection, finished product.

**Example 5** The flowchart of the process of mathematical induction is shown below:

<insert flow chart here> (practical contexts, pose problems, mathematical models, mathematical results, tests, usable results, not conforming to practical situations, modify)

According to this flowchart, combined with concrete real examples, state the process of mathematical modeling.

## Series 2

### Option 2-1

In this module, students shall learn common terminologies of logic, conic section and equation, vectors in space (abbreviated as space vectors) and solid geometry.

Using terminologies of logic correctly is a basic quality possessed by all modern citizens of a society. Irrespective of engaging in thinking, communication and various kind of work, we need to use terminologies of logic correctly to represent our own thinking. In this module, based on the foundation of the stage of obligatory education, students learn common terminologies of logic, realize the function of terminologies of logic in expression and argumentation, deploy these terminologies of logic to express mathematical contents correctly, and to engage in communication better than before.

In this module, based on the foundation of studying preliminary plane analytical geometry in the compulsory curriculum, students shall learn conic section and equation, familiarize with the relationships between conic section and quadratic equation, master the basic geometric properties of conic section, feel the function of conic section in depicting the realistic world and solving practical problems. Combined with the real examples of curves and the equations acquired, students are familiar with the correspondence relationship between curve and equation, and proceed to feel the basic idea of integration of numbers and shapes.

The use of space vectors provides a new perspective to process solid geometric problems. Introduction of space vectors provides a very effective tool to solve problems involving metric and positional relationships of figures in three-dimensional space. In this module, based on the foundation of learning plane vectors, students extend plane vectors and their operations to the three-dimensional space. Students deploy space vectors to solve problems of positional relationships related to straight line and plane, realize the function of vector method in studying geometric figures, and proceed to develop abilities of spatial imagination and geometric intuition.

**Contents and Requirements**

**1. Common Terminologies of Logic (about 8 class hours)**

(1) Propositions and their relationships

(i) Familiarize with converse proposition, negative proposition and converse-negative proposition of a proposition.

(ii) Understand the meanings of necessary condition, sufficient condition, and sufficient and necessary condition; able to analyze the inter-relationships of the four kinds of proposition.

(2) Simple logical connectives

Through mathematical examples, familiarize with the meanings of logical connectives: “or”, “and”, “not”.

(3) Universal quantifier and existential quantifier

(i) Through rich real examples in mathematics and everyday living, understand the meanings of universal quantifier and existential quantifier.

(ii) Able to negate a proposition containing one quantifier correctly.

**2. Conic Section and Equation (about 16 class hours)**

(1) Conic section

(i) Familiarize with conic section and its practical background; feel the function of conic section in depicting the realistic world and solving of practical problems.

(ii) Involve in the process of abstracting an ellipse model from concrete situations (see Example 1); master definition of an ellipse, standard equation and its simple geometric properties.

(iii) Familiarize with definition, geometric shape and standard equation of a hyperbola, and know related properties of hyperbola.

(iv) Able to use coordinates method to solve some simple geometric problems related to conic section (positional relationship between straight line and conic section) and practical problems.

(v) Through learning conic section and equation, students proceed to realize the idea of integration of numbers and shapes.

(2) Curve and equation

Combined with real examples of curves and their equations acquired, students are familiar with the correspondence relationship between curve and equation, and proceed to feel the basic idea of integration of numbers and shapes.

**3. Space Vectors and Solid Geometry (about 12 class hours)**

(1) Space vectors and their operations.

(i) Involve in the process of extending vectors and their operations from plane to space.

(ii) Familiarize with concept of a plane vector; familiarize with basic theorems and associated meanings of space vectors; master orthogonal decomposition of space vectors and representation of their coordinates.

(iii) Master linear operation of space vectors and representation of their coordinates.

(iv) Master the inner product of space vectors and representation of their coordinates; able to use the inner product of vectors to judge whether vectors are collinear or perpendicular.

(2) Application of space vectors.

(i) Understand direction vector of straight line and normal vector of a plane.

(ii) Able to use vector language to express the perpendicular and parallel relationships between two straight lines, a straight line and a plane, and two planes.

(iii) Able to use vector method to prove some theorems related to positional relationships of lines and planes (including theorem of three perpendiculars; see Example 1, Example 2, and Example 3).

(iv) Able to use vector method to solve computational problems involving included angles between two straight lines, a straight line and a plane, and two planes, and realize the function of vector method in studying geometric problems.

**Remarks and Recommendations**

1. During teaching of common terminologies of logic, teachers should pay particular attention to the following problems:

(i) Propositions considered here refer to those with clearly specified conditions and conclusions. Students need only to become familiar with converse proposition, negative proposition and converse-negative proposition of a proposition. The emphasis is on the interrelationships amongst the four types of propositions, as well as the necessary condition, sufficient condition, and sufficient and necessary condition of a given proposition.

(ii) Regarding the meanings of logical connectives “or”, “and”, and “not”, students are only required to become familiar with them through real examples in mathematics. Teachers should help students to express correctly related mathematics contents.

(iii) Regarding the use of quantifiers, the emphasis is on the understanding of their meanings. There is no need to chase after formal definitions of quantifiers.

(iv) During the process of guiding students to use common terminologies of logic, attention should be paid to enable students to master usage of common terminologies of logic, to correct logical mistakes appeared, and to realize the accuracy and clarity regarding the use of common terminologies of logic to express mathematical contents. Mechanical memorization of terminologies of logic and abstract explanations should be avoided. The use of truth table is not required.

2. When introducing conic section, teachers should deploy rich examples (e.g. orbit of a planet, locus of a projectile, mirror surface of a torchlight) to enable students to become familiar with the background and application of conic section.

Teachers should demonstrate the process of obtaining an ellipse through cutting a cone by a plane so as to deepen students’ understanding of conic section. Whenever resources and conditions permit, schools should show sufficiently the function of modern educational technology and use computers to demonstrate the conic section obtained as a result of cutting a cone by a plane (see Example 1 of the exemplary cases in optional study 1-1).

3. Teachers can demonstrate practical applications of conic section, e.g. trajectory of a lead ball thrown, orbit of a satellite.

4. Teaching of curve and equation should be limited to those curves that have been learned before. Teachers should enable students to realize the correspondence relationship between curve and equation, and feel basic idea of integration of numbers and shapes. For those students who are interested, teachers can guide students to become familiar with eccentricity of conic section and the unified equation For those schools where resources and conditions permit, schools should exhibit sufficiently the function of modern educational technology. Using software, teachers can demonstrate to students the influences due to the changes of parameters on the curves represented by the equations. Students proceed further to understand relationship between curve and equation.

5. Regarding teaching of space vectors, teachers should guide students to use method of analogy so as to allow students to involve in the process of extending

vectors and their operations from the plane to the three dimensional space. Teaching of mathematics should pay attention to the influences brought about by the increase in the number of dimension.

6. During teaching, teachers can encourage students to choose between vector method and synthetic method so that they can solve solid geometric problems from different perspectives.

**Exemplary Cases**

**Example 1** In a given triangular prism $ABC - A_1B_1C_1$, $\angle ACB = 90^\circ$, $\angle BAC = 30^\circ$, $|BC| = 1$, $|AA_1| = \sqrt{6}$, $M$ is mid-point of side $CC1$. To prove: $AB_1 \perp A_1M$.

**Example 2** Given rectangle $ABCD$ is perpendicular to rectangle $ADEF$ using $AD$ as the common side, but they are not lying on the same plane. Points $M$, $N$ are located respectively on diagonals $BD$ and $AE$ such that $|BM| = 1/3 \ |BD|$, $|AN| = 1/3 \ |AE|$. Prove that $MN \parallel$ plane $CDE$.

**Example 3** In a given unit cube $ABCD - A_1B_1C_1D_1$, $E$ and $F$ are respectively the mid-points of sides $B_1C_1$ and $A_1B_1C_1D_1$. Find:

(i) the angle formed between $AD_1$ and $EF$;

(ii) the angle formed between $AF$ and plane $BEB_1$.

(iii) the size of the dihedral angle $C_1$-$DB$-$B_1$.

## Option 2-2

In this module, students shall learn derivative and its application, inference and proof, extension of number system and introduction of complex number.

Creation of calculus is a milestone in mathematics development. Its development and widespread application marked a new era of transition towards mathematics of the recent times. It furnishes important methods and strategies for studying variables and functions. Concept of derivative is a core concept in calculus and it has very rich practical background and widespread application. In this module, through a large quantity of real examples, students shall involve in processes depicting mean rate of change and instantaneous rate of change so as to understand the meaning of derivative. Students familiarize with the function of derivative in studying properties such as the monotone and extreme values of functions. Students familiarize with preliminary concept of definite integral and this would build up the foundation for further study of calculus. Through learning of this module, students realize the rich meaning implicit in the ideas of derivative, feel the function of derivative in solving practical problems, and are familiar with the cultural value of calculus.

“Inference and Proof” is a basic thinking process in mathematics. It is the most commonly used thinking method for study and everyday living. Inferences generally include plausible reasoning and deductive reasoning. Plausible reasoning is a deduction process involving prediction of some results based on known facts and correct conclusions (including definitions, axioms and theorems), results of experiments and practical work, as well as personal experiences and intuition. Induction and analogy are common plausible reasoning thinking method. During the problem solving process, plausible reasoning is useful for guessing and discovering conclusions, as well as exploring and providing lines of thoughts. This facilitates development of innovative consciousness. Deductive reasoning is a process involving drawing of new conclusions based on known facts and correct conclusions (including definitions, axioms and theorems), as well as those obtained through strict logical rules. Development and elevation of deductive reasoning or logical proof abilities are important objectives in the mathematics curriculum. Relationships between plausible reasoning and deductive reasoning are intimate and they reinforce each other. Proof generally comprises of logical proof and proof by experiments and practices. Correctness of mathematical conclusion has to be guaranteed through logical proof, i.e. through correct usage of inference rules to arrive at a conclusion based on the foundation of correct premises. In this module, through recall of knowledge already learned, students proceed to realize plausible reasoning, deductive reasoning, as well as the connection and differences between these two types of reasoning. Students realize characteristics of proof, familiarize with basic methods of mathematical proof, including methods of direct proof (e.g. analytic method, synthetic method, mathematical induction) and methods of indirect proof (e.g. prove by contradiction). Students should feel the function of logical proof in mathematics and everyday living, develop habits such that proposals are reasonable and arguments are supported by evidences.

Process of extension of number system exhibits the discovery and creation processes in mathematics. At the same time it exhibits the objective demand and background of the formation and development of mathematics. Introduction of complex number at the stage of senior secondary education is another extension of number system. In this module, students shall familiarize with processes of extension of number system in problem contexts, as well as the necessity of introducing complex numbers. Students learn some basic knowledge of complex number and realize the function of rational thinking in the extension of the number system.

**Contents and Requirements**

**1. Derivative and its application (about 24 class hours)**

(1) Concept of derivative and its geometric meaning

(i) Through analyses of a large quantity of real examples, students involve in processes transiting from mean rate of change to instantaneous rate of change, familiarize with concept of derivative and its practical background, know that instantaneous rate of change is derivative, and realize the ideas and inner meanings of derivative (see Example 2, Example 3 in exemplary cases section of optional study 1-1).

(ii) Through graph of functions understand intuitively the geometric meaning of derivative.

(2) Operation of derivatives

(i) In accordance with the definition of derivative, find the derivative of function $y = c, y = x, y = x^2, y = x^3, y = 1/x, y = \sqrt{x}$.

(ii) Able to use the formulae of derivatives of given basic elementary functions and arithmetic operations to find the derivative of simple functions; able to find the derivative of composite function (limited to functions of the form $f(ax + b)$ ).

(iii) Able to use table of derivative formulae.

(3) Application of derivative in studying function

(i) Combined with real examples, borrow geometric intuition to explore and familiarize with the relationship of monotonicity of a function with its derivative (see Example 4 in exemplary cases section of optional study 1-1); able to use derivative to study the monotonicity of functions, and able to find the monotone intervals of polynomial functions with degrees not exceeding 3.

(ii) Combined with graph of a function, familiarize with the necessary condition and sufficient condition that a function takes on extreme values at some points; able to use derivative to find the maximal value and minimal value of a polynomial function with degree not exceeding 3, as well as to find the maximum values and minimum values of a polynomial function with degree not exceeding 3 within a closed interval; realize the generality and effectiveness of method of derivative in studying properties of a function.

(4) Examples of optimization problem in everyday life

For example, through maximization problems such as maximization of profit, minimization of material use, and highest efficiency, students realize the function of derivative in solving practical problems (see Example 5 in exemplary cases section of optional study 1-1).

(5) Basic theorems of definite integral and calculus

(i) Through real examples (e.g. find the area of a trapezium with curved sides, work done by forces), students become familiar with the practical background of definite integral in problem contexts; borrow geometric intuition to realize basic ideas of definite integral, and become familiar with preliminary concepts of definite integral.

(ii) Through real examples (e.g. the relationship between distance traveled and velocity of an object moving with variable velocity within a certain time period), students familiarize intuitively with the meaning of the basic theorem of calculus (see Example 1).

(6) Mathematical culture

Collect background information of the era when calculus was created, as well as information of related leading figures, and engage in exchanges; realize the meaning and value of calculus on development of human culture. For concrete requirements, please see the section "mathematical culture" in *Standards* (see page ???)

**2. Inference and Proof (about 8 class hours)**

(1) Plausible reasoning and deductive reasoning

(i) Combined with real mathematics examples acquired and real examples in everyday life, students familiarize with the meanings of plausible reasoning, able to use induction and analogy to carry out simple inferences, as well as realize and know the function of plausible reasoning in mathematical discoveries (see Example 2, Example 3 in exemplary cases section of optional study 2-2).

(ii) Combined with real mathematics examples acquired and real examples in everyday life, students realize the importance of deductive reasoning, master the basic format of deductive reasoning, and are able to use deductive reasoning to carry out some simple inferences.

(iii) Through concrete examples, students familiarize with the connection and differences between plausible reasoning and deductive reasoning.

(2) Direct proof and indirect proof

(i) Combined with real mathematics examples acquired, students familiarize with the two basic methods of direct proof – analytic method and synthetic method; familiarize with the thinking processes and characteristics of analytic method and synthetic method.

(ii) Combined with real mathematics examples acquired, students familiarize with one basic method of indirect proof – method of proof by contradiction; familiarize with the thinking processes and characteristics of method of proof by contradiction.

(3) Mathematical induction

Familiarize with the principle of mathematical induction; able to use mathematical induction to prove some simple mathematical propositions.

(4) Mathematical culture

(i) Through real examples introduce the followings: Euclid's Elements, Marx's Capital, Jefferson's Declaration of Independence, Newton's Three Laws. Realize the thinking of axiomatization.

(ii) Introduce the function of computer in areas of automatic deduction and mathematical proof.

**3. Extension of number system and introduction of complex number (about 4 class hours)**

(i) Familiarize with process of the extension of number system in problem contexts; realize the contribution of practical needs and the inner contradictions within mathematics (operation rules of numbers, theory of equation) to the process of extension of number system; feel the function of human rational thinking and the connection of mathematics and realistic world.

(ii) Understand basic concepts of complex number, as well as the sufficient and necessary conditions of equality of complex numbers.

(iii) Familiarize with algebraic representation method of complex number, and its geometric meaning.

(iv) Able to carry out arithmetic operations of complex numbers in algebraic representation; familiarize with the geometric meanings of the addition and subtraction operations of complex numbers in algebraic representation.

**Remarks and Recommendations**

1. In this module, concept of derivative is introduced through practical background and examples of concrete applications. In teaching, teachers can guide students to involve in processes transiting from mean rate of change to instantaneous rate of change and know that derivative is instantaneous rate of change. Students can study real examples of application of derivative such as growth rate, expansion rate, efficiency, density and velocity. Through feeling the function of derivative in studying function and solving practical problems, students realize ideas of derivative and its inner meanings. Treatment of special topic in this way aims to help students understand the background, thinking and function of derivative intuitively.

2. In teaching, teachers should prevent treating derivative merely as studying some rules and steps and ignore its ideas and values. Teachers should enable students to recognize that any rate of change of events can be described using derivative.

3. During the process of solving concrete problems, teachers should guide students to make a comparison between the derivative method and elementary methods when studying function so that students can realize the generality and validity of derivative method in studying properties of function.

4. During teaching, teachers should guide students to use plausible reasoning to explore and guess some conclusions in mathematics through the use of real examples. Teachers should also use deductive reasoning to ascertain the correctness of conclusions obtained, or to use counter-examples to overthrow wrong conjectures. The emphasis is to understand plausible reasoning and deductive reasoning through the use of concrete examples, and not to quest for abstract representation of concepts.

5. Contents included in this module comprise of a summary of basic methods of proof. During teaching, teachers should guide students to recognize the characteristics of different methods of proof through the use of real examples so that students realize the necessity of proof. Regarding skills of proof there is no need to demand too much from the students.

6. Teachers should make use of concrete real examples to enable students to become familiar with the principle of mathematical induction. There is a need to control the difficulty level of proof.

7. During teaching of concept of complex number and its operations, teachers should pay attention to avoid complicated computation and skills training. For those students who are interested, teachers can arrange some contents that allow for extension, e.g. find the roots of $x^3 = 1$, and introduce basic theorems of algebra.

**Exemplary Cases**

**Example 1** An object follows the pattern of $s = s(t)$ to move along a straight line. We already know that at any moment $t_0$ its velocity $v(t_0)$ (instantaneous velocity or instantaneous rate of change) is the derivative of $s = s(t)$ at $t_0$, i.e. $v(t_0) = s'(t)$. Now consider $s(t)$, its overall change of distance from $t = a$ to $t = b$. We divide the interval $a$ □$t$□$b$ into $n$ small intervals. We can also assume that the length of each interval is equal to each other, the length is □$t_i$. Considering any one small interval, we assume that the rate of change of $s(t)$ approximates to a constant. Therefore, we can say:

$$\triangle s \sim \text{rate of change of } s(t) \times \text{time}$$

In the first interval, from $t_0$ to $t_1$, we assume that the rate of change of $s(t)$ approximates to $s'(t_0)$. We have:

$\triangle s_0 \sim s'(t_0)$ □$t_0$, □$t_0 = t_1 - t_0$

Similarly, in the second interval, from $t_1$ to $t_2$, we assume that the rate of change of $s(t)$ approximates to $s'(t_1)$. Therefore, we have:

$\triangle s_1 \sim s'(t_1)$ □$t_1$, □$t_1 = t_2 - t_1$, etc.

We add all the approximate values of changes in position obtained from all the small intervals together and obtain:

total change of $s$ = <insert formula here>

We can write the total change of position of $s(t)$ at $t_0 = a$ to $t_n = b$ as $s(b) - s(a)$. On the other end, when the divisions are smaller and smaller when $n$ tends to infinity, the limiting value of summation:

<insert summation formula here>

is the definite integral <insert integral> or <insert integral>. This corresponds to the total change of position of $s(t)$ from $t = a$ to $t = b$. Therefore, we have the following conclusion:

$s(b) - s(a)$ = <insert integral> =<insert integral>

That is to say, the rate of change of the definite integral gives the total change.

In particular, when object moves at uniform velocity, i.e. $v(t) \equiv v$,

$s(b) - s(a) = v\,(b - a) =$ <insert integral>

When object is moving at uniform acceleration, i.e. $v(t) \equiv \alpha t$ ($\alpha$ is a constant),

$s(b) - s(a) =$ <insert formula> = <insert integral>

Generally, when $f(t)$ is a continuous function, and $f(t) = F'(t)$, then

<insert integral formula>

This is known as the basic theorem of calculus. The proof given here is not rigorous. Nevertheless, it reflects the basic idea of the basic theorem of calculus, and reflects the connection between differentiation (derivative) and integral.

## Option 2-3

In this module, students shall learn enumeration principle, statistical cases and probability.

Problem of enumeration is one of the important research targets in mathematics. Enumeration principles, classified and fractional, are the most basic and important methods of solving enumeration problems. These two principles are also called basic enumeration principles, because they provide ideas and tools for solving many practical problems. In this module, students shall learn enumeration principles, permutation, combination, binomial theorem and its applications, familiarize with the connection between enumeration and realistic everyday living. They are able to solve simple enumeration problems.

Based on the foundation of probability learning in the compulsory curriculum, students shall learn some discrete random variable distribution columns and associated contents of mean and variance. Students begin to learn to use ideas of discrete random variable and methods to describe and analyze some random phenomena. At the same time, students are able to use the acquired knowledge to solve some simple practical problems, and proceed to realize the function of probability models and characteristics of using probability to think about problems. Students begin to form consciousness of using conception of randomness for observing and analyzing problems.

Based on the foundation of statistics learning in the compulsory curriculum, and through discussion of typical exemplary cases, students are familiar with some common statistical methods and their applications, proceed to realize the basic ideas of using statistical methods to solve practical problems, as well as know the function of statistical method in decision making.

**Contents and Requirements**

**1. Enumeration Principle (about 14 class hours)**

(1) Classified enumeration principle, fractional enumeration principle

Through real examples, the classified enumeration principle and the fractional enumeration principle are summarized. Based on the characteristics of concrete problems, students are able to select classified enumeration principle or fractional principle to solve some simple practical problems.

(2) Permutation and combination

Through real examples, understand concepts of permutation and combination; able to use enumeration principle to derive formulae of number of permutations and number of combinations, and are able to solve some simple practical problems.

(3) Able to use enumeration principle to prove binomial theorem (see Example 1); able to use binomial theorem to solve simple problems related to binomial expansions.

**2. Statistics and Probability (about 22 class hours)**

(1) Probability

(i) Regarding analyses of concrete problems, students comprehend concepts of discrete random variable that takes on finite values and its distribution column, and know the importance of distribution column in depicting random phenomena.

(ii) Through real examples (e.g. lottery draws), students understand hypergeometric distribution and its derivation, and are able to carry out simple applications (see Example 2).

(iii) In concrete situations and contexts, students familiarize with conditional probability and concept that two events are mutually independent, comprehend model of $n$ times of independent and repeated trials and the associated binomial distribution, and are able to solve some simple practical problems (see Example 3).

(iv) Through real examples, understand concepts of mean value and variance of discrete random variable that takes on finite number of values, able to calculate mean value and variance of simple discrete random variable, and are able to solve some practical problems (see Example 4).

(v) Through practical problems, and borrowing intuition (e.g. histogram of practical problems), know characteristics of curve of normal distribution and the meaning it represents.

(2) Statistical exemplary cases

Through typical exemplary cases, students learn some of the common statistical methods, and are able to apply these methods preliminarily to solve some practical problems.

(i) Through exploration of typical exemplary cases (e.g. “Is there any relationship between lung cancer and smoking?”), familiarize with the basic ideas, methods and preliminary applications of test of independence (only 2 x 2 cross-tabulations are required).

(ii) Through exploration of typical exemplary cases (e.g. “Quality control”, and “Is the new medicine effective?”), familiarize with basic ideas, methods and preliminary applications of inference principle and hypothesis testing (see Example 1 of exemplary cases section in optional study 1-2).

(iii) Through exploration of typical exemplary cases (e.g. “Classification of insects”), familiarize with the basic ideas, methods and preliminary applications of cluster analysis.

(iv) Through exploration of typical exemplary cases (e.g. “Relationships between human weight and height”), familiarize with the basic ideas, methods and preliminary applications of regression.

**Remarks and Recommendations**

1. Classified enumeration principle and fractional enumeration principle are two basic methods of thinking that deal with the problem of enumeration. In teaching, teachers should guide students to analyze and process problems according to the enumeration principles, and mechanical application of formulae should be avoided. At the same time, in this part of teaching, teachers should avoid enumeration problems that are complicated and skill-based to an excessive extent.

2. The main aim of studying a random phenomenon is to familiarize with all possible outcomes happened and the probability of each outcome happened. Distribution column is used to describe the patterns and regularities of probability of values taken on by the discrete random variables. Binomial distribution and hypergeometric distribution are two probability models that have widespread applications. Teachers are required to introduce these two probability models through the use of real examples, and there is no need to pursue formalized description. During teaching, teachers should guide students to use some of the knowledge acquired to solve some practical problems.

3. During teaching of exemplary statistical cases, teachers should encourage students to involve in the processes of data processing, develop students’ intuitive feeling of data, recognize the characteristics of statistical methods (e.g. mistakes may be made in statistical inference, the randomness of outcomes of estimation), and realize the immense application of statistical methods. Teachers should try their best to provide students opportunities for practical activities. They can integrate with mathematical modeling activities, select one exemplary case and require students to practice it personally. Regarding contents of exemplary statistical cases, teachers can require students to familiarize with basic ideas and preliminary applications of a few statistical methods. There is no demand on the theoretical foundation so as to avoid students to engage in sheer memory and mechanical application of formulae in

calculations.

4. During teaching, teachers should encourage students to use calculators, computers and other modern technologies to process data. For those schools that are better resourced, teachers can deploy some common statistical software to solve practical problems.

5. Teachers can introduce ancient Chinese achievement in mathematics “Yang Hui Triangle” during the teaching of binomial theorem. Teachers can make use of the exemplary statistical cases to introduce the widespread application of statistical methods learned in daily living. This would enrich students’ knowledge of the value of mathematical culture.

**Exemplary Cases**

**Example 1** Binomial theorem and its proof

$(a + b)^n$ is multiplication of $(a + b)$ together $n$ times. When each $(a + b)$ is multiplied with others, there are two choices, i.e. choose $a$ or $b$. Using fractional enumeration principle, we know that there are $2^n$ terms in the expansion (including like terms), and each term is of the form $a^k b^{n-k}$, $k = 0, 1, \ldots, n$. For every term $a^k b^{n-k}$ , it is obtained by choosing $a$ in $k$ of the $(a + b)$, $b$ in $n$-$k$ of $(a + b)$. The number of times it occurs is equivalent to the number of combinations ${}_nC_k$ of choosing $k$ number of $a$ from $n$ number of $(a + b)$. When the like terms are grouped together, the binomial expansion is obtained. This is called binomial theorem.

**Example 2** One game is designed and played in the gala party of one class of senior secondary three students. One pocket contains 10 red balls and 20 white balls. These balls are all identical except for color. A player draws 5 balls out of the pocket, and wins the first prize when 4 red balls are drawn. What is the probability of winning the first prize?

The number of combinations of drawing 5 balls out of 30 balls is: ${}_{30}C_5$ = 142 506; Therefore,

$P$(first prize) = <insert equation here>

If $X$ represents the number of red balls drawn, then $X$ satisfies $N = 30$, $M = 5$, $n = 10$, $m = 4$ of the hypergeometric distribution. Therefore,

$P(X = m)$ = <insert equation here>

**Example 3** A fair coin is tossed randomly 100 times. This is equivalent to 100 repeated trials. For each trial there are two possible outcomes (i.e. Head and Tail). The probability that the Head comes out is 1/2.

If $X$ represents the number of times that Head comes out, then $X$ satisfies $n = 100$, p = 1/2 of the binomial distribution. Therefore,

$P(X = k)$ = <insert equation here>

From these we can see that: the probability of “tossing a fair coin randomly 100 times and 50 Heads come out" is $P(X = 50)$ = <insert equation here>.

One misconception when students learn probability is that since the probability of obtaining Head when a coin is tossed is 1/2, it would be certain that there are 50 Heads obtained when the coin is tossed 100 times, or the probability of this event happened should be very large. Calculation revealed that this probability is about 8% only.

**Example 4** According to weather forecast, the probability that there will be a small flood in some place next month is 0.25, and that the probability for a big flood is 0.01. Suppose that there is a piece of large-scale equipment in a construction site and there are three plans to protect it from the floods.

Plan 1: Remove the equipment. This costs 3800 dollars.

Plan 2: Construct a wall for protection. This costs 2000 dollars. But this wall cannot stop the big flood. When the big flood comes and the equipments are destroyed, the loss is 60000 dollars.

Plan 3: No plan at all. Hope that no flood will come. When there is a big flood the loss is 60000 dollars. When there is a small flood the loss is 10000 dollars.

Try to compare and decide which plan is better.

## Remarks of Series 3 and Series 4

Series 3 and 4 comprise of a number of special topics, each carries one credit.

Series 3 includes six special topics, namely, selected topics of history of mathematics, information security and cryptogram, the geometry of the sphere, symmetry and group, Euler's formula and classification of closed surfaces, trisection of an angle and extension of a number field. Series 4 consists of ten special topics, namely, selected topics of geometrical proofs, matrix and transformation, sequence and difference, coordinates system and parametric equation, selected topics of inequalities, elementary number theory, optimum seeking method and preliminary experimental design, overall planning (critical path) method and preliminary graph theory, risk and decision making, switching circuits and Boolean algebra.

The contents of series 3 and series 4 are relatively rich. Due to the development of the curriculum, these contents will be further extended, enriched and improved.

Contents involved in series 3 and series 4 are all foundation of knowledge of mathematics. Not only students who would like to further themselves in areas such as science and technology, and economics are encouraged to enroll, but also those who would like to develop in areas such as the humanities and social sciences are encouraged as well.

Series 4 and series 5 are meant for those who find mathematics interesting and those who would like to enhance their qualities in mathematics. All contents entailed are the basic knowledge of mathematics, and reflect some important mathematics thinking. Some special topics are extension of contents of the secondary curriculum, whereas some special topics introduce some methods of application through the use typical real cases. Learning of these special topics facilitates students' life-long learning, broaden students' horizon in mathematics, and elevate students' knowledge of scientific, application and cultural values in mathematics. Students' foundation in mathematics is further grounded and application awareness further elevated.

Treatment of special topics should be in-depth and easy to be understood by an ordinary person. It moves a step further to promote students' analytical and problem solving abilities, enable students to master and realize some important concepts, conclusions and methods of thinking, realize the functions of mathematics, and develop application awareness.

Regarding learning of series 3 and series 4, teachers should promote multifarious learning formats. Teachers can conduct lectures, or allow students to engage in autonomous exploration and cooperative exchanges under the guidance of the teachers, or encourage students to read independently and write the special topic summary reports themselves. Teachers should try to enable students to realize that "doing mathematics" is an effective means of learning mathematics well. Independent thinking is the foundation of "doing mathematics".

Evaluation formats of series 3 and series 4 are different. In accordance with the characteristics of series 3, evaluation of this part of learning should adopt a combination of quantitative and qualitative approaches.

## Series 3
## Selected Topics of History of Mathematics

**Contents and Requirements**

Through animating, rich cases, students familiarize with some important events during the mathematics development processes, important outcomes and important people, familiarize preliminarily with the processes of formation and development, realize the function of mathematics on human civilization, foster students' interests in mathematics learning, deepen students' understanding in mathematics, have a feel of the serious attitudes of mathematicians and the persevering spirits of exploration.

Complete a summary study report. Students should write their study reports on the historical trajectory of developments in mathematics, as well as historical events and leading figures felt interested by the students.

These special topics consist of a number of options, the contents of which reflect the characteristics of development of mathematics at different era. Historical facts should be emphasized, and through the historical facts mathematical thinking methods can be introduced. The number of topics chosen should not be less than six. The following special topics may be chosen for study.

1. Early arithmetic and geometry: Counting and measurement

* Mathematics recorded in Rhind Papyrus (ancient Eygpt)

* Mathematics recorded in clay tablets (along the two rivers district)

* Chinese Zhou Bi Suan Jing, Gou Gu Theorem (Zhao Shuang’s Diagram)

* Development of the decimal place value system

2. Ancient Greek mathematics

* Pythagoras polygonal numbers, from Gou Gu Theorem to Gou Gu numbers, problem of incommensurable quantities

* Euclid’s Element, deductive logic system, Fifth Postulate Problem, geometric construction using straight edge and compass, the deep influence of thinking of axiomatization on science of the recent times

* Archimedes’s work: mensuration

3. Precious stones of ancient mathematics in China

* Mathematics in Jiu Zhang Suan Shu (method equations, elimination by addition and subtraction, positive and negative numbers)

* Chinese remainder theorem (Sun Zi Theorem)

* Introduction of ancient Chinese mathematicians

4. Formation of plane analytical geometry: Marriage of numbers and shapes

* Functions and curves

* Meanings of methodology of Descartes

5. Formation of calculus: Ground-breaking achievement of the era

6. Two great mathematicians of the recent times: Euler and Gauss

* Euler’s intuition in mathematics

* Characteristics of mathematics in Gauss’s era (rigor in mathematics)

7. Age-old puzzles: Galois’s solution

* From Abel to Galois (a student mathematician)

* Three geometric construction problems

* Formation of algebra of the recent era

8. Cantor’s set theory: Thinking about infinity

* Infinite set and potency

* Russell's paradox and foundation of mathematics (Godel’s completeness theorem)

9. Development of randomized thinking

* Genesis of probability theory

* Opportunities provoking development of statistics of the recent times

10. Developmental processes of algorithmic thinking

* Historic background of algorithm

* Algorithm in computer sciences

11. Development of mathematics of the present times in China

* Brilliant historical processes of endeavors and struggles of modern Chinese mathematicians to achieve first rate standard in the world.

**Remarks and Recommendations**

1. There is no need to quest for systematic and complete account of history of mathematics development in these special topics. Through students' lively and animating languages and cases that are of interest to the students, students realize important ideas of mathematics and its developmental trajectories. Arrangement of contents of these special topics can adopt a variety of formats. Teachers can search for the history of mathematics development from ancient to the present times. Teachers can also start from realistic mathematics problems that are familiar to the students to pursue their roots and origins, as well as to recall important events and leading figures in mathematics development. For example, teachers can start from "How many methods of enumeration do we have?" and trace the history of the method of enumeration (Babylonian sexagesimal system, British duodecimal system, computer's binary system, decimal system, binary system and Chinese Ba Gua – Eight Trigrams). In addition, teachers can start from π which is very familiar to the students, and talk freely about the results done by Zu Chong Zhi, and use computers to compute π to as many digits after the decimal point as possible.

2. The contents provided above are just provided merely for selection. Teachers can arrange contents of the special topics appropriately in accordance with the practical situations. Teachers should emphasize the ideas implicit in the contents, trajectories of mathematics development, and the scientific spirits of painstaking endeavors of the mathematicians. Selection of contents should tie in with the levels of the students, and the display formats should be rich in both texts and diagrams so as to arouse interests in students.

3. Teaching methods should be multifarious and flexible. Teachers can adopt formats such as storytelling, discussion and exchanges, information search, as well as report writing. Teachers should encourage students to write down their study reports on the trajectories of history of mathematics development, and the historical events and leading figures felt interested by the students.

## Information Security and Cryptogram

Number theory and algebra have a lot of important applications in modern information theory and information security. This special topic will introduce some knowledge pertaining to elementary number theory (e.g. divisibility of numbers and congruence), as well as some important applications of number theory on modern information security. This would enable students to realize the application of mathematics in information sciences, elevate students' level of appreciation and interests in mathematics learning.

**Contents and Requirements**

1. Knowledge related to elementary number theory

(i) Familiarize with divisibility of numbers and congruence, complete residue system modulo *m* and reduced residue system modulo *m*, Euler's theorem and Fermat's Small theorem, factorization of large numbers.

(ii) Familiarize with definition and computational formulae of Euler's function, Wilson's theorem and its applications in the decision of prime numbers, primitive root and exponent, existence of primitive root modulo *p*, discrete logarithm problem.

2. Application of number theory in information security

(i) Familiarize with related concepts in communication security (e.g. plaintext, cyphertext, cipher key) and basic problems in communication security (e.g. confidentiality; digital signature; management, distribution and sharing of cipher keys).

(ii) Familiarize with one classic example of cryptogram: stream cipher (using method of congruence modulo *m*)

(iii) Understand public key infrastructure (concept of one-way function, as well as methods of encryption and digital signature (based on RSA's protocol of factoring of large numbers).

(iv) Understand the application of discrete logarithm on the exchange and distribution of cipher keys: Diffi-Hellman Protocol.

(v) Understand the application of discrete logarithm in encryption and digital signatures: El Gamal algorithm.

(vi) Familiarize with the application of Lagrange's interpolation formula in sharing of the cipher keys.

3. Complete a study summary report

The report should consist of two aspects of contents: (1) Summary of knowledge; knowledge and understanding of contents related to information security; realize the function of mathematics (number theory and algebra) in information security. (2) Extension; through reading extra-curricular materials, students proceed to engage in exploration and thinking of some contents and its applications.

**Remarks and Recommendations**

1. Editing of teaching materials and teaching of these special topics should be in-depth but easy to be understood. During teaching, teachers should pay attention to

the introduction of history of related contents (e.g. development of communication technology) and their backgrounds, help students understand problems needed to be solved in information security, know how to use public key infrastructure to solve these problems, realize ideas of factoring of large numbers, discrete logarithm and their influences on modern information security.

2. Where resources and conditions permit, teachers should guide students to use computer to think about the following problems, and carry out computer programming and experimentation.

(i) Calculate the greatest common factor using the division algorithm;

(ii) Solve the congruence equation $ax \equiv b \pmod{n}$;

(iii) Decide whether a given large whole number is prime or not (use Wilson theorem);

(iv) Factoring of large numbers.

## The Geometry of the Sphere

We are living on earth. The earth surface resembles a sphere very much. Therefore, in practical everyday living, there are plenty of applications of the knowledge of geometry of the sphere (abbreviated as spherical geometry). For example, measurement of the land (astronomical objects), aviation, and satellite positioning, amongst others, all need knowledge of the spherical geometry. Theoretically, spherical geometry is another geometric model of Euclidean plane geometry. It is a very important non-Euclidean mathematical model. Spherical geometry possesses special functions in theoretical research of geometry.

This special topic enables students to become familiar with a new mathematical model—spherical geometry. Students begin to learn some basic knowledge of spherical geometry and some of its practical applications. Through comparison of differences and connection between spherical geometry and Euclidean geometry, students feel that there exist in nature plenty of rich and colorful mathematical models. Analogy is an important method of thinking used in the learning of this special topic. Spatial imagination and geometric intuition are important abilities key to the learning of this special topic well.

**Contents and Requirements**

1. Through rich and practical problems (e.g. measurement, aviation, satellite positioning), students realize the necessity of introduction of knowledge of spherical geometry.

2. Through comparison of spherical figures and plane figures, students feel the differences and similarities of spherical geometry and Euclidean geometry. For example, the great circle on the sphere is equivalent to straight line on the plane and

the shortest spherical distance is the minor arc of the great circle. Theorem of power of a point with respect to a sphere can be compared with that of a circle.

3. Through analyses of real examples, students realize that spheres possess symmetric properties similar to that on the plane.

4. Familiarize with some basic figures on the spheres: great circle, small circle, spherical angle, spherical lune, pole and equator, spherical triangle, polar symmetric triangle of spherical triangles (abbreviated as polar spherical triangles).

5. Through comparisons of Euclidean geometry and spherical geometry, explore what properties of plane Euclidean figures may be extended to that on the sphere, and give reasons. Based on these, students understand Side-Side-Side, Side-Angle-Side, and Angle-Side-Angle Triangles Congruence Theorem (*SSS*, *SAS* and *ASA* for short.) on the sphere.

6. Understand the area formula ($S = A + B + C - \pi$) of the unit spherical triangle, and realize that the interior angles of a spherical triangle is greater than 180°.

7. Familiarize with Angle-Angle-Angle Triangle Congruence Theorem (*AAA* for short.) on the sphere.

8. Use the spherical triangle area formula to prove Euler's formula, experience the relationship between spherical geometry and topology.

9. Use vector product to explore and prove the spherical *cosine* theorem ($\cos c = \cos a \cos b + \sin a \sin b \cos C$) and the spherical Pythagorean theorem (i.e. corresponding to the case when $C = \pi/2$ in the spherical *cosine* theorem); able to derive the spherical *sine* theorem ($\sin A/\sin a = \sin B/\sin b = \sin C/\sin c$) from the spherical cosine theorem.

10. Experience that when the radius of the sphere increases indefinitely, the sphere approaches to that of a plane. Trigonometry formulae become corresponding formulae on the plane.

11. Begin to become familiar with another model of non-Euclidean geometry – Poincare model.

12. Complete a study summary report. The reports should comprise of three parts of contents: (1) Summary of knowledge. Regarding overall structure and understanding of contents of this special topic, students should state which formulae (or theorems) are similar in both spherical geometry and plane geometry, and which formulae are fundamentally different. Students are able to explain why a small piece of spherical surface after taking the radius into account can be envisaged as a plane. (2) Through information search, survey research, interviews and exchanges, independent thinking, students proceed to think about relationships between geometry and space in the realistic world. (3) Realization and feelings as a result of the learning of spherical geometry.

**Remarks and Recommendations**

1. The main points of this special topic are to develop students' spatial imagination and geometric intuition abilities.

2. During teaching, teachers should enable students to feel solidly that they can make use of knowledge of spherical geometry to solve (or explain) some practical problems in everyday living and production. During the introduction of spherical geometry, teachers should enable students to arrive at related conclusions of spherical geometry through an analogy of plane Euclidean geometry and spherical geometry. Students are enabled to think about similarities and differences between plane geometric models and non-Euclidean geometric models (such as those of spherical geometry).

3. Introduce spherical geometry and Euler's formula. The main aim is to broaden students' horizon and enable them to become familiar with some non-Euclidean models. This helps students master modern methods of mathematical thinking greatly.

4. Spherical geometry entails a great deal of symmetry properties (transformations) of spatial figures. For those schools where resources and conditions permit, teachers can use CAI multi-media technologies as sufficiently as practicable.

## Symmetry and Group

Symmetry, such as axisymmetry and central symmetry, is a very important property in nature. Group is a mathematical concept used for depicting symmetry, and group theory is an important research target in modern mathematics.

Starting with real examples of symmetric transformation of plane figures, students become familiar with concepts of transformation group, learn methods of expression of a group, learn how to find out symmetric groups of some relatively simple geometric figures, and proceed to realize the important function of groups on study of symmetric property of events and objects, as well as study of other mathematical objects.

**Contents and Requirements**

1. Through rich symmetric figures, students feel that everyday life and the realistic world are replete with a large quantity of symmetric phenomena.

2. Familiar with motion of a rigid body and its basic properties.

3. Through analyses of different types of symmetry of figures and motion of rigid bodies, attempt to obtain ideas used in depicting symmetry of different figures.

4. Combined with simple concrete figures, try to find out all symmetric transformations.

5. Combined with real cases of concrete figures, form concept of composition of symmetric transformation progressively, and understand the closeness of composition

of symmetric transformation.

6. Combined with real cases of concrete figures, know through operations that symmetric transformation satisfy associative law.

7. Combined with real cases of concrete figures and through operations, understand concept of identical transformation, concept of inverse transformation and its properties, and able to find an inverse transformation of a symmetric transformation of some concrete figures.

8. Through concrete real examples, establish concept of transformation group, and proceed to become familiar with concept of an abstract group.

9. Able to borrow geometric intuition to find out some geometric figures and some symmetric groups of simple chemical molecule models that possess some degrees of symmetry.

10. Through concrete examples, familiarize with one method of group representation – method of multiplication table.

11. Starting from concrete real examples, familiarize with one method of constructing relatively complicated groups from relatively simple groups – direct product.

12. Familiarize with some important applications of group theory in realistic everyday living, such as crystal classification theorem.

13. Examine other forms of symmetric transformation, such as algebraic expressions. Through processes of solving equations of second and third degrees, familiarize with the meaning of the symmetric group of roots of an algebraic equation, as well as familiarize with the scientific historical fact that Galois has used method of group theory to arrive at the solution of an equation in surd form, and feel the important function of group theory in modern mathematics.

14. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of overall structure and contents of this special topic, description of mathematics of symmetry and knowledge of concept of a group. (2) Through information search, survey research, interviews and exchanges, and independent thinking, students proceed to explore the popularity of symmetry in nature and the function of group to depict symmetry. (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

1. Because of the fact that symmetric transformation, and operations related to composition of transformations (multiplication) are relatively abstract concepts, there is a need to learn these through the use of concrete real examples and appropriate contexts. Teachers should not start from the abstract definitions.

2. As far as the secondary students are concerned, group is a totally new object for

study. Symmetric transformation group treats symmetric transformation as an operational system for purposes of scrutiny, and this is very much different from the operational system of numbers and algebraic expressions studied. Therefore, in this special topic, teachers can only use relatively simple concrete groups as examples. The main point of teaching is to enable students to become familiar with the function of group in depicting symmetry, and try hard not to explicate the abstract definitions and properties of group. At the same time, through analyses of concrete geometric figures, students are required to find out symmetric group of some simple geometric figures, and have a feeling of the meanings of group during the processes of their operational practices.

3. Classification of crystals and Galois's theory of equation are two important results of application of group theory. In this special topic, teachers may not be able to prove in detail the crystal classification theorem and Galois's theorem of equations. But teachers can introduce these two results so as to enable students to feel the research methods and characteristics of modern mathematics. Therefore, to do well this introductory job is one aim of teaching of this special topic.

## Euler's Formula and Classification of Closed Surfaces

Using transformation to classify geometric figures is an important topic in geometry. It discloses that under different transformations invariant properties and quantities of geometric figures are basic thinking methods of this type of problem. This special topic discusses primarily Euler's formula and invariant topological quantities such as Euler characteristic so as to deploy them to classify curves and curved surfaces.

**Contents and Requirements**

1. Revise transformations already learned, and use them to classify plane figures.

(i) Revise transformations such as translation, rotation, motion of a plane, reflection, congruence, homothety, magnification and contraction, similarity, as well as classification of plane figures.

(ii) Explore what geometric properties are invariant in the above transformations.

(iii) Realize some basic characteristics of transformations: one-one correspondence, continuity.

2. Euler's formula

(i) Through processes of exploring and discovering Euler's formula, understand Euler's formula.

(ii) Understand topological proof of Euler's formula.

(iii) Deploy Euler's formula to solve some problems (e.g. exploration of the number of regular polyhedrons).

(iv) Explore relationships amongst number of faces, edges and vertices of non-Euler polyhedrons.

3. Understand concepts of triangulation of curved surfaces.

4. Able to undertake triangulation of some curved surfaces, and calculate their Euler characteristics.

5. Familiarize with the intuitive meaning of topological transformation.

6. Know some invariant topological quantities, and is able to use them to classify curves and closed surfaces; familiarize with the results of classification of some curves and closed surfaces.

7. Familiarize with some applications of topological thinking (e.g. cabling on a plane problem, one-stroke drawing problem, Brouwers fixed-point theorem and point of economy stability problem, and the four color problem).

8. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of overall structure and contents of this special topic, as well as knowledge of thinking of mathematical transformation; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to understand invariant quantities of transformation and ideas on the classification of curved surfaces; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

1. This part of content is rather abstract. Students should first revise the geometric transformation learned at the secondary stage and analyze the invariant geometric properties of these transformations. As a result, students realize ideas of transformation and invariant quantities of transformation.

2. Teachers should guide students to explore processes of the discovery of Euler's formula, as well as understanding of the proof of Euler's formula. Teachers should help students realize the creative work done by the mathematicians. This special topic is a very good exemplar.

3. Triangulation is a very important thinking method used for studying topological properties of figures. Teachers should guide students to involve in processes of using the triangulation method to study properties of curved surfaces. Through operations and practices, students learn and master the thinking methods of triangulation.

4. Topological transformation is a very abstract concept. Teachers should pay attention to students' images of topological transformation and intuitive understanding. For example, students can visualize topological transformation as rubber sheet transformation. Teachers should not guide students to quest for formal definition of topological transformation.

5. Teachers should pay attention to the introduction of topological thinking method when introducing application of topology. There is no need for a rigorous exposition.

### Trisection of an Angle and Extension of a Number Field

Trisecting the angle, doubling the cube, and squaring the circle are known as the three great ancient Greek geometric problems. There are important meanings underlying the thinking methods of solving these problems, and these are not only present in mathematics, but also in the history of human thinking as well.

This special topic, through discussing the trisecting the angle problem, enables students to become familiar with the basic thinking method of solving this type of problem. At the same time, students can make use of this method to handle the doubling the cube problem, as well as to tackle the problem that it is not possible to have a regular heptagon constructed merely using compass and straight edge. On the other hand, teachers can introduce how to use algebraic method to discuss the viability of constructing a regular heptadecagon (i.e. one can use compass and straight edge geometric construction method to construct a regular heptadecagon). Through the above discussion, students are able to realize and understand the mathematics thinking methods implicit in the problems, and to have the analytical and problem solving abilities elevated.

**Contents and Requirements**

1. Familiarize with the three ancient Greek geometric construction problems. Through trisecting the angle problem, familiarize with how the problem is posed. Under the prerequisite that there is no limitation on the use of compass and straight edge, students become familiar with several different methods for solving the trisecting the angle problem.

2. Understand the basic considerations of solving the trisecting the angle problem – depicting the scope of geometric constructions using compass and straight edge.

3. Given line segments $a$, $b$, able to use compass and straight edge geometric construction method to construct line segments $a + b$, $a – b$, $ab$, and $a/b$.

4. Given any specified line segment and treated this segment as a measurement unit, any positive rational number can be geometrically constructed (i.e. one can construct a line segment with length equal to that rational number using compass and straight edge only).

5. Through closed operations of addition, subtraction, multiplication and division of rational numbers, familiarize with concepts of the rational number field and the general number field.

6. Let $F$ is a number field, <insert formula>. Prove that <insert formula> is also a number field, and $F$ is a subset of the set <insert formula>. Familiarize with concept of field extension.

7. Give some concrete examples of number fields and extension fields.

8. Given a line segment of length $a$, able to use compass and straight edge geometric construction to construct a line segment of length $\sqrt{a}$.

9. Able to tackle the trisecting the angle problem algebraically.

10. Proof: It is not possible to use compass and straight edge geometric construction method to trisect an angle of 60°.

11. Use the above method to discuss the doubling the cube problem, as well as the problem that it is not possible to have a regular heptagon constructed merely using compass and straight edge.

12. Realize the thinking method underlying the solution of the three great ancient Greek geometric construction problems and the function they play on the understanding of human thinking.

13. Familiarize with the de Moivre formula of complex number multiplication. Able to use algebraic method to discuss the viability of the geometric construction of a regular heptadecagon (i.e. one can use compass and straight edge geometric construction method to construct a regular heptadecagon).

14. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; basic considerations for the solution of the problem of trisecting the angle, and clear explication of the proving process; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to realize methods of tackling geometric problems algebraically and ideas of handling geometric construction problems; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

1. The ideas behind this special topic and requirements on exposition of proofs are rather high. Teachers should demand students to learn to grasp the overall considerations of problem solving. During proving, students should be demanded to be systematic and clear in thoughts. Teachers should develop students' abilities of expression and argumentation.

2. During the teaching process, teachers should guide students to explore some problems related to this special topic.

3. Through studying this special topic, students know that the function of mathematics is not limited to problem solving. Instead, mathematics exerts its important influences on the world views and helps the formation of correct human thinking methods.

**Series 4**

**Selected Topics of Geometrical Proofs**

Selected geometric proofs are helpful for developing students' logical deduction abilities. The geometric proving processes not only involve logical deduction procedures, but also include a large quantity of observations, explorations, and processes of creative discoveries. This special topic starts by revising the properties of similar figures so as to prove some important theorems on the relationships of circles and straight lines. Through further exploration of the properties of conic sections, students' spatial imagination, geometric intuition, and application of synthetic geometric methods for problem solving are elevated accordingly.

**Contents and Requirements**

1. Revise definitions of similar triangles and their properties, and familiarize with the triangle proportionality theorem. Prove the projection theorem of a right angled triangle (i.e. the perpendicular height theorem and the theorem of the sides adjacent to the right angle).

2. Proof the central angle theorem (also called inscribed angle theorem), as well as the decision and property theorems of the tangent of a circle.

3. Proof the intersecting chords theorem, the property theorem and decision theorem of an inscribed cyclic quadrilateral, the tangent chord theorem.

4. Familiarize with the meaning of parallel projection. Through positional relationships of the cylinder and the plane, realize parallel projection. Prove that the intersecting curve between the plane and cylinder surface is an ellipse (the special case is a circle).

5. Through observation of how a plane intersects with the conical surface, realize the following theorem:

**Theorem:** In space, use line $l$ as axis, the straight line $l'$ intersects $l$ at point $O$, the included angle is $\alpha$, $l'$ revolves around $l$, and a conics with $O$ as vertex and $l'$ as generator is formed. Take any plane $\pi$, if the angle enclosed between this plane and $l$ is $\beta$ (when $\pi$ and $l$ are parallel, denote $\beta = 0$), then:

(1) $\beta > \alpha$, plane $\pi$ and the conics intersect to form an ellipse;

(2) $\beta = \alpha$, plane $\pi$ and the conics intersect to form a parabola;

(3) $\beta < \alpha$, plane $\pi$ and the conics intersect to form a hyperbola.

<insert figure here>

6. Using the Dandelin's double balls (both balls are situated inside the cone, one ball is placed on top of the plane $\pi$, another is placed below $\pi$, both balls are tangent with the plane $\pi$ and the conics), prove the situations as described in theorem (1).

7. Try to prove the following results: (i) In note 6, one Dandelin ball intersects

with the conics to form a circle, and this is parallel with the base of the cone, denote the plane where this circle is situated is π'; (ii) if the line of intersection between plane π and plane π' is *m*, then select any one point on the ellipse in note 5(1), and the tangent point between the Dandelin ball and the plane π is *F*, then the ratio of the distance between point *A* and *F* and the distance between point *A* to the straight line *m* is a constant *e* which is less than 1. (*F* is called focus of the ellipse, *m* is called the directrix of the ellipse, and the constant *e* is called eccentricity).

8. Explore the proof of theorem (3), realize the limit of the results of π when β approaches α indefinitely.

9. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of overall structure and contents of this special topic, as well as knowledge of mathematical proofs; (2) Extension. Through information search and independent thinking, students proceed to explore some contents of this special topic and its applications; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

Editing and teaching of this special topic should be treated in depth but easy to understand. Regarding proof of the two propositions stated in notes 6 and 7 and other related contents, there are rich ideas of methods of thinking underneath. These are useful for helping students realize the function of spatial imagination and geometric intuition abilities in problem solving, as well as helping elevate students' ability in deploying geometric knowledge in an integrated manner to solve problems. During teaching, teachers should encourage students to engage in independent thinking, autonomous attempts and explorations. If necessary, teachers can provide appropriate guidance, encourage students to write a summary project report, and try as far as possible to express clearly their thinking and argumentation processes.

For those schools where resources and conditions permit, teachers can use modern computer technology to display dynamically the Dandelin's double ball method, and help students use geometric intuition to engage in reasoning.

## Matrix and Transformation

Matrix is a basic tool used for studying transformation of figures (vectors). It has widespread applications and many mathematical models can be represented using matrices.

Through transformation of plane figures, this special topic discusses multiplication and properties of square matrices of second order, as well as concepts such as inverse matrix and characteristic vector of a matrix. This topic would use transformation and mapping perspectives to enable students to understand the meaning of system of linear equations, and make a start to demonstrate the widespread application of matrices.

**Contents and Requirements**

1. Introduction of second order matrices.

2. Multiplication of second order matrix with a plane vector (column vector), transformation of plane vectors.

(i) Use perspective of mapping and transformation to know the meaning of multiplication of a matrix with a vector.

(ii) Prove that matrix transforms a straight line on a plane into a straight line, i.e. to prove:

$$\boldsymbol{A}(\lambda_1\boldsymbol{\alpha} + \lambda_2\boldsymbol{\beta}) = \lambda_1\boldsymbol{A}\boldsymbol{\alpha} + \lambda_2\boldsymbol{A}\boldsymbol{\beta}$$

(iii) Through transformation of given figures (e.g. square) on a plane using a large quantity of matrices, know that matrices may be used in the following transformations: identity, reflection, dilation, rotation, shear transformation, and projection.

3. Composition of transformations – Multiplication of second order square matrices.

(i) Through real examples of transformations, familiarize with the meaning of multiplication of two matrices.

(ii) Through transformation of concrete geometric figures, state that matrix multiplication does not satisfy the commutative law.

(iii) Verify that multiplication of second order matrices satisfies associative law.

(iv) Through concrete transformation of geometric figures, state that multiplication does not satisfy cancellation law.

4. Inverse matrix and second order determinant

(i) Through transformation of concrete figures, understand the meaning of an inverse matrix; through concrete projection transformation, state that inverse matrix may not exist.

(ii) Able to prove the uniqueness of inverse matrix and simple properties such as $(\boldsymbol{AB})^{-1} = \boldsymbol{B}^{-1}\boldsymbol{A}^{-1}$, and become familiar with their meanings in transformations.

(iii) Familiarize with the meaning of second order determinants; able to use second order determinant to find the inverse of a matrix.

5. Second order matrix and system of linear equations in two unknowns

(i) Able to use various viewpoints of transformation and mapping to know the meanings of system of linear equations.

(ii) Able to use the inverse matrix of a coefficient matrix to solve system of equations.

(iii) Through concrete coefficient matrices, able to state from the geometric viewpoint the existence and uniqueness of solutions of system of linear equations.

6. Invariant quantities of transformation

(i) Master the meanings of characteristic values of a matrix and characteristic vectors; able to explain the meanings of characteristic vectors from the geometric transformation viewpoint.

(ii) Able to find the characteristic values and characteristic vectors of a second order square matrix (characteristic values are in the form of two different real numbers only).

7. Application of matrices

(i) Use the characteristic values and characteristic vectors of a matrix $\boldsymbol{A}$ to come up with a simple representation of $\boldsymbol{A}^n\boldsymbol{\alpha}$, and is able to use this to solve problems.

(ii) Familiarize preliminarily with matrices of third or higher orders.

(iii) Familiarize with the application of matrices.

8. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of overall considerations, structure and contents of this special topic, and proceed to know the ideas of transformation; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to explore matrix transformation and its applications; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

1. Only concrete second order square matrices are discussed in this special topic. There is no need to discuss the general $m$ x $n$ order matrix, as well as the $(a_{ij})$ form of representation.

2. Introduction of matrix should start from concrete real examples. Through concrete real examples, students know that some geometric transformations can be represented using matrices. This would enrich students' understanding of the geometric meanings of matrices. Students are guided further to use the mapping viewpoint to know matrix and solution of system of linear equations.

3. Students are requested to understand multiplication of matrices intuitively through transformation of figures. Through concrete real examples, students understand the multiplication laws of matrix multiplication.

4. Using concrete real examples, students should understand the practical meaning of an inverse matrix, characteristic values, and their invariant properties. Combined with concrete examples, students are able to use system of linear equations or determinants to solve the inverse matrix and characteristic values of a simple second order matrix. The uniqueness theorem of inverse matrix should be sensibly understood within the context of concrete geometric transformations.

5. At the same time when the fundamental knowledge of second order matrix is studied, teachers can introduce promptly some extension knowledge of matrix in accordance with the realistic teaching situations (e.g. third or higher order matrices). Students are not required to master these contents. They need to have some affective knowledge only. These will help students become familiar with related knowledge of matrices comprehensively, and facilitate their learning in the future.

6. This part of contents should enable students to know that matrix arises because of the needs of practical everyday living. Matrix has widespread application in practical problems. Experiencing abstract mathematics helps people think about problems and their solutions.

## Sequence and Difference

Due to the popularity and development in information technology, applications of discrete mathematics are increasingly widespread. Recurrence relations and difference equations are important tools used for the description of variations of discrete variables. From the theoretical point of view, they are important topics which have widespread applications.

This special topic studies preliminarily recurrence relations and simple difference equations, so that students can master some methods of discrete variable analyses for problem solving purposes.

**Contents and Requirements**

1. Recurrence relations of a sequence

(i) Through some concrete examples, understand concepts of recurrence relation of a sequence.

(ii) Understand first and second order recurrence relation of a sequence, and the meanings of them in describing variations of a sequence. Combined with the graph of the sequence (treated as a function), familiarize with recurrence relation and sequence relationships (i.e. increasing and decreasing, extremal values, concavity and convexity of the graph of sequence).

2. First order linear difference equation $x_{n+1} = k\,x_n + b$

(i) Through some concrete real examples, realize that $x_{n+1} = k\,x_n + b$ is a very important mathematical model.

(ii) Understand that in the equation $x_{n+1} = k\,x_n + b$, when $b = 0$ (when equation is homogenous), its solution is a geometric progression; when $k = 1$ (when difference is a constant), its solution is an arithmetic progression.

(iii) Know the general solution and particular solution of $x_{n+1} = k\,x_n + b$, familiarize with the relationship of solution of equation and that of the corresponding homogeneous equation $x_{n+1} = k\,x_n$; able to give the general formula of the solution of

$x_{n+1} = k\, x_n + b$.

3. System of first order difference equations in two unknowns (<insert equations>)

(i) Through some real cases, know that system of first order difference equations is a very important model for describing the realistic world.

(ii) Familiarize with the general solutions and particular solutions of first order difference equations, as well as relationships with the general solutions of the corresponding homogeneous equations.

(iii) Given an initial value, able to use iteration method to find the solution of system of first order difference equation; able to write down the flowchart of the solution algorithm.

(iv) Given a system of concrete equations, able to proceed to discuss when $n$ approaches infinity the tendency of changes of the solution of the sequence (convergence, divergence, and period).

4. Through concrete real examples (e.g. growth of species), realize that difference equation $x_{n+1} = k\, x_n\, (1 - x_n)$ is a very useful mathematical model. Borrowing computational tool, use the method of iteration to discuss the changes of $x_n$ corresponding to different special values of $k$ (e.g. $0 < k \leqq 1$, $1 < k \leqq 3$, $k$=3.4, $k$=3.55, $k$=3.7); begin to become familiar with the complexity of non-linear problems.

5. Application

(i) Able to use difference equation and system of difference equations to solve some simple practical problems.

(ii) Realize preliminarily thinking of discretization of continuous variables, and able to use it to discuss some simple problems.

6. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of overall considerations, structure and contents of this special topic, and knowledge of mathematical methods of depicting changes of discrete variables; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to explore difference equations and their applications; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

1. Regarding teaching processes and editing of teaching materials, teachers should help students understand concepts of recurrence relations and meanings of difference equations through the use of a large quantity of real examples. There is a need to treat the contents in depth and easy to be understood by the students.

2. Through discussion of first order linear difference equation, students are able to understand the structure of solutions of difference equation, i.e. the relationships

amongst the general solution, particular solution and the general solution of homogeneous equation. This would not only solve the system of difference equation, but also are helpful to proceed to learn system of linear equations and ordinary differential equation.

3. Teachers should pay attention to students' abilities of using difference equations to solve practical problems. In particular, teachers should encourage students to establish difference equation from realistic problems, and to integrate with practical problems to guide students to discuss the practical meaning of solutions.

4. Method of iteration is one of the commonly used mathematical methods to solve problems. Teachers should enable students to integrate with concrete problems to realize the meaning and function of method of iteration.

5. During the process of learning concepts of recurrence relations, teachers should be conscious of contrasting concepts of recurrence relations and derivatives. Students should master meaning and function of concepts of recurrence relations, and proceed to become familiar with the idea of discretization of continuous variables.

## Coordinates System and Parametric Equations

Coordinate system is the foundation of analytical geometry. In this coordinate system, we can use ordered pairs of real numbers to ascertain the position of points, and proceed to use equations to depict geometric figures. In order to facilitate the use of algebraic method to depict geometric figures and describe natural phenomena, there is a need to establish different coordinate systems. Polar coordinate system, cylindrical coordinate system, spherical coordinate system are all different from the rectangular coordinate system. For some geometric figures, the use of these systems may simplify the construction of equations.

Parametric equation uses parameters as a means to represent the coordinates of equation of points of a curve. It is another way of representation of the same curve using coordinates system. It is more convenient for some curves to be represented as equations in parametric form than the ordinary equation form. Learning of parametric equation can help students proceed to realize the flexibility of mathematical methods in problem solving.

This special topic is an integrated application and consolidation of contents of preliminary geometry, plane vectors and trigonometry. Polar coordinates system and parametric equations are important contents of this special topic. Regarding cylindrical coordinates system and spherical system, only simple familiarization is required. Through learning of this special topic, students shall master basic concepts of polar coordinates and parametric equations, familiarize with the various representation of curves, realize the process of abstracting mathematical problems from practical problems, develop interests and abilities of exploration of mathematics problems, realize the value of mathematics in practical applications, and elevate application consciousness and practical abilities.

**Contents and Requirements**

1. Coordinates system

(i) Recall methods of depicting the positions of points in the plane rectangular system, and realize the function of coordinate system.

(ii) Through concrete examples, familiarize with the changing situations of plane figures due to the influence of dilation transformation in plane rectangular coordinates system.

(iii) Able to use polar coordinates to depict position of points in a polar coordinates system; realize the difference between polar coordinates system and rectangular coordinates system in depicting position of points; able to convert from polar coordinates into rectangular coordinates, and vice versa.

(vi) Able to produce the equation of simple figures in the polar coordinates system (e.g. straight line that passes through the pole, circle that passes through the pole or with centre coincides with the pole). Through comparing the equations of these figures in the polar coordinates system and rectangular coordinates system, students realize the meanings of selecting appropriate coordinates system in depicting plane figures using equations.

(v) Borrowing concrete real examples (e.g. seats of the spectator stand of a circular sports stadium, longitudes and latitudes of Earth), students become familiar with methods of depicting points in space using the cylindrical coordinates system and spherical coordinates system. They are able to compare these methods with those using the rectangular coordinates system, and realize the differences amongst them.

2. Parametric Equations

(i) Through analyzing the relationship of time and position of moving objects in motion of projectiles, able to write down the parametric equations of the locus of projectile, and realize the meanings of the parameters.

(ii) Analyze the geometric properties of straight line, circle, and conic section, and select appropriate parameters to write down their parametric equations.

(iii) Give examples to illustrate that it would be more convenient to represent some curves using parametric equations than ordinary equations, and feel the superiority of parametric equations).

(iv) Borrowing teaching aids and computer software, observe locus of fixed point of a circle rolling on a straight line (i.e. cycloid) and locus of fixed point of a straight line rolling on a circle (i.e. involute); familiarize with processes related to the formation of cycloid and involute, and is able to derive their parametric equations.

(v) Through reading materials, familiarize with the formation processes of other types of cycloids (amplitude of variation of cycloid, amplitude of variation of involute, epicycloid, hypocycloid, ring cycloid); familiarize with real examples of cycloids in practical applications (e.g. Brachistochrone is a cycloid, ellipse is a special kind of hypocycloid – Cardanao's rotating disks, circular cycloidal gears and involute gears, principles of cycloid and design of mechanical devices and installations such as harvesters and ploughs, astroid and the door of public buses); familiarize with the function of cycloids in depicting the locus of motion of planets.

3. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of overall considerations, structure and contents of this special topic, proceed to know ideas of integrating numbers and figures, and think about relationships of this special topic with other contents in the senior secondary curriculum; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to explore applications of parametric equations and cycloids; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

1. Teaching of coordinates system should emphasize that students are able to use ordered pairs of numbers (coordinates) to depict the position of points on plane and in space. In different coordinates systems, the meanings exhibited by these numbers are all different. There are different formats of the equation of the same geometric figure when it is represented in different coordinates system. Therefore, it would be more convenient to present the equation of figure by selecting appropriate coordinates system.

2. In the teaching of coordinates system, teachers can guide students to attempt to establish the coordinates system themselves, state the rules of establishing the coordinates system, stimulate students' divergent thinking and creative thinking, and use concrete real examples to state clearly what conveniences are incurred as a result of using particular coordinates system.

3. Students should introduce parametric equations through analyses of concrete physical phenomena (e.g. locus of motion of projectiles) so that students can become familiar with the function of parameters.

4. Teachers should pay attention to encourage students to use knowledge already acquired, such as plane vector and trigonometric function, and to choose appropriate parameters to establish the parametric equation of curves.

5. Teachers can organize students to form interest groups so that they can cooperate to study properties of cycloid, and collect real examples of application of cycloid.

6. Teachers can use computers to display cardioids, spirals, rose curves, foliums, cycloids and involutes so as to enable students to appreciate the beauty of these curves.

**Selected Topics of Inequalities**

There are large quantities of unequal and equal relationships in nature. Unequal relationships and equal relationships are basic mathematical relationships. They play important functions in mathematics research and applications.

This special topic shall introduce some important inequalities and their applications, mathematical induction and their simple applications. This special topic focuses in particular on the geometric meaning and background of inequalities and their proofs. This would deepen students' understanding of mathematical essence of inequalities, and increase students' logical thinking abilities and problem solving abilities.

**Contents and Requirements**

1. Recall and revise the basic properties of inequalities and some basic inequalities.

2. Understand the geometric meaning of absolute value; able to use the geometric meaning of inequalities involving absolute values to prove the following inequalities:

(i) $|a+b| \square |a| + |b|$;

(ii) $|a-b| \square |a-c| + |\mathrm{c}-b|$;

(iii) Able to use geometric meaning of absolute value to solve the following types of inequalities:

$|ax+b| \square c$;

$|ax+b| \geqq c$;

$|x-c| + |x-b| \geqq a$.

3. Know the different forms of Cauchy Inequality; understand its geometric meanings.

(i) Proof of Cauchy inequality in vector format: $|\boldsymbol{\alpha}|\ |\boldsymbol{\beta}| \square |\boldsymbol{\alpha} \cdot \boldsymbol{\beta}|$

(ii) Proof: $(a^2+b^2)(\mathrm{c}^2+\mathrm{d}^2) \geqq (ac+bd)^2$

(iii) Proof: <insert formula here>

(This is commonly known as Plane Triangle Inequality).

4. Use method of completing squares of parameters to discuss the general situation of Cauchy inequality.

<insert formula here>

5. Use vector recursive method to discuss Rearrangement Inequality.

6. Familiarize with the principle of mathematical induction and its scope of application; able to use mathematical induction to prove some simple problems.

7. Able to use mathematical induction to prove Bernoulli Inequality:

$$(1 + x)^n > 1 + nx \ (x > -1, n \text{ is a positive whole number}).$$

Familiarize with the situation that Bernoulli inequality holds when n is a real number.

8. Able to use the above inequalities to prove some simple problems; able to use Arithmetic-Geometric Mean Inequality and Cauchy Inequality to evaluate the limiting values of particular functions.

9. Through some simple problems students become familiar with some basic methods of proving inequalities: comparison, synthesis, analysis, contradiction, and magnification.

10. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; summary of mathematics thinking methods and mathematics background implicit in inequalities introduced in this special topic; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to explore applications of inequalities; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

During teaching of this special topic, teachers should guide students to familiarize with that there are in-depth meaning and background associated with important inequalities. For example, there are clear geometric backgrounds associated with inequalities mentioned in this special topic. During learning, students should master these geometric backgrounds, and understand the real properties of these inequalities.

Algebraic identical transformations, as well as methods of magnification and reduction, are commonly used in proving inequalities. For example, methods of comparison, synthesis, analysis, contradiction and magnification, under many circumstances need predecessors to create techniques for us. For those people who would like to engage in research in some mathematics areas it is important for them to grasp these skills. But, for the majority who would like to learn inequalities, it is often very difficult for them to grasp the essential properties of mathematics from these complicated algebraic identical transformations. What is more important for these people is to understand the mathematics thinking and background of these inequalities. Therefore, this special topic strives to use geometry and other methods to prove these inequalities so as to enable students to understand these inequalities more easily, as well as the mathematics thinking of proofs. Difficulty of identical transformations, and skills in particular, should not be excessively demanded. Teaching of inequalities should not be drowned in excessive formalization and skills of complicated identical transformations. Teachers and editors of teaching materials are advised not to choose those algebraic identical transformations that are complicated, or those problems or exercises that are too much skill-based.

3. Mathematical induction is a very important method of thinking. Through analyses of some simple problems, teachers should help students grasp this kind of thinking method. When using mathematical induction to solve problems, students often need to undertake some algebraic identical transformation. Teachers and editors of teaching materials are advised not to choose those algebraic identical transformations that are complicated, or those problems or exercises that are too much skill-based so as to avoid diluting understanding of ideas of mathematical induction.

## Elementary Number Theory

Number theory is not only age-old but also fundamental mathematics. At present, there are still a large number of questions remained to be solved. Some solved problems play an important function to propel the development of modern mathematics, and form new important branches that are directly related to mathematics. These new branches have important applications in modern information technology. In our everyday life, we often encounter problems related to number theory.

Students of this special topic shall learn knowledge related to whole numbers and divisibility through the use of concrete problems. Students explore how to use the division algorithm to evaluate simple indeterminate equation, simple congruence equation, and system of congruence equations. Students realize the thinking methods therein and become familiar with some important achievements of ancient mathematics in China.

**Contents and Requirements**

1. Through real examples (e.g. week), students know division algorithm, understand concepts and meanings of congruence and residue class, explore the operational properties of residue class (addition and multiplication), and understand its practical meanings. Students realize the differences and similarities between operations of residue class and traditional number operations (e.g. occurrence of zero divisors).

2. Understand concepts of divisibility, factors and prime numbers, familiarize with methods that ascertain prime numbers (Eratosthenes' sieve); know that there are infinitely many prime numbers.

3. Familiarize with the test of divisibility of whole number in decimal system representation; explore the test of divisibility of whole numbers by 3, 9, 11 and 7; able to check mistakes in whole number operations (addition and multiplication).

4. Through real examples use division algorithm to explore the method of finding the greatest common divisor of two whole numbers; understand the concept of

relative prime; use method of division algorithm to prove that: if *bc* is divisible by *a*, and *a*, *b* are relative prime, then *c* is divisible by *a*. Explore the properties of common divisors and common multiples. Familiarize with the fundamental theorem of arithmetic.

5. Through real examples understand the model of linear indeterminate equation; use division algorithm to solve linear indeterminate equation. Also, attempt to write down the procedural block diagram of the algorithm, and wherever conditions permit demonstrate the algorithm on the computer.

6. Through real examples (e.g. Chinese remainder theorem) to understand the model of system of linear congruence equations.

7. Understand Chinese remainder theorem (Sun Zi Theorem) and its proof.

8. Understand Fermat little theorem (when $m$ is a prime number, $a^{m-1} \equiv 1 \pmod{m}$) and Euler theorem ($a^{\phi(m)} \equiv (\bmod\ m)$), where $\phi(m)$ is the number of numbers amongst 1, 2, …, $m-1$ that are relatively prime with $m$), as well as their proofs.

9. Familiarize with the application of number theory in cryptogram – public cipher key.

10. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of the contents and overall structure of this special topic, as well as knowledge of basic properties of positive whole number and its research methods; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to explore application of number theory; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

1. Although students are relatively conversant with the operations of expression of factorization of whole number, they are actually not that familiar with the theory underlying these operations. Teachers may explicate only some of the main methods and their properties. Other properties may be completed by the students themselves through discussion and autonomous exploration.

2. The idea of moving from particular solution to the general solution in the demonstration of Sun Zi Theorem is the same as that of establishing Lagrange interpolation formula. Therefore, inclusion of the interpolation formula is helpful to increase students' attention so that they are conscious of the connection with related contents.

3. Zero divisors may appear in the ring of the residue classes, and these are good for broadening the computational viewpoints of the students. Since understanding of this topic may be difficult for the students, teachers are advised to decide themselves whether to arrange activities for students to explore the issue further.

4. Methods and properties of the divisibility of polynomials are almost parallel to that of whole numbers. Teachers can arrange students to explore the issue further. Division of polynomial in vertical format is one effective method to carry out division of polynomial, and it is similar to that of the whole number. Teachers can list these as appendices.

### Optimum Seeking Method and Preliminary Experimental Design

In production and scientific experiments, people seek to achieve objectives of best qualities, high production, and low consumption. They need to choose the best combination of related factors (abbreviated as the best point), and optimum seeking method deals with the choice of best combination (the best point) problems. In the many practical situations, it may be very difficult to express relationships of experimental results and factors in mathematical forms, or the expression is very complicated. Optimum seeking method and experimental design are commonly used mathematical methods to solve these problems. During the 60's in the twentieth century, the renowned mathematician Hua Luo Geng personally organized and promoted the use of optimum seeking method. This method has been used widely in the industrial departments all over the country, and the results were also very fruitful.

Speaking simply, optimum seeking method is a mathematical method dealing with reasonable ways of arranging experiments so as to find the best point quickly. Experimental design is also a kind of mathematical method. Generally speaking, it considers methods of arranging experiments under conditions of multiple factors. It helps us obtain better combination of factors and better designs through the use of a smaller number of experiments.

This special topic will combine with concrete real examples to introduce in a preliminary fashion one-factor and two-factor optimum seeking methods, as well as method of orthogonal experimental design involving multiple factors. Teachers explain these methods in simple terms and help students understand the basic thinking of these methods, as well as think about how to use these methods to solve simple practical problems.

**Contents and Requirements**

1. Through rich exemplary cases in everyday living and production, students are able to feel the large quantity of optimum seeking problems in realistic everyday living.

2. Through analyses and solving of concrete practical problems, students are able to master fractional method and 0.618 method, as well as understand the scope of their applications. Students can use computers (or calculators) to carry out experiments, and think about and attempt to use these methods to solve some practical

problems so as to realize the thinking behind optimum seeking method.

3. Familiarize with Fibonacci sequence{ $\boldsymbol{F}_n$ }. Understand that under the condition that frequency of experiments is known the method of fraction can be proved to be the best amongst known methods. Through continued fraction students know the relationship between $\boldsymbol{F}_{n-1}/\boldsymbol{F}_n$ and golden section.

4. Through some concrete real examples, students are able to know the bisection method, climbing method, batch experimentation method, as well as processing methods dealing with multimodal situations using objective function.

5. Through rich real examples, students become familiar with optimum seeking problems of multiple factors. Also, they become familiar with the use of some methods of optimum seeking to handle two-factor problems, and proceed to realize the thinking method of optimum seeking.

6. Through rich exemplary cases in everyday living and production, students are able to feel the large quantity of experimental design problems in realistic everyday living.

7. Through analyses of concrete exemplary cases (number of factors does not exceed 3, and their levels do not exceed 4), students understand the process of using method of orthogonal experimental design to solve simple problems. Students familiarize with the ideas and methods of orthogonal experimental design, and are able to use this method to think about and solve some simple practical problems.

8. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of the contents and overall structure of this special topic, as well as knowledge of experimental design methods and their meanings; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to explore and extend some contents, some results and their applications in greater scope and depth; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

1. This special topic requires students to master some optimum seeking methods. Although rigorous mathematical proof is not required, students should seek to understand the thinking and essential properties underlying these methods.

2. Since this is an application-oriented course, whenever conditions and resources permit, teachers should allow students to do some experiments themselves so that they can master these methods better.

3. Students are able to recognize that they should discuss to adopt effective methods according to the concrete situations of problems, and to integrate these methods with specialized knowledge of concrete problems. At the same time, students are able to compare advantages, disadvantages, and scope of applications of different

methods.

**Overall Planning (Critical Path) Method and Preliminary Graph Theory**

Overall planning method is a basic method in operational research. It is one of the most important methods in modern project management. Through real examples, this special topic shall introduce overall planning method and its application, as well as basic concepts of graph. Teachers provide algorithms of shortest path on graph and the minimum spanning tree so as to enable students to become familiar with graph theory and its applications.

**Contents and Requirements**

1. Methods of overall planning

(i) Through real examples familiarize with thinking of overall planning problems and their widespread applications.

(ii) Through real examples understand basic concepts of overall planning method.

(iii) Through real examples master methods of drawing overall planning (critical path) diagrams.

(iv) Able to compute the parameters of the overall planning (critical path) diagrams: earliest start time of activities, latest finish time of activities, duration of task sequence.

(v) Able to find the critical paths in the overall planning (critical path) diagrams, and master the algorithms of finding critical paths; understand the importance of critical paths.

(vi) Able to use overall planning (critical path) method to analyze and process simple practical problems.

2. Preliminary graph theory

(i) Through real examples familiarize with basic concepts of graphs and the function of graphs in depicting relationships of practical problems.

(ii) Through real examples familiarize with spanning tree of a graph; master the algorithms of spanning tree of a graph and minimum spanning tree.

(iii) Through real examples familiarize with problem of shortest path of a graph; master algorithm for finding the shortest path of graph.

(iv) Familiarize with other problems in graph theory, and know the complexity of algorithms.

3. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of the overall considerations and structure of contents or part of contents (overall planning method and graph theory) in this special topic; knowledge of the mathematical thinking methods implicit

in the contents; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to explore and extend some contents, some results and their applications in greater scope and depth; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

1. Overall planning (critical path) method can be applied widely. During learning students are not only required to master that method but also develop application consciousness. Students are able to integrate with practical everyday living, and are conscious of collecting practical problems that may be tackled by the overall planning method.

2. Teachers should allow students to recognize that during solving of practical problems some complicated factors may arise (randomness of time, variation of cost, deployment of manpower). Some ready-made method may not be totally applicable and there is a need to combine with other mathematical tools to handle the problems.

3. In teaching of preliminary graph theory, teachers on one hand should allow students to recognize the importance of studying these models and that graphs and networks are important mathematical models of practical problems, and on the other hand, emphasize the introduction of algorithms. Students are required to express clearly these algorithms and become familiar with the issue of complexity of algorithm.

## Risk and Decision Making

In our everyday living and economic activities, such as personal procurement, job hunting, investment, industrial and commercial plans for production or execution, and departmental or national plans for some endeavors, there are needs to report progress for purposes of decision making. Follow-up actions that are most beneficial can be adopted accordingly. Since progress of events and information are often influenced by random factors, it may not be certain to make speculation and the decisions made are often carrying some risks. In such situations, decision makers often have a number of action plans to adopt. Statistical decision making method can furnish optimal action plan and this would decrease the loss immensely due to blind decision making. Therefore, statistical decision method and statistical decision making analyses will increasingly contribute immensely to development and progress of a society.

In a modern society, citizens should possess reasonable decision making abilities. Therefore, students at the secondary stage of schooling should master knowledge and methods of some simple statistical decision making methods and form preliminary decision making consciousness. This special topic is established with this aim in mind.

**Contents and Requirements**

1. Through analyses of real examples in everyday living and economic activities, students are conscious of risks, understand the necessity and importance of decision making under risks, as well as concepts of decision making under risks.

2. Through real examples students understand loss/gain functions and loss/gain matrices, explore ways and methods of decision making, and understand the meanings of conclusion of decision making.

3. Learn how to use decision tree to represent related information of problems needed decision making, able to use decision tree in reverse to carry out decision making.

4. Through real examples understand sensitivity analyses of decisions under risks; able to carry out sensitivity analyses of decision making.

5. Through real examples familiarize with Markov decision processes and the associated decision making methods.

6. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of the overall considerations, structure and contents, as well as knowledge of methods and meanings of decision under risks; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to explore and extend some contents, some results and their applications in greater scope and depth; (3) Realization and feelings as a result of the learning of this special topic.

**Remarks and Recommendations**

1. The whole special topic should be expanded using a number of real examples so as to help students understand the widespread applications of decision making. Students understand that results of decision making are associated with risks, and these results are valid and possess practical meanings. Concepts introduced should be kept to a minimum. Teachers should introduce this special topic using concrete examples, and extend and summarize on this foundation when necessary.

2. Students should understand concepts of decision making under risks using real examples, and learn preliminary methods of decision making. Students can then proceed through concrete examples to introduce loss/gain functions and loss/gain matrices, understand the function of decision tree, as well as master decision making methods using decision tree.

3. Through decision making processes students encounter probability estimation and imprecision. As a result, they understand the necessity of sensitivity analyses.

4. Markov decision models have widespread applications. Through examples, senior secondary students can understand and master these methods. Teachers should

not expand this special topic at the level using general theories and methods.

### Switching Circuits and Boolean Algebra

One basic characteristic of mathematics is concerned with high level of abstraction that is associated symbols and formalization. Different practical problems, after abstraction and generalization, can come up with the same mathematics concepts, operational rules, and even the same theory in mathematics. In the opposite, the same mathematics concept, operation rule and mathematics theory can be applied to practical problems that look entirely different superficially.

Boolean algebra was first introduced by G. Boole in 1847 to study the mathematical theory of propositional calculus. Later, the American electrical engineer Shannon pointed out that we can use Boolean algebra to study switching circuits and related problems.

This special topic started with the background involving three persons who are controlling a circuit of light bulbs. One concrete problem is posed based on this design, and students are asked to transform the mathematics of circuit design into circuit algebra and circuit polynomial, and then proceed to study circuit algebra and circuit polynomial mathematically. In this way, students completely solve the initial problem posed to them and present a complete mathematical model of circuit algebra. This is one practical application of Boolean algebra. Students can feel the abstract process of mathematization, as well as the value of application of theory of mathematics.

The states of the new circuit $\{0, 1\}$ are formed using the +, ×, and complement (<insert formula>) operations in the circuit connected in series, in parallel or in the form of an inverter. Besides, the truth and falsity of new propositions composed of the “OR”, “AND” or “NOT” (“Negation”) of simple propositions in this special topic are obtained by the +, ×, and complement (<insert formula>) operations of the original propositions. Both would then have the same roots and lines of development. These operations and operations of numbers and polynomials learned in secondary mathematics are also similar to some extent. Therefore, learning of this special topic is good for secondary students to know in-depth properties of numbers and polynomials.

**Contents and Requirements**

1. Through switching circuit know about circuits and the two states of circuits and their representation. Know what is meant by circuits connected in series or in parallel, and what is meant by an inverter, and how their states are ascertained.

2. Through analyses of switching circuits, students know that states of the new circuit are formed by the operations of the states of the original circuits. Students

should master the two concepts “states” and “operation of states”.

3. Through knowledge of states and operation of states, students are able to abstract concepts of Boolean algebra, circuit function and circuit polynomial. Students recognize the mathematical processes of abstraction and generalization from practical problems, as well as the thinking method involving the use of mathematical theory to solve practical problems.

4. Students understand that any circuit can be represented by a circuit function, and this function can be achieved through the use of a circuit polynomial.

5. Through learning of propositional calculus, students familiarize with what is meant by a proposition and the values it may take. Students know what is meant by “OR-proposition” and “AND-proposition” of two propositions, what is meant by “NOT-proposition” (negation proposition), as well as how the values of these propositions can be ascertained.

6. Compare the relationships between switching circuits and propositional calculus; able to use simple examples to state clearly the relationships. Compare the operations between Boolean algebra and that of the rational number system. Consider their common points, different points and similarities.

7. Complete a study summary report. The report should consist of three aspects of contents: (1) Summary of knowledge; understanding of the contents and overall structure of this special topic, as well as knowledge of the underlying mathematical thinking methods; (2) Extension. Through information search, survey research, interviews and exchanges, independent thinking, students proceed to explore and extend some contents, some results and their applications in greater scope and depth; (3) Realization, feelings and opinions as a result of the learning of this special topic.

**Remarks and Recommendations**

1. This special topic should exhibit sufficiently how practical problems can be transformed into mathematical problems, as well as the use of mathematical methods to solve practical problems. Teachers should make visible that different practical problems after abstraction and generalization may come up with identical mathematics concepts or even identical mathematical theory. Therefore, teachers can use concrete examples to introduce switching circuits, enable students to use mathematical expressions to represent circuit diagrams, and train students to draw circuit diagrams in accordance with the mathematical expressions. Students understand that any circuit can be represented using circuit functions, and circuit functions can be achieved using circuit polynomials.

2. Through mathematical expressions of concrete circuits, students compute the states of all sorts of circuits so as to master and understand the structure of circuit algebra – the totality of all sorts of combination of states represented by each of the

letters of the mathematical expression, and new states are formed through operating the states when the letters assume different values. Students proceed to understand the rule of operation of Boolean algebra.

3. Teachers can require students to use the mathematical theory learned to the adding unit and logical mathematical unit of computers. Students can write a dissertation or a summary report as a result of learning this special topic.

## 3. Mathematical Exploration, Mathematical Modeling, Mathematical Culture

Mathematical exploration, mathematical modeling and mathematical culture are important contents threaded through the whole senior secondary mathematics curriculum. These contents are not packaged independently. They infuse into every modules or special topics. At the senior secondary stage of schooling, teachers should arrange at least one relatively comprehensive mathematical exploration and mathematical modeling activities. Teaching requirements of mathematical exploration, mathematical modeling and mathematical cultures are explicated below:

### Mathematical Exploration

Mathematical exploration is the study of topics of exploratory nature in mathematics. It refers to students who confront with some mathematical problems and engage in processes of autonomous exploration and learning. Processes involved are: observe and analyze mathematical facts, raise meaningful mathematical problems, guess and obtain appropriate mathematical conclusions and regularities, provide explanations and proofs.

Mathematical exploration is a new way of mathematics learning introduced to the senior mathematics curriculum. It helps students become familiarize with mathematical concepts and processes of arriving at conclusions, begin to understand relationships between intuition and rigor, begin to research into mathematics, experience the passion of creation, establish rigorous scientific attitudes and strenuous scientific spirits. This would also help students develop habits of doubt and reflection, abilities to discover, pose, and solve problems, and creative consciousness and practical abilities.

**Requirements**

1. Selection of topics is key to the completion of exploratory studies. Selection of topics should facilitate students to understand mathematics, experience the processes of researching into mathematics, develop consciousness of discovering and exploring problems, and encourage students to liberate their imagination and creative powers. Topics chosen should possess a certain degree of open-endedness. Prerequisite knowledge should not exceed students' current scope of acquired knowledge.

2. Topics for exploration should be multifarious. It can be extended to include studies of some mathematical results, connection and analogies amongst different mathematical content areas, as well as discoveries and exploration of mathematical results new to the students.

3. Topics for exploration can be drawn from examples, or discovered and established from background materials provided by the teaching materials. Likewise, they can be drawn from examples, or discovered and established from background materials provided by teachers. Teachers should encourage students to discover and pose their own problems for studying during processes of learning mathematical knowledge, skills, methods and thinking.

4. During processes of mathematical exploration, students should be able to enquire about information, collect data, and read literature.

5. During mathematical exploration, students should develop independent thinking and brave questioning habits. At the same time, students learn how to cooperate well with peers, establish serious scientific attitudes and strong spirits against hardships and difficulties.

6. During mathematical exploration, students begin to know mathematics concepts and processes of drawing conclusions, experience processes of researching into mathematics and passion for creativity. Abilities of discovering, posing and solving problems are elevated. Imagination and creative spirits are also liberated as well.

7. For students at the senior secondary level, teachers should arrange at least one exploratory activity, and this should be integrated organically inside and outside the curriculum.

*Standards* will not stipulate concrete arrangements of mathematical exploration in terms of contents and number of hours required. Schools and teachers should coordinate the contents and timing of exploratory activities in accordance with their own practical situations. For example, teachers can arrange mathematical exploratory activities when contents of approximate solution of equations and application of derivatives are delivered to the students.

**Remarks and Recommendations**

1. Teachers should try their best to serve as creators of topics of mathematical exploration. They should possess relatively broad mathematical perspectives, familiarize with extended knowledge related to secondary mathematics and the associated inner mathematical thinking, and think seriously some of the problems therein. They seek to deepen mathematical understanding, increase mathematical abilities so as to guide students to prepare sufficiently for the mathematical exploration, and accumulate resources that may be used to guide students to carry out mathematical exploration.

2. Teachers should become organizers, guides and collaborators when their students undertake mathematical exploration. Teachers should provide students with relatively rich examples of topics of mathematical exploration and the associated background materials, as well as guide and help but do not replace students to discover and pose topics for exploration. In particular, teachers should encourage and help students discover problems independently and pose problems, organize and encourage students to form working groups to solve problems collaboratively, guide and help students develop habits of reading relevant reference books and materials, as well as seeking and validating information via the computer network. On one hand teachers should guide students to think independently and help them establish perseverance and bravery to overcome difficulties. On the other hand, teachers should guide students to think from an independent standpoint to seek help by various means from others. When needs arise, teachers should become equal partners with the students, and should possess confidence to carry out exploration with students together.

3. Teachers should undertake specially tailored guidance in accordance with students' individual differences. At the same time when innovation is encouraged, teachers should permit students to liberate their power of imagination and creativity from a modeling perspective.

4. Results of exploration should be completed in project report or dissertation format. Project reports should comprise of project name, problem background, observation and analyses of facts, guessing of results and their validation, scenario of collaboration, realization and critique of results of exploration, quotation of bibliographic information, etc.

5. Exchange of results of exploration may be done using the small group presentation, class presentation, and oral defense formats. Evaluation of the achievement of the exploratory learning can be made through discussion between teachers and students, as well as amongst students themselves. Evaluation seeks to

encourage students' exploratory spirits from a positive standpoint, ascertain the creative work done by the students, and at the same time point out existing problems and inadequacies.

6. Reports of mathematical exploration and the associated comments can be incorporated in students' portfolios. The information therein can be used to reflect students' mathematics learning processes and form a basis for making recommendations for the students. For those reports and dissertations rated as outstanding teachers should praise, award, recommend to journals for presentation, edit for publication, as well as recommend to higher institutes of learning, etc.

7. Examples and background materials of certain topics of mathematical exploration should be available in appropriate sections of the teaching materials. Teachers can provide some exemplary cases completed by other students. These provide teachers with references and recommendations on how to guide exploratory learning.

## Mathematical Modeling

Mathematical modeling is a problem solving process involving the deployment of mathematical thinking, methods and knowledge. It has become important and basic contents of mathematics education at various levels of schooling. Mathematical modeling can be exhibited using the following block diagram:

<insert diagram here: Practical Contexts, Pose Problems, Mathematical Models, Mathematical Results, Validation, Usable Results, Modify, Not Congruent with Practical Contexts.

Mathematical modeling is a new format of mathematics learning. It provides students with opportunities for autonomous learning. It facilitates students to experience the values and function of mathematics in solving practical problems, experience the relationships of mathematics with everyday living and other subjects, experience the processes of synthesizing knowledge and methods to solve practical problems, and enhance application awareness. It helps stimulate students' interests in learning, and develop students' innovative awareness and practical abilities.

### Requirements

1. In mathematical modeling, problems are essential. Problems of mathematical modeling are multifarious, and come from students' everyday living, realistic world, and other subject areas. At the same time, the knowledge, thinking, methods involved in problem solving should be

related to the senior secondary mathematics curriculum.

2. Through mathematical modeling, students shall know and involve in the whole process of solving practical problems in the above-mentioned block diagram, experience the connection between mathematics and everyday living, as well as with other subjects. Students feel the practical values of mathematics. Their application awareness is enhanced and practical abilities increased.
3. Every student should discover and pose problems in accordance with their daily experiences. Tackling similar problems, students can utilize their special talents and personalities to explore methods of solving problems from different angles and levels. As a result, students acquire experiences of synthetic application of knowledge and methods during the problem solving processes, and develop innovative consciousness as well.
4. When students are engaged in the processes of discovering and solving problems, students learn to acquire information through various means such as enquiries.
5. During mathematical modeling, students should adopt multifarious ways of cooperation in problem solving, develop habits of cooperative exchanges with others, and acquire good affective experiences.
6. Teachers should arrange at least one mathematical modeling activity for the students at the senior secondary stage of schooling. Teachers should integrate the curriculum inside and outside organically so that activities of mathematical modeling and synthetic practical activities can be integrated organically.

*Standards* will not stipulate concrete arrangements of mathematical modeling in terms of contents and number of hours required. Schools and teachers should coordinate the contents and timing of mathematical modeling activities in accordance with their own practical situations. For example, teachers can arrange mathematical modeling activities when contents of statistics, linear programming, and sequences are delivered to the students.

**Remarks and Recommendations**

1. Schools and students can decide the number of times and time arrangement of mathematical modeling activities in accordance with the practical situations of the schools and students. Teachers can pose some mathematical modeling problems for students to choose in accordance with the teaching contents and students' practical situations. Teachers can also provide some practical situations

to guide students to pose problems. In particular, students are encouraged to discover and pose problems from their everyday living.

2. Mathematical modeling can adopt project groups as one mode of learning. Teachers can guide and organize students to learn how to think independently, share labor work amongst themselves, discuss and engage in exchanges, and seek help from others. Teachers should become working partners and mentors of the students.
3. During mathematical modeling, teachers should encourage students to use computational tools such as computers and calculators. Teachers should provide appropriate guidance when necessary.
4. Teachers should guide students to complete the mathematical modeling reports. Contents of the report should include the problem background, design of the problem solving plan, processes of problem solving, processes of cooperation, evaluation of results, as well as reference to bibliography.
5. When evaluating the performance of the students while they are engaging in mathematical modeling, attention should be paid on processes and participation. There is no need to demand excessive rigor in the mathematical modeling processes and the accuracy of the results. Contents of evaluation should emphasize the followings:
   - Innovativeness: There is originality in the posing of problems and solution plans.
   - Reality: Problems arise from the realities of the students.
   - Authenticity: It can be ascertained that students personally participate in the production, and the data are real.
   - Reasonableness: The mathematical methods used in the mathematical modeling processes are appropriate. The solution processes are normal.
   - Validity: There are certain practical meanings in the modeling results.

   Not all aspects need to quest for perfection. When one of the above aspects can attain satisfactory standard, the performance should be ascertained.
6. Evaluation of mathematical modeling can adopt different formats: oral defense, presentation and experience sharing session. Qualitative evaluation can be conducted through exchanges between teachers and students, and amongst students themselves. Students are praised if they can demonstrate their sparkling points.
7 Evaluative reports of mathematical modeling can be incorporated into students' portfolios. The information therein can be used to reflect students' mathematics learning processes and form a basis for making recommendation for the students. For those reports and dissertations rated as outstanding, teachers should praise,

award, recommend to journals for presentation, edit for publication, as well as to recommend to higher institutes of learning, etc.

8. In the teaching materials, mathematical modeling problems that suit the levels of the students and the associated background materials should be available for references by the students and the teachers. Exemplary cases may be provided in the teaching materials to stimulate students' interests in learning.

## Mathematical Culture

Mathematics is an important constituent of human culture. Mathematics is a product signifying the progress of the human society, and is a driving force propelling the development of a society. Through learning of mathematical culture at the senior secondary stage of schooling, students become familiar with the reciprocal functional relationship between mathematical science and development of human society, and realize the scientific values of mathematics, including application values and human values. Students' horizons are also broadened as they look for historical trajectories of mathematics progress. Knowledge of how mathematical innovativeness is energized can be enlightened as well. When students are enlightened within an excellent culture, students shall comprehend the aesthetic values of mathematics so as to elevate their cultural qualities and innovative consciousness.

### Requirements

1. Mathematical culture should integrate as far as possible organically with the contents of senior mathematics curriculum. Teachers should choose and introduce historical events and figures that have immense contribution to the development of mathematics; that can reflect the importance of mathematics on the progress of human society; and that reflect at the same time the propelling function of societal development on development in mathematics.

2. Through learning of mathematical culture, students are familiar with the reciprocal functional relationship between development of human society and development in mathematics; know the formation and consequent developmental patterns and regularities of mathematics; familiar with processes of how human can comprehend the objective world from the viewpoint of mathematics. Students seek for knowledge and truths and develop affection and brave exploratory attitudes. They realize that mathematics is systematic, rigorous, and widely applicable everywhere. Students are familiar with the relative nature of truth. Their mathematics learning is elevated as well.

3. Topics for optional studies are listed below:

(1) Formation and development of numbers;

(2) Euclid’s Elements and ideas of Axiomatization;

(3) Formation of plane analytic geometry and idea of “marriage of number and shape”

(4) Calculus and ideas of a limit;

(5) Non-Euclidean geometry and the Relativity problem;

(6) Formation of topology;

(7) Binary system and computers;

(8) Complexity of computation;

(9) Data and reliability in advertisement;

(10) Design of logo and geometric figures;

(11) Mathematical problems derived from Golden Section;

(12) Mathematics in arts;

(13) Infinity and fallacy;

(14) Television and image compression;

(15) Mathematics in CT Scans: Radon Transform;

(16) Military and mathematics;

(17) Mathematics in finance;

(18) Coastal lines and fractal;

(19) Reliability of systems.

**Remarks and Recommendations**

1. Teachers should adopt multifarious teaching modes. For example, while teachers teach mathematical knowledge they can introduce the related background cultures, or give special talks. Teachers can encourage and guide students to seek information, read and collect materials and references on some special topics. Based on these collected information, students can write small mathematics essays and edit some general science articles with various formats, and engage in exchanges afterwards.

2. Teachers should integrate related contents with an intention to emphasize the scientific, cultural and aesthetic values of mathematics.

3. Teachers should try their best to present the related topics in iconic formats. For example, teachers can use pictures, slides, videos and computer software.

4. Teachers should exploit and deploy educational resources sufficiently inside and outside schools. They should reach out to engage in exchanges with colleagues teaching other subjects (including subjects in the humanities), so as to foster better the integration and diffusion amongst the various subjects.

5. Teachers can work together with teachers of other subjects to examine students’ performance when they enquire references, read information, write essays and reports, and engage in communication. Those work that are rated as outstanding

should be encouraged, displayed, and recommended.

6. Regarding contents of teaching materials pertaining to mathematical culture, attention should be paid to introduce important ideas in mathematics, as well as outstanding mathematical results. In particular, for those topics on humanistic spirits involving people and events threaded through moral education and human thinking, studying materials should be: precise, animating, interesting, natural, comprehensive and yet easy to comprehend, popular and yet easy to understand.

# Section Four – Implementation Recommendations

## 1. Teaching Recommendations

There are relatively big changes starting from rationale, contents to implementation in the new wave of mathematics curriculum reform. Teachers are key persons to achieve the objectives of mathematics curriculum reform. Teachers should start to recognize the rationale and objectives of mathematics curriculum reform, as well as the role and function they play in the curriculum reform process. Teachers are not only implementers of the curriculum, but also important source of power in undertaking curriculum research, as well as building and exploiting resources. Teachers are not merely transmitters of knowledge, but also are students' guides, organizers and collaborators. For the sake of implementing the new curriculum better, teachers should explore and study constructively, and elevate their professional qualification in mathematics, as well as qualities in the educational sciences.

Mathematics teaching should exhibit the basic rationale of curriculum reform. Instructional design should consider sufficiently the characteristics of mathematics as a subject, psychological characteristics of senior secondary students, the learning needs and different levels and interests of the students. Teachers should deploy a multitude of teaching methods and strategies, guide students to learn constructively and autonomously, master fundamental knowledge and basic skills as well as methods of mathematical thinking exhibited in these knowledge and skills, develop application consciousness and innovative consciousness, have relatively more comprehensive knowledge in mathematics, elevate mathematical literacy, form constructive affection and attitudes, as well as build up a solid foundation for future development and further study. During teaching teachers should grasp properly the following aspects:

**i. Supervise students to select curriculum appropriately, and devise study plans that are based on the development of the students.**

Because of the need to exhibit characteristics of basic rationale that is modern, fundamental, selective and multifarious so as to enable students to learn different mathematics and acquire different developments in mathematics, senior mathematics curriculum has set up a framework consisting of a compulsory series and four optional study series. During teaching, teachers should encourage students to devise mathematics learning plans and select mathematics curriculum autonomously in accordance with the plans and requirements of the prescribed national curriculum, as well as their own potentials, interests and favorites. Teachers should provide concrete guidance according to students' different foundations, levels, interests and directions of development.

**ii. Help students build up foundation and develop abilities.**

Teachers should help students understand and grasp fundamental knowledge and basic skills in mathematics, and develop abilities. Concretely speaking:

(1) Emphasize understanding and mastery of basic concepts and ideas.

During teaching, teachers should emphasize understanding and mastery of basic concepts and basic ideas. For those core concepts and basic ideas (e.g. function, conception of space, operation, integration of number and figure, vector, derivative, statistics, conception of randomness, algorithm), there is a need to form a thread connecting the starting and finishing points of senior mathematics teaching so as to help students deepen their understanding progressively. Because of the highly abstract characteristics of mathematics, teachers should pay attention to aspects regarding the formation and progression of basic concepts. During teaching, teachers should guide students to involve in processes that abstract mathematical concepts from concrete real examples, as well as understand the nature of concepts progressively during the preliminary applications.

(2) Emphasize training of basic skills.

Master proficiently some basic skills is very important to learn mathematics well. In the senior mathematics curriculum, teachers should emphasize operation, graph construction, inference, data processing, as well as training of basic skills such as using scientific calculators. Attention should be paid to avoid training that is intricate and too much skill-based.

(3) Scrutinize fundamental knowledge and basic skills timely.

Due to the development of the era and mathematics itself, fundamental knowledge and basic skills of senior mathematics are changing everyday and teaching should scrutinize fundamental knowledge and basic skills in a timely manner. For example, contents such as statistics, probability, derivative, vector, and algorithm have become fundamental knowledge in mathematics. Some original fundamental knowledge need to deploy new rationale so as to organize teaching. For example, teaching of solid geometry can be conducted from different perspectives – from the whole to the parts and vice versa, from the concrete to the abstract, from the general to the specific, and attention is directed to the use of vector method (algebraic method) to handle related problems. Teaching of inequalities should pay attention to the geometrical background and its applications. Transformation of trigonometric identities should enhance its connection with the vectors, as well as simplification of corresponding operations and proofs. It is noteworthy that oral and written mathematical expression is basic to learning mathematics well, and attention is very much needed during teaching. Teachers should cancel or diminish complicated calculation, artificial skill-based problems, and contents that are excessively intricate and detailed so as to conquer tendencies of "disorganization and disorientation of the double basics".

**iii. Pay attention to connections, and elevate knowledge of mathematics holistically**.

Developments of mathematics are not only internally but also externally powered. During senior secondary teaching, teachers should pay attention to the connection amongst the different branches and contents of mathematics, connection between mathematics and everyday living, and connection between mathematics and other subject areas.

Senior mathematics curriculum is presented in the form of modules and special topics. Therefore, teaching should pay attention to the bridging of the connection amongst the different parts of contents. Through analogy, associated thinking, transfer of knowledge and application, students are enabled to realize the organic relationships amongst different parts of knowledge. They have a feel of the holistic nature of mathematics, proceed to understand the nature of mathematics, and elevate problem solving abilities. For example, teaching should pay attention to: the connection amongst functions, equations and inequalities; connection amongst transformation of vectors and trigonometric identities, vectors and geometry, and vectors and algebra; diffusion of ideas of algorithm in related contents and application in different contents. Apart from these, teachers should pay attention to the connection of mathematics with other subjects and the realistic world. As an example, teaching should emphasize the relationship of vectors with forces and velocities, as well as relationship of derivative with rate of changes existed in the realistic world.

**iv. Pay attention to the connection between mathematical knowledge and the practicalities, and develop students' application awareness and abilities.**

During mathematics teaching, teachers should pay attention to develop students' application awareness. Through a variety of rich examples teachers can introduce mathematical knowledge to guide students to apply to solve practical problems, involve in exploration and problem solving processes, and realize the application value of mathematics. Teachers should help students realize that: mathematics is of concern to me and my practical everyday living; mathematics is useful; I will apply mathematics; I can apply mathematics.

Concerning teaching of contents of mathematics, teachers should guide students to apply mathematical knowledge directly to solve some simple problems, e.g. deploy knowledge of function, sequence, inequality, and statistics to solve problems. Through activities of mathematical modeling, teachers should guide students to discover problems in practical contexts and situations, formulate mathematical models, and attempt to use mathematical knowledge and methods to solve problems. Teachers may also introduce students the widespread application of mathematics in society, encourage students to pay attention to cases of mathematics application so as to broaden their outlook and horizon.

**v. Concern about the cultural values of mathematics, and foster formation of students' scientific perspectives.**

Mathematics is an important constituent of human culture and is a product related to the growth of human society. It is a source of power to drive development of the society. During teaching, teachers should guide students to become familiar with the mutual functional relationship between mathematical sciences and the human society, realize the scientific, application and humanistic values of mathematics, broaden outlook and horizon, find out the historical trajectories of mathematical development, elevate cultural literacy, nourish habits of rational cognitive thinking such as truth seeking, argumentation, criticism and doubt, as well as spirits of perseverance in the pursuit of truth and principles.

During teaching, teachers should strive to integrate the contents of senior mathematics curriculum, introduce some historical events and leading figures that have important contribution to mathematics development, reflect the progress of mathematics in human society and the contribution of mathematics on the development of human civilization. Likewise, mathematics teaching should reflect how social development may foster mathematics development. For example, teachers can make use of geometry teaching to introduce methods of thinking behind Euclid's axiomatic system and indicate how these methods influence greatly human's rational thinking, development in mathematics and science, and progress of the society. Likewise, during teaching of analytical geometry and calculus, teachers can introduce how Descartes invent analytical geometry, how Newton and Leibniz jointly invent calculus. Teachers explain in what ways these inventions propel science, society, and progress of human thinking after the Renaissance. During teaching of number system, teachers can introduce development of number system and the process of its extension so as to allow students to have a feel of the internal and external dynamics of mathematics, as well as the contribution of human rational thinking on the formation and development of mathematics.

**vi. Improve ways of teaching and learning so as to allow students to learn autonomously.**

Enrichment of ways and improvement of methods of student learning are basic rationale pursued in the senior secondary curriculum. Students' mathematics learning activities should not be restricted to the memorization of concepts, conclusions and skills. Instead, imitation and reception, independent thinking, autonomous exploration, practical activity, cooperative exchange, reading and self-learning are all important ways of mathematics learning. Attention must be paid to personal participation and interaction between teachers and students. Rationale of education and aspects of exploitation and deployment of subject matter contents and curricular resources pose great challenges to our teachers. During teaching, teachers should explore constructively ways of teaching that are most appropriate for the senior secondary students in accordance with the rationale and objectives of the senior mathematics curriculum, cognitive traits of the students and characteristics of mathematics. The following aspects are worthy of particular attention:

(1) New contents have been added to the senior mathematics curriculum. Concerning teaching of these contents, teachers should grasp the scope and levels set in *Standards*. For example, concerning contents of algorithm teachers should pay attention to enable students to realize ideas of algorithm and elevate abilities of logical thinking. Teachers should not treat algorithm in a simple manner as learning of a programming language and design of computer program. At the same time, teachers should allow students to have hands-on experiences (or programming) using concrete real examples to help students understand ideas and function of algorithm. There are new changes in the arrangement and requirement of the traditional contents in *Standards*.

For the sake of understanding, grasping and effectively undertaking teaching, teachers should carry out necessary investigation and research so as to elevate their professionalism in mathematics and qualities in the educational science.

(2) During teaching, teachers should encourage students to participate actively in instructional activities. Not only instruction and guidance are provided, but also students' autonomous exploration and cooperative exchanges are involved. Teachers need to create appropriate problem contexts, encourage students to discover

regularities and patterns in mathematics and ways of solving problems, so as to enable students to involve in the processes of formation of knowledge.

(3) Enhance the function of intuition in geometry and emphasize the function of graph and figure in mathematics learning. In teaching of geometry and related contents, teachers should borrow intuition in geometry to uncover the nature and relationship of objects under examination. For example, teachers can borrow intuition in geometry to comprehend conic section, as well as comprehend concept of derivative and relationship of monotonicity of function with their derivative.

(4) During mathematics teaching, learning how to express in a formalized way is a basic requirement. Teachers should not restrict themselves to formalized expression but pay attention to uncover the nature of mathematics. For example, some concept teaching (e.g. function) can start from knowledge and real examples, and then concepts abstracted as stringent definitions. For example, teaching of some contents (e.g. statistics) has been carried out through case studies in order to learn the underlying thinking methods, as well as to understand the associated meaning and function. Another example is that teaching of concept of derivative can be undertaken through concrete examples to enable students to involve in processes transiting from average rate of change to instantaneous rate of change. As a result, students can become familiar with the practical background of the concept of derivative, as well as realize that derivative is actually instantaneous rate of change. Students realize the thinking behind and inner meaning of derivative.

(5) Teachers can adopt different ways of teaching and learning for different kind of contents. For example, teachers can engage students in data collection, sample survey, practical investigation, autonomous exploration, and cooperative exchanges, as well as adopt methods of reading comprehension, discussion and exchanges, and dissertation.

(6) In accordance with different contents, objectives and students' practical situations, teachers should reserve appropriate space and time for students' extension and development, and engage students further in investigation and research of related topics. For example, general concepts of inverse function and computation of geometric probability model are contents that may be used for extension and development. Contents used for development and extension should not be set as examination requirement.

(7) Teachers should respect fully personalities of the students and their differences in mathematics learning. Teachers should adopt appropriate methods of teaching and stimulate students' interests in mathematics learning during processes of mathematics learning and problem solving. Furthermore, teachers should help students nourish good habits of learning, form constructive investigative attitudes,

work hard, overcome difficulties bravely, and form styles of learning that strive for personal advancement.

(8) Teachers should reflect their teaching continuously, improve their methods of teaching, elevate their own levels of teaching, and form personalized styles of teaching.

**vii. Deploy modern information technology appropriately, and elevate teaching quality.**

Teachers should pay attention to the organic integration of information technology and mathematics curriculum, the main principle of which is to facilitate studying of the nature of mathematics. For example, preliminary algorithm has been included as contents in the compulsory study series and teachers should pay due attention to their integration with other contents. Other examples are that data processing in statistics and approximate solutions of equations have exhibited the idea of integration of information technology and mathematics curriculum. Teachers should pay particular attention on these issues. Regarding integration of information technology with mathematics curriculum, there is still ample room for development. Teachers can engage in constructive and meaningful investigation in this area.

Widespread application of modern information technology has engendered great influences on the contents of mathematics, as well as mathematics teaching and learning. During teaching, teachers should deploy information technology to present curriculum contents that in the past are difficult to handle. At the same time, teachers should use scientific calculators, computers and software, and Internet as far as possible, as well as all sorts of mathematics education technological platforms. Teachers should improve ways of student learning, guide students to borrow information technology to learn related mathematics contents, as well as investigate and study some meaningful and worthwhile mathematical problems.

## 2. Evaluation Recommendations

Evaluation of mathematics learning should not only emphasize mastery of students' knowledge and skills and elevation of students' abilities, but also pay attention to the change of affection, attitudes and values; not only emphasize discrimination of levels of student learning, but also pay attention to the liberation of subjective inner dynamics of individuals during the learning processes; not only emphasize knowledge of quantification, but also pay attention to analyses of qualities; not only emphasize student evaluation by the educational practitioners, but also pay attention to students' self-evaluation and peer-evaluation. In a nutshell, teachers should make evaluation central to the whole process of mathematics learning, not only fulfill the discrimination and selection function of evaluation, but also make visible the encouraging and developmental function of evaluation.

Evaluation of mathematics teaching should facilitate the creation of good environment conducive to education, monitoring of processes conducive to teaching and learning, as well as promoting mutual growth of both students and teachers.

**i. Pay attention to the evaluation of students' mathematics learning processes.**

Compared with outcomes, processes can reflect better developmental changes of each student, and make visible the growth trajectories of the students. Therefore, evaluation of mathematics learning should not only emphasize outcomes, but also processes as well. Concerning evaluation of students' mathematics learning processes, teachers may include students' interests and attitudes in participating mathematical activities, confidence in mathematics learning, habits of independent thinking, consciousness of cooperative exchanges, and levels of development of cognition in mathematics, etc.

Some concrete recommendations of contents of evaluation and requirements are shown below:

- Through evaluation of mathematics learning processes, teachers should try their best to guide students to recognize the values of mathematics, engender constructive attitudes, motivation and interests in mathematics learning.
- Independent thinking is a basic characteristic of mathematics learning. During evaluation, teachers should pay attention to whether students are willing to think, good at thinking, and insist to think, as well as improve methods and processes of thinking continuously.
- Concerning evaluation of learning processes, teachers should pay attention to whether students take the initiative to participate in mathematics learning activities actively, whether students are willing or are able to engage in exchanges of experiences of mathematics learning with peers, as well as to cooperate with others to explore mathematical problems.
- Students' confidence in learning mathematics well, dedication, hard work, and perseverance in overcoming difficulties are good habits of the mind. As such they are important contents of the evaluation of mathematics learning processes.
- Evaluation should pay special attention to examine whether students are able to abstract mathematical knowledge from practical situations, as well as apply mathematical knowledge to solve problems.
- Evaluation should pay attention to examine whether students can comprehend

and express mathematical contents systematically.

- Evaluation should pay attention to whether students can reflect their processes of mathematics learning systematically, and improve methods of learning.

**ii. Evaluate correctly students' fundamental knowledge and basic skills.**

Students' understanding of fundamental knowledge and mastery of basic skills are basic requirements of mathematics learning, and are basic contents for the evaluation of student learning. Evaluation should pay attention to the understanding of the nature of mathematics and mastery of methods of thinking, avoid emphasizing mechanical memorization in a superficial manner, imitation and complicated skills.

Shown below are some concrete recommendations and requirements of the contents of evaluation:

- Concerning evaluation of mathematics understanding, teachers should pay attention to whether students can independently raise a certain quantity of examples and counter-examples to illustrate the problems. In particular, evaluation of core concepts of learning should be emphasized during the whole process of senior mathematics learning.
- Evaluation should pay attention to whether students can establish the interrelationship of different kinds of knowledge, grasp the structure and system of mathematics.
- Concerning evaluation of basic skills in mathematics and premised on students' understanding of methods, teachers should pay attention to whether students can engage in appropriate selection of methods to tie in with problem characteristics, and move a step further to use them proficiently.
- Precision, elegance and formalization are characteristics of the mathematical language. Whether students can deploy mathematical and natural languages appropriately are also important contents of evaluation.

**iii. Pay attention to the evaluation students' abilities.**

Acquisition and elevation of student abilities are of vital importance to autonomous learning and realization of sustained development. In this regard, evaluation should be clearly directed. Abilities are exhibited through mastery of knowledge and increased levels of deployment. Therefore, evaluation of abilities should thread its way through mathematical knowledge construction and problem solving processes.

How to evaluate abilities is not only an important issue encountered in curriculum reform, but also a challenge as well. Listed below are some aspects needed attention, using evaluation of abilities of posing, analyzing and solving problems mathematically as examples:

- During everyday mathematics learning, especially activities of mathematical exploration and mathematical modeling; whether students possess problem consciousness, and whether they are good at discovering and posing problems.
- Whether students can select effective methods and strategies to collect information, connect related knowledge, pose consideration of problem solving solution paths, establish appropriate mathematical models, and proceed to try to solve problems.
- Whether students can engage in independent thinking and cooperative exchanges with others during the problem solving processes.
- Whether students can question, modify and perfect problem solving plans.
- Whether students can use written or oral language to express the plan and outcome of problems to be solved in a relative accurate manner, and exchange views with others. Also, whether students can undertake analyses, discussion and application according to the practical requirement of the problems.
- Evaluation should pay attention to whether students can undertake self-evaluation and peer-evaluation of problems posed and solved by themselves.
- During evaluation, teachers should pay attention to ascertain students' development and progress in mathematics learning, and their strengths and specialties.

**iv. Implement multifarious evaluation that promotes student development.**

Concerning meanings of multifarious evaluation that promotes student development, there are a number of aspects worthy of attention, including multifarious involvement of the evaluators, methods, contents and targets of evaluation. Teachers should choose according to the purpose and contents of evaluation.

Multifarious involvement of evaluators refers to combination of methods of evaluation conducted through teacher evaluation, self-evaluation, peer evaluation, parent evaluation and related personnel in the society. Multifarious use of methods entails combination of qualitative and quantitative approaches, written and oral formats, inside and outside classroom situations, as well as process and outcome evaluation. Multifarious contents of evaluation include knowledge, skills and abilities, processes and methods, affection, attitudes, values, as well as qualities of the body and mind. Multifarious targets of evaluation refer to that different student would have different standards of evaluation, so that students' individual differences and choices for optional studies are respected. Teachers will not use one single standard to consider and examine all sorts of student situations.

Concrete recommendations of formats of evaluation are listed below:

- Evaluation should respect students as target of evaluation. Premised on this condition, evaluators should participate in mathematics education activities and pay attention to the communication amongst the evaluators.
- Written tests are still important format of quantitative evaluation. However, attention should be paid on the examination of understanding of mathematical concepts, mastery of mathematics thinking methods, depth of mathematics thinking, exploration and levels of innovation, as well as abilities of applying mathematics to solve practical problems.
- Quantitative evaluation can adopt system of percentiles or grades. Students should be promptly informed outcome of evaluation as feedback. Teachers should avoid using marks to rank students.
- Qualitative evaluation should deploy formats such as remarks or growth records. Language used should be encouraging, objective and comprehensive enough to describe the situations of the students.
- Teachers should pay attention to students when they are engaged in the processes of doing mathematics, and should fully demonstrate the function of mathematics assignment in student evaluation. There should be multifarious formats of assignments, including regular exercises, open-ended and exploratory mathematics problems, mathematics experiments, mathematical modeling, thematic projects, and summary reports of special topics. Presentation format of these outcomes are also multifarious, e.g. question and answer, experience sharing of mathematics learning, small mathematics papers, written or oral research reports, experiments and surveys. Both quantitative and qualitative evaluation may be applied to these assignments. Process of evaluation should be constructive, simple to carry out and with initiative, so as to avoid the burden of the students.
- Teachers should emphasize the use of modern education technology strategies in learning, e.g. calculators and computers.

In a nutshell, through multifarious evaluation, teachers can achieve encouragement and evaluation of students from multiple viewpoints better than before, work hard to enable students to experience success, and promote student development effectively.

**v. Undertake evaluation in accordance with students' different choices.**

Students can select different combination of senior mathematics curricula for study in accordance with individual conditions, interests and ambitions (refer Choice Recommendations for Students). Schools and teachers should undertake evaluation in accordance with the different options chosen by the students.

- After choosing the curriculum options by the students, schools and teachers can set up corresponding files for the students. When students complete the curriculum modules or special topics studies, schools should file outcomes of learning that reflect students' level of attainment.
- When students change their curriculum options, schools and teachers should promptly help students accomplish evaluation of the program just completed. Teachers should assist students to opt for the different series of the curriculum.
- Evaluation conducted by the schools and teachers constitute the basis for evaluating students when they seek jobs in the society or enroll in institutes of higher learning. Institutes of higher learning should set admission examination questions in accordance with their different requirements, as well as the five different curricular combinations in *Standards*. Scope of the admission examination includes compulsory series, optional study series 1, optional study series 2, and optional study series 4. Regarding evaluation of optional study series 3, a combination of quantitative and qualitative methods that tie in with the characteristics of the curriculum contents should be deployed, and this should be accomplished by the senior secondary schools themselves. When institutes of higher learning admit students, they should consider students' mathematics learning at the senior secondary stage of schooling thoroughly and comprehensively.

## 3. Teaching Materials Editing Recommendations

Teaching materials are important resources used for the realization of curriculum objectives and implementation of teaching. Editing of senior mathematics teaching materials should be based on the spirits of *Synopsis of Basic Education Curriculum Reform (Trial Version).* There is a need to take into account the basic rationale and requirements of the senior mathematics curriculum so as to guarantee the successful implementation of the curriculum. Teaching materials should facilitate the mobilization of initiatives of the teachers so that they can engage in teaching creatively, as well as facilitate the modes of learning of students so that they can study and develop autonomously.

Editing of teaching materials should make use of the modules in *Standards* as units. A variety of editing styles are encouraged in *Standards*. Editors can adjust appropriately the order of the teaching contents as prescribed in the modules, and different teaching materials can have their own styles and characteristics. In particular, attention should be paid to the following problems during editing of teaching materials.

**i. Selection of contents should exhibit the nature of mathematics, connection with practicalities, and adapt to the characteristics of the students.**

Selection of contents of teaching materials should facilitate the reflection of the nature of corresponding mathematics contents, facilitate students' recognition and understanding in mathematics, stimulate students' interests in mathematics learning, as well as consider sufficiently the psychological characteristics and cognitive level of the students. Teaching materials should be acceptable by others and they exhibit fundamental, modern, typical, and multifarious characteristics.

Senior secondary students already possess relatively rich life experiences and certain degree of scientific knowledge. Therefore, teaching materials should include contents that are of interest to them and are intimately related to their practical everyday living, such as those commonly encountered phenomena in the realistic world and real examples in other areas of science. Mathematics concepts and conclusions should be displayed, mathematics thinking and methods should be made visible, and mathematics applications should be reflected in the teaching materials. Students feel that mathematics is around them and its applications proliferate everywhere. Using teaching of contents of statistics as an example, teachers can choose exemplary cases with rich everyday life background to display the widespread application of statistical thinking and methods. Through orbits of planetary motion, convex and concave mirrors, the meaning and application of conic sections can be exemplified. Through rate of change of velocities, rate of expansion of volumes, and efficiency and density, teachers can introduce concept of derivative within a variety of rich realistic backgrounds.

**ii. Exhibit developmental processes of knowledge formation, and promote students' autonomous exploration.**

Presentation of curriculum contents should pay attention to reflect the pattern of development of mathematics, and cognitive pattern of ourselves as well. The rule is that presentation should start from concrete to abstract, and from particular to general.

For example, when introducing general concepts of function, teachers should start from the concrete functions already acquired by the students (first degree function, second degree function) and functional relationships in everyday living (e.g. changes of temperature, meter charges of taxi fares). General concepts and properties of functions should be abstracted so that students can comprehend concepts of function in a progressive manner. Regarding contents of solid geometry, teachers can make use the points, lines and planes of cuboids as embodiments. Based on the foundation of intuitive perception, students recognize the positional relationships of points, lines and planes in space.

Teaching materials should pay attention to the creation of situations and contexts. Teachers can start from concrete real examples to demonstrate how mathematical knowledge occurs and processes develop so as to allow students opportunities to discover problems, pose problems, and involve in processes of discovery and creation. Students become familiar with the trajectories of the knowledge building processes.

**iii. Exhibit connection amongst related knowledge, and help students recognize and understand mathematics comprehensively.**

Different parts of mathematical knowledge are interrelated, and student learning should be progressive and developmental. Editing of teaching materials should pay attention to these problems. Because contents of the mathematics curriculum have been divided into a number of modules, teachers should not ignore the interconnection amongst them.

Because of the need to develop students to recognize the internal connection of mathematics, teaching materials should comprise of different mathematical contents and communication should be set up amongst them. This is to deepen students' mathematical knowledge and their understanding of the nature of mathematics. For example, teachers can make use of the graphs of function of second degree to compare and study solution of quadratic equations and inequalities in one unknown. Teachers can also compare the images of function of first degree and arithmetic progression, as well as images of exponential function and geometrical progression so as to discover the interrelationship amongst them.

Contents listed in *Standards* are chosen in accordance with the various needs of the students, as well as presented as a number of series and levels. Teaching of related contents in different modules should pay due attention to their requirements and sequences. Students can upgrade themselves in a progressive and spiral manner that is based on the foundation of knowledge already acquired. For example, statistical contents in the compulsory series of the mathematics curriculum should be taught through a large number of real examples. Anchored upon the foundation of the stage of obligatory education, students become familiar with the idea of random sampling and the use of sample statistics to estimate population parameters. They learn some methods of data processing as well. Statistical contents in the optional studies are taught through a number of exemplary cases so that students proceed to learn common statistical methods, and deepen their knowledge of mathematical ideas and the function of statistics in daily production activities of the society.

**iv. Pay attention to special treatment of new rationale and contents when teaching materials are edited.**

Based on the new rationale of current curriculum reform, some new curriculum contents and treatment methods have been added. There is a need to pay attention to their treatment during editing of the teaching materials in accordance with the contents requirement as stipulated in *Standards*.

Algorithm is amongst the new contents in the senior mathematics curriculum. Teaching materials should make prominent the ideas of algorithm, provide concrete examples, enable students to involve in imitation, investigation, design of procedural block diagrams and operational processes so as to realize basic properties of algorithmic thinking. Teachers should not treat contents of algorithm simply in the form of programming language learning and design of computer programs. At the same time, there is a need to pay attention to integrate algorithm with appropriate curriculum contents and infuse algorithmic thinking into problem solving exercises so that students' knowledge of algorithm can be deepened. For example, teachers can infuse contents of algorithm into contents of solution of inequalities of second degree in one unknown.

Mathematical exploration, mathematical modeling and mathematical culture are new learning activities set up in *Standards*. When editing these teaching materials, teachers should insert these activities appropriately in related teaching contents. Teachers should pay attention to provide related recommended topics, background materials and exemplary cases, help students design their own learning activities, complete project assignments or summary thematic project reports.

When editing of optional curriculum series 3 and 4, teachers should exploit and explore constructively, meaningfully and creatively in accordance with the characteristics of the series and the concrete requirements of the special topics.

**v. Diffuse mathematical culture, and exhibit spirits of humanism.**

During editing of teaching materials, teachers should diffuse values of mathematical culture into the various contents of the curriculum. Teachers can adopt multifarious formats, such as integration of mathematical culture with concrete mathematical contents, or introduction of special topics pertaining to the various issues of mathematical culture. Teachers can also furnish a list of references and related resources for home reading. This would facilitate students to consult, collect and organize.

**vi. Design of contents should exhibit a certain degree of flexibility.**

During editing of teaching materials, there should be a certain degree of flexibility in the design of contents. For example, in accordance with the characteristics and interests of the students, teachers can include in the senior secondary mathematics curriculum related contents for extended studies. These contents may be in the form of problems needed investigation, problems needed further elaboration, or some important mathematical thinking methods. When contents are selected and arranged, there is a need to pay due attention to different ways of thinking so as to reflect the fundamental properties of mathematics. There is no requirement to evaluate these contents.

**vii. Reflect integration of modern information technology and mathematics curriculum.**

Information technology has diffused into teaching of mathematics to keep in pace with the era. How to make use of information technology to help students learn mathematics better than before is a question needed attention and further thinking during editing of teaching materials. One principle of using information technology is to facilitate understanding of fundamental properties of mathematics. Teachers can encourage students to use calculators or computers to process specific contents of the teaching materials. This would help students understand mathematics concepts, explore mathematics conclusions, and encourage students to deploy modern technologies as means to process complicated computations and solve practical problems. This would save up some time and energy to explore and discover patterns and regularities in mathematics, as well as develop creative spirits and practical abilities. On the other hand, modern information technology not only possesses great potentials improving students' ways of learning, but also through integration of curricular resources infuses with various contents in the mathematics curriculum. For example, algorithm may be infused into related mathematics contents. Teachers can guide students to collect information via the World Wide Web to study mathematical culture, and to realize the human values of mathematics.

## Appendix

Table of names of foreign mathematicians and
corresponding Chinese translation used in *Standards*

| | |
|---|---|
| 阿貝爾 | Abel |
| 阿基米德 | Archimedes |
| 貝努利 | Bernoulli |
| 畢達哥拉斯 | Pythagoras |
| 布爾 | Boole |
| 布勞威爾 | Brouwer |
| 笛卡兒 | Descartes |
| 棣弗-赫爾曼 | Diffie-Hellman |
| 棣莫弗 | de Moivre |
| 斐波那契 | Fibonacci |
| 費馬 | Fermat |
| 伽利略 | Galileo |
| 伽羅瓦 | Galois |
| 蓋莫爾 | El Gamal |
| 高斯 | Gauss |
| 歌德爾 | Godel |
| 傑弗遜 | Jefferson |
| 卡丹 | Cardanao |
| 開普勒 | Kepler |
| 康托 | Cantor |
| 柯西 | Cauchy |
| 拉東 | Radon |
| 拉格朗日 | Lagrange |
| 萊布尼茨 | Leibniz |
| 羅素 | Russell |
| 馬爾可夫 | Markov |
| 莫比烏斯 | Mobius |
| 牛頓 | Newton |
| 歐幾里得 | Euclid |
| 歐拉 | Euler |
| 龐加萊 | Poincare |
| 申農 | C.E. Shannon |
| 威爾遜 | Wilson |