Full-time Obligatory Education

Mathematics Curriculum Standards

(Experimental Version)

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Section One – Forward

Mathematics is a process of grasping the objective world qualitatively, as well as gauging its attributes quantitatively. The process goes through abstract generalization, method and theory formulation, and wide-range applications. Since the middle of the 20th Century, there have been immense changes in mathematics itself. This was particularly so when mathematics was integrated with calculators and computers. As a result of this integration, research areas, methodologies and application areas were replete with developments of mathematics. Mathematics helps us a lot explore the regularities of the objective world, as well as make appropriate choices and decisions in a modern society overwhelmed by a flood of intricate information. Simultaneously, mathematics provides us means of communication effectively and quickly. As mathematics is a technology widely applied everywhere, it helps us collect, organize, and describe information, establish mathematical models, as well as solve problems, resulting in direct generation of wealth for the society.

The initial starting point of the mathematics curriculum for the stage of obligatory education is to promote all-round, continuous and harmonious development in school children. The mathematics curriculum not only considers the characteristics of mathematics itself, but also follows children's psychological rules and patterns of mathematics learning. It emphasizes that learning starts from children's past experiences, allowing them the opportunity to experience personally the processes within which authentic problems are abstracted to form mathematical models that may be used for explanation and application. Apart from understanding mathematical knowledge, children also progress and develop in areas like cognitive thinking, attitudes and emotions, and values.

1. Basic Rationale

(1) The **mathematics curriculum** for the stage of obligatory education should exhibit fundamental, universal and developmental characters. It seeks to realize:

- ☺ Everybody learns valuable mathematics;
- ☺ Everybody can acquire mathematics essential to him or her;
- ☺ Different person can have different types and levels of development in mathematics.

(2) We cannot do without **mathematics** in our daily living, work and study. Mathematics helps us process data, engage in computation, draw inferences and prove hypotheses. Mathematical models can describe natural and social phenomena effectively. Mathematics casts the foundation upon which all major technological advances are anchored, and it furnishes languages, ideas and methods for the other sciences. Mathematics has special contribution in raising our inferential and abstraction capacities, imagination and creativity, etc. Mathematics is one of the human cultures, and its contents, thinking, methods and languages are important constituents of modern civilization.

(3) Contents of **mathematics learning** for school children ought to be realistic, meaningful and challenging. These contents should facilitate school children to engage actively in mathematical activities, such as observation, experimentation, guessing, hypothesis testing, inference making, and communication. Contents should be presented in different ways so as to satisfy the diverse learning needs of the students. Effective mathematics learning activities cannot simply rely on imitation and memorization. Instead, hands-on practical work, autonomous investigation and cooperative exchanges are important modes of mathematics learning. Because of the differences in cultural environment, family background, and ways of thinking, our children's mathematics learning activities should be animating, lively, active, self-initiative and filled with individual characters.

(4) **Mathematical instructional activities** should be based on children's cognitive developmental levels and built upon the knowledge foundation of their past experiences. Teachers should stimulate children's initiatives in learning, provide ample opportunities for children to participate and acquire widespread experiences in mathematical activities, help children genuinely understand basic concepts, mathematical thinking and methods, as well as master basic knowledge and skills in their autonomous explorations and cooperative exchanges. The children themselves are masters of their own mathematics learning, whereas teachers are organizers, facilitators and collaborators of mathematics learning.

(5) The primary aim of **evaluation** is to understand fully school children's processes of mathematics learning, stimulate children's learning and improve teacher's instruction. There is a need to establish an assessment system with multiple ways and objectives of evaluation. Not only learning outcomes, but also processes are emphasized in the evaluation of mathematics learning. Teachers should pay attention to both levels of children's mathematics learning and the emotions and attitudes

exhibited during mathematical activities. Teachers need to help children understand their self-concepts and build up their confidence.

(6) Development of **modern information technology** has great impact on the values, objectives, contents and pedagogy of mathematics education. Design and implementation of mathematics curriculum should pay attention to the use of modern information technology. In particular, the effects of calculators and computers on contents and methods of mathematics learning should be sufficiently considered. There is a need to exploit and provide richer learning resources to the children, and to use modern information technology as a powerful tool for mathematics learning and problem solving. Teachers should endeavor to change school children's learning styles so that they are willing to invest more energy in realistic, investigative mathematical activities.

2. Design Considerations

(1) About Stages of Schooling

Full-time Obligatory Education Mathematics Curriculum Standards – Experimental Version (thereafter abridged as *Standards*) thoroughly considers the contents of the nine years school curriculum in order to demonstrate that the stage of obligatory education is regarded as a coherent whole. At the same time, the nine years of schooling time are concretely divided into three stages of schooling according to children's physiological and psychological characteristics. The three stages are:

The first stage of schooling (grade 1 ~ 3), the second stage of schooling (grade 4 ~ 6), and the third stage of schooling (grade 7 ~ 9).

(2) About Objectives

In accordance with *Synopsis of Basic Education Curriculum Reform (Trial Version)*, and taking into account of characteristics of mathematics education, *Standards* clarifies the overall objectives applicable to the stage of obligatory education. Furthermore, it elucidates the objectives in greater details along four dimensions: Knowledge and Skill, Mathematical Thinking, Problem Solving, Affection and Attitudes.

Standards not only uses terminologies such as Knowledge, Comprehension, Mastery, and Flexible Application to depict verbs associated with the Knowledge and Skill objectives, but also deploys Feeling, Experiencing, and Exploring to depict verbs linked with mathematical activity levels of the Process objectives. By doing this, *Standards* is able to convey clearly its requirements on school children in mathematical thinking, problem solving, as well as areas such as affection and attitudes.

Knowledge and Skill Objectives
Familiarity (Knowledge) – Able to know or give examples from concrete situations to describe the characteristics and meanings of objects; able to identify objects from concrete situations in accordance with the objects' given characteristics.
Comprehension – Able to describe the characteristics and developments of objects; able to assert clearly the connections and discriminations of one object with related objects.
Mastery – Able to apply the objects in new contexts (given the solid foundation of comprehension).
Flexible Application – Able to apply knowledge in an integrated manner; apply methods flexibly and appropriately to accomplish designated mathematical tasks.

Process Objectives
Involving (Feeling) – Acquire some initial experiences in designated mathematical activities.
Experiencing (Realizing) – Begin to know the characteristics of objects in concrete mathematical situations so as to acquire some experiences.
Exploring – Through observations, experimentation and inferences discover some characteristics of objects, as well as the connections and discriminations with other objects in some autonomous mathematical activities.

(3) About Contents of Learning

At each stage of schooling, there are altogether four content domains in *Standards*, namely Numbers and Algebra, Space and Figures, Statistics and Probability, Practical and Integrated Applications. School children's mathematical activities are emphasized, and curricular contents seek to develop number sense, symbol sense, space concepts, statistical concepts, application awareness and inferential abilities in the learning process.

Number Sense is primarily demonstrated in situations such as – understand the meaning of numbers; able to use different ways to represent numbers; able to grasp

the relative magnitude relationships of two numbers in concrete contexts; able to use numbers to represent and exchange information; able to choose an appropriate algorithm to solve problems; able to estimate outcomes of computation, and to justify the validity of the outcomes.

Symbol Sense is primarily demonstrated in situations such as – able to abstract relationships of quantities and patterns of changes in concrete contexts, and subsequently use symbols to represent these patterns and relationships; understand the meaning of symbols that represent the relationships of quantities and patterns of changes; able to exchange relationships amongst symbols; able to choose an appropriate procedure and method to solve a problem represented in symbol terms.

Space Concept is primarily demonstrated in situations such as – able to visualize geometrical figures from concrete objects, and vice versa, as well as to transform the geometrical forms into three-view drawing or net, and vice versa; able to construct models or draw figures according to the given conditions; able to decompose relatively complicated figures into basic figures, and to analyze its basic elements and inter-relationships; able to describe motions and changes of real objects and geometrical figures; able to use an appropriate method to describe the relative positions of objects; able to deploy figures to describe problems visually so as to think in an intuitive manner.

Statistical Concept is primarily demonstrated in situations such as – able to think about problems related to information data from the statistical point of view; able to collect, describe and analyze data to make justifiable decisions, and are aware of the role of statistics in decision making; able to query the sources of data, methods of data processing, and the outcomes derived from these sources and methods appropriately.

Application Awareness is primarily demonstrated in situations such as – realize that there is a large quantity of mathematical information in our daily living, and mathematics applies widely in the phenomenal world; attempt actively to find problem solving strategies from the mathematical point of view when confronted with practical problems by using the acquired knowledge and methods; locate actively the actual background when confronted with new mathematical knowledge, and investigate the worth of its applications.

Inferential Ability is primarily demonstrated in situations such as – obtain

mathematical conjectures through observation, experimentation, induction and analogy, as well as find evidences to confirm or to give counter-examples; able to express one's thinking processes clearly and systematically, as well as to give evidences to substantiate our claims; able to use mathematical languages to discuss and query logically when communicating with others.

Because of the need to exhibit the flexible and selective characters of the mathematics curriculum, *Standards* merely prescribes the basic attainment levels of the stages of schooling. Editors of teaching materials, schools, and teachers in particular should practice individually configured education in accordance with children's aspirations and possibilities of development. *Standards* has not stipulated how the contents should be arranged and displayed, and there are various forms of editing the curricular contents.

(4) About Implementation Recommendations

Standards proposes recommendations targeted at teaching and learning, evaluation, editing of teaching materials, deployment and exploitation of curriculum resources. These recommendations are meant to be references for all those concerned so as to guarantee that *Standards* can be smoothly implemented.

In order to explain and explicate the corresponding curricular objectives or curriculum implementation recommendations, *Standards* provides some exemplary cases for our references.

Section Two – Overall Objectives

1. Overall Objectives

Through mathematics learning at the obligatory stage of schooling, students are able to:

☺ Acquire important mathematical knowledge (including mathematical facts, mathematical activity experiences), basic mathematical thinking methods, and necessary application skills that are essential for adapting to future social life and further development;

☺ Begin to know how to deploy various ways of mathematical thinking to observe, analyze realistic world, solve problems encountered in daily living and studies in other disciplinary areas;

☺ Realize the intimate relationships between mathematics and nature, as well as between mathematics and human society, know the worth of mathematics, increase understanding of mathematics, and gain confidence in learning mathematics with good results;

☺ Possess some degree of creative spirits and practical abilities, and develop sufficiently in areas of general abilities, affection and attitudes.

The overall objectives are concisely stated below:

Knowledge and Skills
- Involve in processes of how authentic problem situations are abstracted as number and algebra problems; master fundamental knowledge and basic skills pertaining to numbers and algebra; able to solve simple problems. - Involve in and explore the processes of how shapes, sizes and positions of objects and figures are related and transformed; master fundamental knowledge and basic skills pertaining to space and figures; able to solve simple problems. - Involve in the processes of problem posing, data collection and processing, decision making and prediction; master fundamental knowledge and basic skills pertaining to statistics and probability; able to solve simple problems.
Mathematical Thinking
- Involve in the processes of applying mathematical symbols and figures to describe the phenomenal world; establish initial number sense and symbol sense; develop abstract mathematical thinking. - Enrich knowledge of space and objects in the phenomenal world, establish initial space concepts; develop iconic mathematical thinking. - Involve in how information is described by data processing and organization, as

well as how inferences are made; develop statistical thinking.
- Involve in how observation, experimentation, guessing, and proving are done in mathematical activities; develop reasonable analogical and induction abilities and initial mathematical deduction ability; able to present one's ideas systematically and clearly.

Problem Solving

- Begin to learn how problems can be posed and comprehended from the mathematical perspectives; able to apply knowledge and skills acquired for problem solving in an integrated manner; develop application awareness.
- Formulate some strategies for problem solving; experience that problems can be solved in a variety of ways; develop practical abilities and creative spirits.
- Learn how to cooperate with others; able to communicate with others about one's processes and products of thinking.
- Begin to form an awareness of evaluation and reflection.

Affection and Attitudes

- Able to participate positively in mathematical learning activities; demonstrate curiosity and eagerness in mathematics learning.
- Experience success in mathematical activities; develop strong will in overcoming difficulties; develop confidence in mathematics learning.
- Begin to know the intimate relationships between mathematics and human lives, as well as the values and influences of mathematics on human civilization; experience that mathematical activities are full of explorations and creative productions; sense the rigor of mathematical knowledge and appreciate the certainty of mathematics conclusions.
- Form an attitude that is pragmatic and realistic; develop habit of doubts and engage independent thinking.

The above-mentioned four objective areas should be regarded as an intimately interrelated organic whole. They are very important for human development, and should be realized in rich and wonderful mathematical activities. In particular, development of mathematical thinking, problem solving, affection and attitudes cannot be separated from knowledge and skills acquisition. Likewise, knowledge and skills acquisition needs to satisfy one prerequisite, i.e. to facilitate the realization of the other three objective areas.

2. Stage Objectives

Knowledge and Skills

First Stage (Grade 1~3)

☺ Involve in the process of abstracting numbers in everyday living; know numbers up to ten thousand, decimal fractions, simple fractions and common quantities; familiarize with the meanings of the four rules of arithmetic operations; master the necessary operational skills (including estimation).

☺ Involve in the process of knowing simple geometrical forms and plane figures intuitively; familiarize with simple geometrical forms and plane figures; feel phenomena of translation, rotation and symmetry; begin to describe the relative positions of objects; acquire elementary skills in measurement (including making guesstimates), figure recognition and construction.
☺ Experience the process of data collection, organization, description and analyses; master some simple processing skills; begin to feel uncertain phenomena.

Second Stage (Grade 4~6)
☺ Involve in the process of abstracting numbers and relationships of simple quantities in realistic world; know numbers up to one hundred millions; familiarize with the meanings of fractions, percentages and negative numbers; master the necessary operational skills (including estimation); explore implicit rules and patterns in given objects and events; able to use equations to express simple quantitative relationships; able to solve simple equations.
☺ Involve in the process of exploring the shapes, sizes, motions and relative positions of objects, forms and figures; familiarize with the basic characteristics of simple geometrical forms and plane figures; able to transform simple figures from one form into another; able to ascertain the tentative positions of objects; develop skills in measurement (including making guesstimates), figure recognition and construction.
☺ Involve in and feel the process of data collection, organization, description and analyses; master some simple processing skills; experience whether events are happened with equal chances, whether the rules of the game are fair or not; able to calculate the chances for some simple events to happen.

Third Stage (Grade 7~9)
☺ Involve in the process of abstracting symbols in concrete contexts; know rational numbers, real numbers, algebraic expressions, equations, inequalities, and functions; master the necessary operational skills (including estimation); explore quantitative relationships and change patterns in concrete problems, as well as able to use algebraic expressions, equations, inequalities and functions to express these relationships and patterns.
☺ Involve in the process of exploring the basic properties, transformations, and relative positions of objects, forms and figures; master the basic properties of triangles, quadrilaterals and circles, together with basic properties of translation, rotation, axisymmetry and similarity; begin to know projection and three-view drawings; master basic skills in figure recognition and construction; realize the necessity of proofs; able to prove basic properties of triangles and quadrilaterals; master basic inference skills.
☺ Engage in the process of data collection, description and analyses so as to make judgments and involve in exchange of ideas; Feel the necessity of sampling; realize the idea of using sample to estimate the population; master the necessary skills in data processing; proceed to enhance knowledge of probability; know the relationship between frequency and probability; able to calculate the probability for some events to happen.

Mathematical Thinking

First Stage (Grade 1~3)
☺ Able to use everyday experiences to explain numbers encountered, and begin to

know how to use numbers in a concrete manner to describe simple phenomena encountered in the realistic world.

☺ Develop space concepts during the processes of exploring the shapes, sizes, relative positions and motions of simple objects and figures.

☺ With the help of the teachers begin to learn how to choose useful information to undertake simple induction and analogy.

☺ Able to engage in simple, systematic thinking during the problem solving processes.

Second Stage (Grade 4~6)

☺ Able to explain appropriately numbers and data encountered in the realistic world, and able to use numbers, alphabets and diagrams to describe and solve simple problems encountered in the realistic world.

☺ Develop space concepts during the processes of exploring relative positions of objects, characteristics of figures, transformation of figures, and design of symbols and patterns.

☺ Begin to develop reasonable inferential abilities by collecting useful information to undertake induction, analogy and guessing in accordance with the problem solving requirements.

☺ Able to engage in systematic thinking, and justify the appropriateness of conclusions in a convincing manner during the problem solving processes.

Third Stage (Grade 7~9)

☺ Able to explain and infer appropriately relatively large amount of numbers and data encountered in concrete contexts, and able to use algebraic expressions, equations, inequalities and functions to depict the inter-relationships amongst objects and events.

☺ Begin to establish space concepts and develop geometrical intuition during the processes of exploring characteristics of figures, transformation of figures, also during the activities involving mutual conversion between plane figures and spatial geometrical objects.

☺ Able to make reasonable inferences or bold guesses after collecting, selecting and processing mathematical data.

☺ Able to use concrete examples to examine mathematical conjectures so as to increase the credibility of these conjectures, or falsify them.

☺ Able to realize the necessity of proofs, begin to develop deduction abilities.

Problem Solving

First Stage (Grade 1~3)

☺ With the help of the teachers able to discover and pose simple mathematical problems in everyday living.

☺ Familiarize with that there are different ways of solving the same problem.

☺ Have experiences in cooperating with peers to solve problems.

☺ Begin to acquire how to express the general problem solving processes and the solutions.

Second Stage (Grade 4~6)

☺ Able to discover and pose simple mathematical problems in the realistic world.

☺ Able to explore effective methods to solve problems, and attempt to seek other methods.

☺ Able to deploy calculators to solve problems.
☺ Begin to learn how to cooperate with peers during the processes of problem solving.
☺ Able to express the problem solving processes, attempt to explain the obtained solutions.
☺ Are aware of the needs to reflect and analyze the problem solving processes.

Third Stage (Grade 7~9)
☺ Able to discover and pose mathematical problems taking into account of the concrete contexts and situations.
☺ Attempt to seek methods to solve problems from different angles, and able to solve the problems effectively; attempt to evaluate the differences of the methods.
☺ Experience the importance of cooperating with others during the processes of problem solving.
☺ Able to use words, alphabets, diagrams and tables to express clearly the problem solving processes, attempt to justify the solutions.
☺ Acquire experiences of problem solving by reflecting the problem solving processes.

Affection and Attitudes

First Stage (Grade 1-3)
☺ With the help and encouragement of others demonstrate curiosity in events around us and in objects related to mathematics; able to participate actively in animating, intuitive mathematical activities.
☺ With the help and encouragement of others able to overcome problems encountered in mathematical activities; acquire experiences of success and are confident in learning mathematics well.
☺ Familiarize with that one can use numbers and forms to describe phenomena; feel the intimate relationships between mathematics and everyday living.
☺ Engage in observation, operation, induction and other processes of mathematics learning; feel the reasonableness of mathematical thinking processes.
☺ Under the guidance of others able to discover and rectify errors promptly in mathematical activities.

Second Stage (Grade 4-6)
☺ Demonstrate curiosity in objects related to mathematics that have bearings with the surrounding environments; able to participate actively in mathematical activities organized by the teachers.
☺ With the direction and encouragement of others able to actively overcome problems encountered in mathematical activities; have successful experiences in overcoming difficulties and deploying knowledge to solve problems; possess certain degree of confidence in obtaining, or awareness of failing to obtain, correct solutions to problems; believe that one can keep on advancement in learning.
☺ Experience the intimate relationships between mathematics and everyday living; know that many practical problems can be solved by mathematical methods, as well as represented and communicated using mathematical languages.
☺ Through observation, operation, induction, analogy, inferences and other mathematical activities, experience the investigative and challenging characters

of mathematical problems; feel the systematic character of mathematical thinking processes, and the certainty of mathematical conclusions.

- ☺ Conscious of querying things that one do not understand and that arise from different perspectives; willing to discuss mathematical problems, discover errors and rectify them promptly.

Third Stage (Grade 7-9)

- ☺ Happy to contact mathematical information in the societal environments; willing to talk about some mathematical issues and topics; able to play an active role in mathematical activities.
- ☺ Dare to confront problems encountered in mathematical activities; have successful experiences in overcoming difficulties independently by oneself and deploying knowledge to solve problems; possess confidence in learning mathematics well.
- ☺ Experience that numbers, symbols and figures are effective means of describing the realistic world; know that mathematics is an important tool to solve practical problems, and to take part in communications; familiarize with the functions of mathematics in promoting advancements in society and developments of rationality of humankind.
- ☺ Know that through observation, experimentation, induction, analogy and inferences one can obtain mathematical hypotheses; experience that mathematical activities are filled with investigative and creative characters; feel the necessity of proofs, the rigor of the proving process, as well as the certainty of the conclusions.
- ☺ Based upon the foundation of independent thinking participate actively in discussion of mathematical problems; dare to express one's viewpoints, as well as respect and understand others' proposals. Able to benefit through communicating with others.

Section Three – Content Standards

This section explicates in turn the content standards of the four domains of study of each of the three stages of schooling, namely Numbers and Algebra, Space and Figures, Statistics and Probability, and Practical and Integrated Applications.

Numbers and Algebra mainly comprises of the following contents: numbers and expressions, equations and inequalities, and functions. They serve as mathematical models for studying quantitative relationships and change patterns. It enables us to know, describe and grasp the realistic world from the point of view of quantitative relationships with precision and clarity.

Space and Figures involves mainly objects in the phenomenal world, as well as relationships and transformation of shapes, sizes and positions of geometrical and plane figures. It is an important tool used for knowing and describing the space around us, as well as for communicative purposes.

Statistics and Probability mainly studies data in our daily living and random phenomena in the objective world. Through data collection, organization, description and analyses, as well as depicting the chances of events going to be happened, appropriate inferences and prediction can be made.

Practical and Integrated Applications will help students deploy acquired knowledge and experiences in an integrated manner. Through autonomous explorations and cooperative exchanges, students solve fairly challenging and integrated problems that are intimately related to their daily living so as to develop problem solving abilities. It deepens students' understanding in such content areas as Numbers and Algebra, Space and Figures, and Statistics and Probability, as well as allows students to realize the interrelationships of these areas.

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Space and Figures	- Knowing Figures - Measurements - Figures and their Transformation - Figures and their Positions	- Knowing Figures - Measurements - Figures and their Transformation - Figures and their Positions	- Knowing Figures - Figures and their Transformation - Figures and their Coordinates - Figures and Proofs
Statistics and Probability	- Statistical Data Activities for Beginners - Phenomenon of Uncertainty	- Simple Statistical Data Processing - Possibility	- Statistics - Probability
Practical and Integrated Applications	- Practical Activity	-Integrated Applications	- Thematic Studies

First Stage of Schooling (Grade 1~3)

1. Numbers and Algebra

At this stage of schooling, students learn numbers up to ten thousand, simple fractions and decimals, and common quantities. They experience the meaning of numbers and computation, grasp basic number computation skills, explore and understand simple quantitative relationships.

In teaching, it is necessary to guide students to relate concrete and interesting things around them. Through rich activities such as observation, manipulation, and problem solving, students experience the function of numbers in expression and communication, as well as to start to develop number sense. There is a need to pay more attention to mental computation (i.e. oral computation), strengthen estimation skills, and promote diversified use of algorithms. However, skill-based training for stand-alone simple purposes should be lessened; complicated calculations and

description of "computational rules" in a programmatic fashion should be avoided.

(1) Concrete Objectives

i. Knowing Numbers

(i) Able to recognize, read and write numbers up to ten thousand; able to use numbers to indicate the quantity of objects, or order and positions of events and objects.

(ii) Know the meaning of the symbols <, =, and >; able to use symbols and words to describe the magnitude of numbers up to ten thousand. [Please refer to example 1]

(iii) Able to name the numerical digits of a number; identify the meanings of numerals corresponding to different numeral digits.

(iv) Able to feel the meaning of large numbers when associated with realistic materials, as well as to do estimation. [Please refer to example 2 & 3]

(v) Able to comprehend the preliminary meaning of fractions when associated with concrete situations; able to recognize, read and write decimals and simple fractions.

(vi) Able to use numbers to represent events and objects in daily living, as well as to engage in communications. [Please refer to example 4]

ii. Number Operations

(i) Coupled with concrete situations, experience the meaning of the four rules of arithmetic operations.

(ii) Able to compute addition and subtraction orally for numbers within 20 in a proficient manner, as well as using tables of multiplication and division. Able to compute addition and subtraction orally for numbers within one hundred.

(iii) Able to compute addition and subtraction involving three-digits numbers, multiplication of a one-digit by a three-digits number and a two-digits number by a two-digits number; as well as division of a three-digits number by a one-digit number.

(iv) Able to compute addition and subtraction of fractions with same denominators (denominator smaller than 10), as well as one-digit decimals.

(v) Able to estimate in concrete situations, and explain the process of estimation.

(vi) Engage in communication processes within which one can share one's algorithms with others.

(vii) Able to solve simple problems in daily living flexibly using a variety of methods, and to judge whether the results are reasonable or not. [Please refer to example 6]

iii. Common Quantities

(i) Acquire knowledge of dollar, ten cents and cent in concrete situations and

contexts, as well as understand their interrelationships.

(ii) Acquire knowledge of clocks and watches and familiarize with 24-hour time system; Coupled with one's life experiences experience that time duration can be long or short. [Please refer to example 7]

(iii) Acquire knowledge of year, month and day and familiarize with their interrelationships.

(iv) Feel and acquire knowledge of gram, kilogram and ton, as well as to convert simple quantities from one to another.

(v) Coupled with realistic daily living, solve simple problems related to common quantities.

iv. Exploring Patterns

Discover simple rules and patterns inherent in given events and objects. [Please refer to example 8]

(2) Exemplary Cases

Example 1 Use terms like *a bit greater than*, *a bit smaller than*, *very much greater than* and *very much smaller than* to describe the magnitude relationships of the following numbers: 50, 98, 38, 10 and 51. Use symbols ">" or "<" to represent their magnitude relationships.

Example 2 What is the approximate thickness of 1200 sheets of paper? What is the approximate number of classes that may be formed comprising of 1200 students? What is the approximate length of 1200 footsteps?

Example 3 Estimate the number of words (or characters) contained in one whole page of a newspaper.

Remark If the page is divided into a number of equal parts, one can estimate number of words in the whole page using any one part's number of words.

Example 4 Please talk about numbers intimately related to daily living, as well as their functions and effects.

Remark Numbers such as student ID, class ID, shoe-size, weight and height.

Example 5 Suppose the fee for entering the park is $8 and a school organizes 97 students to visit the park, decide whether $800 is enough or not?

Example 6 Suppose one small boat can only carry 4 persons, how many boats

are needed for 17 persons? Please consider ways one can allocate persons in a reasonable manner.

Example 7 Estimate the number of beats of the pulse, number of words read, number of times of skipping, number of steps walked in one minute.

Example 8 Please fill in an appropriate number or symbol in the following blanks, and give reasons:

1, 1, 2, 1, 1, 2, ____, ____, ____;

<insert figures here>

[1] Concerning multiplication: 5+5+5 may be written as 5x3 or 3x5. 3x5 is pronounced as 3 times 5. Both 3 and 5 are multipliers (or factors).
Concerning division: It is not required to use terms such as first kind of division (i.e. dividing objects into equal parts by distributing objects one by one), second kind of division (i.e. dividing objects into certain number of equal-sized parts).

2. Space and Figures

At this stage of schooling, students shall acquire simple geometrical objects and plane figures, as well as feel phenomena of translation, rotation and symmetry. Students learn ways to describe objects' relative positions, engage in simple measurement activities, and establish initial conceptions of space.

During teaching, it is important to pay attention to the intimate relationships between knowledge acquired and everyday living. Likewise, during activities needed observation and manipulation, it is important to allow students to experience intuitively simple geometrical objects and plane figures.

(1) Concrete Objectives

i. Knowing Figures

(i) Through real objects and models distinguish amongst cuboid, as well as cubic, cylindrical and spherical objects.

(ii) Recognize shapes of simple objects observed from the front, side and the top. [Please refer to example 1]

(iii) Recognize simple figures such as rectangles, squares, parallelograms and circles.

(iv) Through observation and manipulation, able to use one's own language to describe the characteristics of rectangles and squares.

(v) Able to use rectangles, squares, triangles, parallelograms or circles to construct diagrams and figures.

(vi) Coupled with everyday living situations, acquire knowledge of angles; able to distinguish right, acute and obtuse angles.

(vii) Able to classify simple geometrical objects and figures.

ii. Measurements

(i) Coupled with everyday realities, use different ways to measure length of objects. During these measurement activities, experience the importance of establishing unified measurement units.

(ii) Experience the meanings of kilometre, metre, and centimetre during practical activities. Know decimetre and millimetre. Able to do simple unit conversion. Know how to choose units of length appropriately. [Please refer to example 2]

(iii) Able to estimate the length of some objects, and engage in measuring them.

(iv) Point out and measure the perimeter of concrete figures. Explore and grasp the formulae of a rectangle and a square. [Please refer to example 3]

(v) Coupled with concrete examples, acquire meanings of an area. Able to use units of one's choice to estimate and measure the area of a figure. Experience and acquire knowledge of units of area (centimetre^2, metre^2, kilometre^2, hectare). Able to do simple unit conversion. [Please refer to example 4]

(vi) Explore and grasp the area formulae of rectangle and square. Able to estimate the area of given rectangles and squares.

iii. Figures and their Transformation

(i) Coupled with concrete examples, recognize phenomena of translation, rotation and symmetry.

(ii) Able to draw on a grid paper the results of translating simple figures along the horizontal and vertical directions.

(iii) Through observation and manipulation, acquire knowledge of figures with axes of symmetry. Able to draw on a grid paper the axes of symmetry of simple figures.

iv. Figures and their Positions

(i) Able to use *up*, *down*, *left*, *right*, *front* and *back* to describe the relative positions of objects.

(ii) Given any one of the four directions (east, south, west, and north),

distinguish the directions of the other seven directions (east, south, west, north, north-east, north-west, south-east, and south-west). Able to use these terminologies to describe the direction of an object. Able to read simple route maps.

(2) Exemplary Cases

Example 1 A kettle is placed on a table. Four students observe it from their own directions.

<Insert Picture here>

Indicate which of the following four views are seen by the students.

<Insert Picture here>

Example 2 One metre is equivalent to _____ pencils. The length of the railway from Bejing to Nanjing is approximately 1000 _____.

Example 3 Measure the perimeter of an irregular figure (e.g. a leaf).

Example 4 Use a square paper as a measurement unit to measure the area of the top of the desk.

Example 5 Which of the followings are phenomena of translation and rotation?
(1) rotation of a steering wheel; (2) rotation of on/off switch of a water tap; (3) up/down movements of an escalator; (4) motion of a pendulum.

3. Statistics and Probability

At this stage of schooling, students shall experience statistical data processing and learn some simple methods of data collection, organization and description. Students are able to reply simple questions in accordance with the statistical results. They shall have an initial feeling on the uncertainty and possibility of happening of events.

During teaching, teachers should pay attention and use examples drawn from everyday living. This allows students to have a feel of simple statistical data processing. The focus should be targeted upon intuitive feeling of uncertainty and possibility.

(1) Concrete Objectives

i. Statistical Data Activities for Beginners

(i) Able to compare, order and classify objects in accordance with specified standards or standards of one's choice (e.g. quantity, shapes, color). Experience consistency of results of these activities when standards are the same, whereas in the case of different standards experience variety of results instead.

(ii) Experience processes of data collection, organization, description and analyses.

(iii) Through real examples, acquire knowledge of statistical tables, pictograms and bar charts (one grid represents one unit), and proceed to complete the associated tables and graphs.

(iv) Able to use appropriate methods (e.g. counting, measuring, experimenting) to collect data in accordance with simple questions. In addition, able to record data in statistical tables. [Please refer to example 1]

(v) Through a variety of real examples, familiarize with the meaning of a mean. Able to calculate the mean of simple data (the answer is a whole number).

(vi) Know that one can obtain statistical information from newspaper, magazine and television media.

(vii) Raise and answer simple questions in accordance with data recorded in statistical tables and graphs. Able to exchange ideas with peers.

ii. Phenomenon of Uncertainty

(i) Have an initial experience that the happening of some events is certain, whereas some are uncertain. [Please refer to example 2]

(ii) Able to list all possible outcomes of simple experiments.

(iii) Knows that the possibility of an event happened can be large or small. [Please refer to example 3]

(iv) Able to describe the possibility of happening of simple events, and to exchange ideas with peers. [Please refer to example 4]

(2) Exemplary Cases

Example 1 Examine the number of beats of your pulse after running and have it compared with that before running. Record the measurement data and exchange ideas with your peers.

Example 2 Which of the following phenomena are certain events?

(1) There will be rain here next Wednesday. (2) Someone will walk tomorrow.

Example 3 Randomly draw one ball from a pocket containing four red balls and one black ball. Compare the possibility of drawing a red ball with that of a black ball, which is greater?

Example 4 Use terminologies of "certain", "often", "occasional" and "impossible" to describe the possibility of events happened in daily living.

4. Practical Activities

At this stage of schooling, students begin to acquire some mathematical experiences through practical activities and familiarize with simple application of mathematics in everyday living. Students begin to learn to cooperate and exchange ideas with others, so as to acquire constructive affection in mathematics learning.

During teaching, priority should be given to attention on student participation in activities. Students are guided to think constructively, exchange ideas actively, as well as initiate to collaborate with others. As a result, students are increasingly confident in deploying mathematics to solve simple practical problems, and at the same time realize the roles they play in the community.

(1) Concrete Objectives

i. Involve in practical activities on observation, manipulation, experimentation, survey and inferences. Acquire good affective experiences during processes of cooperation and exchange of ideas.

ii. Begin to acquire some experiences related to mathematical practical activities. Able to deploy the knowledge and methods learned to solve simple problems.

iii. Feel the role and function of mathematics in everyday living.

(2) Example

Example A class would like to tour three scenic spots in the region. The timing is from 8:00-16:00. Please design a tour plan, including the schedule, fees and itinerary.

Remark During the problem solving processes, students are engaged in the following activities:

i. Familiarize with the related information, including the itinerary connecting the scenic spots, type of vehicle and the renting fees, favorite food of classmates, and things needed to be brought during the trip.

ii. Use numbers, figures, statistical graphs and tables to represent the information

gathered.

iii. Calculate the total time needed for the car journey, the touring time spent for visiting each of the three scenic spots, total expenditure, and fees paid by each participating student.

iv. Divide into small groups to design the tour plan and engage in exchange of ideas.

Through problem solving, students can increase their abilities in collecting and organizing data, as well as inculcate awareness of cooperation with other persons.

Second Stage of Schooling (Grade 4~6)

1. Numbers and Algebra

At this stage of schooling, students move one step further to learn whole number, fraction, decimal, percentile, as well as the associated computations. Students develop number sense further, begin to familiarize with negative number and equation, start to use calculators to carry out complicated computation and explore mathematical problems. They possess the ability to solve simple problems in their everyday living.

During teaching, there is a need to develop further number sense through solving practical problems, so as to increase students' understanding of the meaning of computation. Oral computation should be emphasized, estimation should be strengthened, and a variety of algorithms should be encouraged. Students should involve in practical problem solving to abstract quantitative relationships, and deploy knowledge learned to solve problems. There is a need to avoid complicated computations, separation of computation and application, and mechanical programmatic drilling of application problems.

(1) Concrete Objectives

i. Knowing Numbers

(i) Recognize, read and write numbers within one hundred millions in concrete situations. Familiarize with decimal counting system. Able to use ten thousands and one hundred millions to represent large numbers.

(ii) Move a step further to acquire decimal and fraction, and begin to know percentile. Explore the interrelationships of decimal, fraction and percentile, as well as able to carry out conversion (excluding converting a recurring decimal into a fraction).

(iii) Able to compare the magnitudes of decimal, fraction and percentile.

(iv) Familiarize with the meaning of negative number in familiar everyday living context. Able to use negative numbers to represent some problems encountered in everyday living.

(v) Coupled with realistic situations to have a feel on the meaning of large numbers. Able to carry out estimation. [Please refer to example 1]

(vi) Move a step further to experience the application of numbers in everyday living. Able to use numbers to represent objects, and to engage in exchange of ideas. [Please refer to example 2 & 3]

(vii) For all natural numbers within 1 to 100, locate all the multiples of a natural number that is less than 10. Know the characteristics of multiples of 2, 3 and 5. Able to find the common multiples and least common multiple of two natural numbers that are less than 10.

(viii) For all natural numbers within 1 to 100, able to find all the factors of a natural number. Able to find the common factors and greatest common factor of two natural numbers.

(ix) Know whole number, odd number, even number, prime number and composite number.

ii. Number Operations

(i) Able to carry out oral computation of multiplying and dividing a two-digits number by a one-digit number.

(ii) Able to carry out hand computation of multiplying a three-digits number by a two-digits number, as well as dividing a three-digits number by a two-digits number.

(iii) Able to combine with realistic materials to understand order of computation. Able to carry out simple mixed arithmetic operations that involves whole numbers. (mainly two steps, not exceeding three steps)

(ix) Explore and understand operation rules. Able to use operation rules to carry out simple computation.

(v) During concrete computation and solving of simple practical problems, able to experience the inverse relationships of addition and subtraction, as well as multiplication and division.

(vi) Able to carry out simple computation involving decimals and fractions (excluding mixed number) individually. Computations include addition, subtraction, multiplication, division and their mixed operations. (mainly two steps, not exceeding three steps)

(vii) Able to solve simple practical problems related to decimals, fractions and percentiles.

(viii) During the process of solving practical problems, able to select appropriate estimation method, and develop habits of estimation. [Please refer to example 4 to 6]

(ix) Able to use a calculator to carry out more complicated computations, solve simple practical problems, and explore simple mathematical patterns. [Please refer to example 7]

iii. Expressions and Equations

(i) Able to use letters to represent numbers in concrete situations and contexts.

(ii) Able to use an equation to represent equal quantitative relationships in simple

contexts.

(iii) Understand the property of equality. Able to use the property of equality to solve simple equations (e.g. $3x + 2 = 5$, $2x – x = 3$).

iv. Direct Proportion/Inverse Proportion

(i) Understand the meaning of "allocation by proportion" in practical situational contexts, and able to solve simple problems.

(ii) Through concrete problems acquire knowledge of quantities in direct or inverse proportions.

(iii) Able to draw graphs on a grid paper with a rectangular coordinates system to depict data with direct proportional relationships, as well as to estimate the value of a quantity given a value of the other. [Please refer to example 8]

(iv) Able to give real life examples of direct proportion and inverse proportion, and engage in exchange of ideas.

v. Exploring Patterns

Explore the implicit pattern or change tendency of given objects and events.

(2) Exemplary Cases

Example 1 Approximately how long does it take for a heart of a normal person to beat one million times? How many years are equal to one million hours? How thick is one million sheets of paper?

Example 2 A school assigns an ID to each of its students. Use 1 as the last digit to indicate a boy and 2 a girl. 9713321 means that the assigned student is a boy. He entered school in 1997, studied in grade 1 class 3 with student number 32. Hence, indicate what information is contained in 9532012 (year of entry, grade level, class, student number, sex of student)?

Example 3 Do you like mathematics? If 5, 4, 3, 2, 1 are used to indicate the degree of liking from "like most" to "dislike most", which number will you choose? Give reasons. If Michael chooses 2, what does it mean? If Albert likes mathematics to some extent, which number is likely chosen?

Example 4 Aunt Lee would like to buy two bags of rice (each bag costs $35.4), beef ($14.8), vegetable ($6.7) and fish ($12.8). She had 100 dollars. Is it sufficient?

Example 5 What is the approximate value of 9.2 x 7.1? Which is greater? 1 or

the result of 1/2 + 4/7?

Example 6 Guesstimate the mass of one peanut.

Remark One can guesstimate by weighing 50 peanuts. One can also guesstimate by counting how many peanuts weigh 100*g*.

Example 7 Given any four numbers that are mutually different, form a number that is largest and another one smallest, then deduct the largest one by the smallest one. Repeat the steps above on the four digits obtained, what do you discover? (use calculator)

Example 8 A ribbon costs $4 per metre. How much does it cost to buy 2*m*, 3*m*, etc.?

Fill in the table below:

<Insert Table> Length/metre Cost/dollar

Plot the points corresponding to the ordered pair of "length" and "cost" on a sheet of rectangular coordinates grid paper, join the points sequentially, and answer the following questions:

a. Are the points lying on a straight line?
b. Estimate approximately how much does it cost to buy 1.5*m* of ribbon?
c. The length of ribbon bought by Gary is triple of that of Helen. What multiple is the money spent by Gary when compared with that of Helen?

Example 9 Complete the following sequence. Give reasons.

0.5, 1.5, 4.5, _____.

Example 10 Michael decorated the classroom for a gala party with a string of colored balloons in accordance with the sequence: 3 red balloons, 2 yellow balloons, 1 green balloon. Do you know the color of the 16^{th} balloon?

Remark Students may use different methods to solve this problem. For example, use A to indicate a red balloon, B a yellow balloon, and C a green balloon. Then the sequence can be written as

AAABBCAAABBC…..

If we can locate the 16^{th} letter, one can deduce the color of the 16^{th} balloon.

2. Space and Figures

At this stage of schooling, students shall familiarize with some basic characteristics of simple geometrical objects and plane figures. They proceed to learn transformation of figures and methods of locating the positions of objects so as to develop space conceptions.

During teaching, attention should be paid to enable students to explore problems related to space and figures in the realistic world. Likewise, through strategies of observation, manipulation and deduction, students step by step know the shapes, magnitudes, relative positions and transformations of simple geometrical objects and plane figures. Students can inculcate space conceptions through activities such as observing objects, recognizing directions, building models and designing patterns.

(1) Concrete Objectives

i. Knowing Figures

(i) Familiarize with that a line can be ascertained by two points, and two intersecting straight lines determine a point.

(ii) Able to discriminate amongst straight line, segment and ray.

(iii) Experience that amongst all lines connecting two points the segment is the shortest. Know the distance between two points.

(iv) Know concepts of round angle and straight angle, as well as interrelationships of round angle, straight angle, obtuse angle, right angle and acute angle.

(v) Coupled with everyday living situations, familiarize with the parallel and intersection relationships of two lines (including perpendicular relationships).

(vi) Through observation and manipulation, acquire knowledge of parallelogram, trapezium and circle. Able to use compasses to draw circles.

(vii) Know triangles. Through observation and manipulation, familiarize with that the sum of the length of two sides of a triangle is greater than the third side. Sum of interior angles of a triangle is 180 degrees.

(viii) Know isosceles triangle, equilateral triangle, right-angled triangle, acute-angled triangle, and obtuse-angled triangle.

(ix) Through observation and manipulation, acquire knowledge of cuboid, cube, cylinder and cone. Know the nets of a cuboid, cube and cylinder.

(x). Able to discriminate amongst the shapes and relative positions of objects seen from different orientations. [Please refer to example 1]

ii. Measurement

(i) Able to use a protractor to measure the degrees of a specified angle. Able to construct an angle of specified degree. Able to use set squares to construct angles of

30°, 45°, 60° and 90°.

(ii) Use a grid paper or methods such as cut-and-paste to explore and master area formulae of a triangle, parallelogram and a trapezium.

(iii) Explore and master the area and circumference formulae of a circle.

(iv) Able to use grid paper to estimate the area of an irregular figure. [Please refer to example 2]

(v) Through real examples, familiarize with the meanings of volume (including capacity) and units of measurement (metre^3, decimetre^3, centimetre^3, litre, millitre). Able to convert form one unit to another. Feel the practical meanings of 1 metre^3, 1 centimetre^3, 1 litre and 1 millilitre.

(vi) Coupled with concrete situations, explore and master the computational methods of volume and surface area of cuboid, cube and cylinder, as well as volume of cone.

(vii) Explore the measurement methods of some real objects. [Please refer to example 3]

iii. Figures and their Transformation

(i) Use methods such as paper folding to ascertain the axes of symmetry of axisymmetry figures. Able to draw on a grid paper an axisymmetry figure of a given figure.

(ii) Able to use methods such as grid paper to magnify or reduce the size of a figure according to some specified ratio so as to experience the similarity of figures.

(iii) Through observing real examples, acquire knowledge of translation and rotation of a figure. Able to use a grid paper to carry out translation of simple figures, or rotation by 90°.

(iv) Appreciate figures encountered in daily living. Able to deploy flexibly translation, symmetry and rotation to design patterns on a piece of grid paper.

iv. Figures and their Positions

(i) Familiarize with scale. In concrete situations, able to convert a distance on a diagram into actual distance in accordance with the specified scale of the diagram.

(ii) Able to ascertain the position of an object in accordance with the direction and distance.

(iii) Able to describe simple route diagram. [Please refer to example 6]

(iv) In concrete situations, able to use numbers to represent positions, as well as using a pair of numbers to ascertain position on a grid paper. [Please refer to example 7]

(2) Exemplary Cases

Example 1 A set of blocks is shown below:

<Insert diagram>

Indicate the corresponding figure seen from the front, right and from the top.

<Insert diagram>

Example 2 In the diagram below, one small grid represents one square unit. Try to estimate the area enclosed by the curve.

<Insert diagram>

Example 3 How to measure the volume of a potato?

Remark The potato can be immersed into water and measured indirectly.

Example 4 Which of the following figures can coincide with each other after translation?

<Insert diagram>

Example 5 Suppose the gate is situated 50*m* south of the classroom, whereas the library is situated in a direction 60° deviated from the north to the east and at a distance 100*m* away from the classroom. Try to draw an illustrative diagram.

Example 6 Draw an illustrative diagram showing the route of school to your home, indicating the direction and major reference objects.

Example 7 Cindy sits on the 3^{rd} column and 4^{th} row of a classroom. This can be represented as (4, 3). If May sits on the 1^{st} column and 3^{rd} row of the same classroom, how should her position be represented?

3. Statistics and Probability

At this stage of schooling, students shall experience simple statistical data processing. They move a step further to learn methods of collecting, organizing and describing data, as well as to make simple judgment and prediction in accordance with the outcomes of the data analysis. They experience further the meaning of the possibility of an event happened, and are able to calculate the possibility of a simple event happened.

During teaching, teachers should pay attention to the intimate relationships

between contents of learning and realistic everyday living. Attention should be paid to enable students to experience consciously simple statistical data processing, make simple judgment and prediction in the light of data, and engage in exchange of ideas. Experiences on possibilities in concrete situations should not be neglected. There is a need to avoid students carrying out sheer calculation of statistics.

(1) Concrete Objectives

i. Simple Statistical Data Processing

(i) Experience in a simple way processes of collecting, organizing, describing and analyzing data (use calculator if necessary).

(ii) Design a simple questionnaire in accordance with practical problems.

(iii) Through real examples, move a step further to know bar charts (1 grid represents several units). Know broken-line graph and fan-shaped pie chart. According to needs, able to select bar chart or broken-line graph to represent data intuitively.

(iv) Through rich real examples, understand the meanings of mean, median and mode. Able to obtain the mean, median and mode of data, and to explain their meanings from the practical point of view. According to concrete problems, able to select appropriate statistics to reflect the different characteristics of data. [Please refer to example 1 & 2]

(v) Able to acquire some data information consciously from newspapers, magazines and television, and can comprehend simple statistical graphs and tables. [Please refer to example 3]

(vi) Able to design statistical activities, and test some predictions. [Please refer to example 4]

(vii) Able to explain statistical results. Able to make simple judgment and prediction according to the results, and engage in exchange of ideas.

(viii) Begin to experience that people can be misled by data. [Please refer to example 5]

ii. Possibility

(i) Experience equal possibility of events happened, and fairness of the rule of the game. Able to obtain the possibility of simple events happened.

(ii) Design a project that meets specified requirements. [Please refer to example 6]

(iii) Make predictions of the possibility of a simple event happened, and to explicate one's reasons. [Please refer to example 7]

(2) Exemplary Cases

Example 1 The mean height of students in Michael's class is 1.4 metre. The mean

height of students in Karen's class is 1.5 metre. Is it necessary that Michael is shorter than Karen?

Example 2 Select an appropriate statistics to indicate the most favorable color of students in the class.

Example 3 In the special issue of China Daily News celebrating the 1st October 1999 National Day, information on China's city development during the past 50 years of national development was released. A statistical graph showing the number of cities is excerpted below. What information do you obtained in this statistical graph? Exchange your ideas with your peers.
<Insert diagram> Statistical Graph of Number of Cities in China

Example 4 Estimate the number of plastic bags discarded by all families of students of your class. Validate your estimation through practical surveys.

Example 5 There are 15 staff members in a company. It is announced in the job advertisement that the average wage of the staff exceeds 1200 dollars. Please analyze the statistical table below. How do you conceive the figure announced in the company's job advertisement?
<Insert diagram> Rank, Manager, Deputy Manager, Staff, Number of Persons, Monthly Wage/Dollar

Example 6 Label numbers on the six sides of a cube so that the possibility of having a "2" facing up is 1/3.

Remark The numbers labeled on the six faces of the cube can be 1,2,2,3,4,5.

Example 7 Conduct a survey of tournament results of two rival ball teams to reveal the win-lose situations. Predict which team is going to win in the forthcoming contest, and explain your reasons.

4. Integrated Applications

At this stage of schooling, students shall familiarize with the vast relationships between mathematics and everyday livin, through mathematical activities. Students integrate and deploy the knowledge and methods learned to solve simple practical problems. This will deepen their understanding of what has been learned. Students

acquire thinking methods that allow mathematics to be used to solve problems, as well as engage in cooperation and exchange of ideas with others.

During teaching, teachers should guide students to discover the rich mathematical information contained in practical problems from different angles of view, explore a multitude of problem solving methods, as well as encourage students to try to solve some simple practical problems independently.

(1) Concrete Objectives

i. Possess successful experiences that integrate and deploy numbers and operations, space and figures, statistics and probability and their related knowledge to solve some simple practical problems. Begin to build up confidence to use mathematics to solve problems.

ii. Acquire experiences of activities and methods that can be used to integrate and deploy the knowledge learned to solve simple practical problems.

iii. Begin to feel the connections amongst the different branches of mathematical knowledge, experience the usefulness and functions of mathematics.

(2) Exemplary Cases

Example 1 Design an appropriate method of packaging.

i. There are 4 cassette tapes. How many ways are available to package them? Which method uses less paper for the packaging? (You may ignore the overlapping parts)

ii. Suppose you have 8 cassette tapes, which method uses less paper for the packaging? (You may ignore the overlapping parts)

Remark This is a commonly encountered problem in everyday living. Through solving such problems, teachers can inculcate students' abilities to integrate and deploy the knowledge learned to solve practical problems.

Example 2 How high is the TV Tower in Shanghai? How high is the TV Tower in Beijing? How many classrooms of which the heights taken together are equivalent to that of a TV Tower? How many students joining hand in hand of which their span is equivalent to that of a TV Tower? Is there any method to describe the height of the TV Tower in an intuitive manner?

Remark This problem can increase students' feeling and knowledge of large numbers. At the same time, students can learn how to collect data through surveys such as questioning and enquiries.

Third Stage of Schooling (Grade 7~9)

1. Numbers and Algebra

At this stage of schooling, students shall learn knowledge of real number, integral and fraction expression, equation and system of equations, inequality and system of inequalities, and function. They explore the relationships and patterns implicit in numbers, space and practical problems. They begin to master some tools that may be deployed effectively to represent, process and exchange quantitative relationships and patterns of variation. They develop symbol sense, experience the intimate relationship between mathematics and realistic everyday living, enhance awareness of application, and raise the abilities of deploying knowledge and methods of algebra to solve problems.

During teaching, teachers should pay attention to allow students to comprehend quantitative relationships and patterns of variations in practical contexts and situations. Teachers should seek to enable students to build mathematical models of practical problems, and engage in processes of estimating, finding solutions, validating the correctness and reasonableness of solutions. Teachers should enhance the connection of contents like equation, inequality and function, and to introduce the geometrical background of some algebra contents. Complicated operations are to be avoided.

(1) Concrete Objectives

i. Numbers and Expressions

(i) Rational Number

(a) Understand the meaning of rational number. Able to use the points on the number line to represent rational numbers. Able to compare the magnitude of rational numbers.

(b) Make use of the number line to understand the meanings of opposite numbers and absolute values. Able to find the opposite number and the absolute value of a rational number. (No letter is within the absolute value symbol)

(c) Understand the meaning of power. Master addition, subtraction, multiplication, division, power, and simple mixed operations of rational numbers (mainly three steps).

(d) Understand the operation rules of rational numbers; able to use operation rules to simplify computations.

(e) Able to deploy operations of rational numbers to solve simple problems.

(f) Able to explain and predict with justification using information of relatively large numbers. [Please refer to example 1]

(ii) Real Number

(a) Familiarize with the concept of square root of a number, radical of a number (i.e. positive square root of a number) and cube root. Able to use the radical sign to represent the square root and cube root of a number.

(b) Familiarize with that power and root are both inverse operations. Able to use the square root operation to evaluate the square root of a non-negative number. Able to use the cube root operation to evaluate the cube root of a number. Able to use a calculator to evaluate square root and cube root.

(c) Familiarize with the concept of irrational number and real number. Know the one-one correspondence relationship between rational numbers and points on the number line.

(d) Able to use a rational number to estimate the approximate region of an irrational number. [Please refer to example 2]

(e) Familiarize with approximate number and the concept of significant figure. Able to use a calculator to approximate an answer so as to solve practical problems, and give an approximate value to the result obtained in accordance with the requirements of the question.

(f) Familiarize with the concept of radical expression of second degree, and the addition, subtraction, multiplication and division operation rules. Able to use these concepts and operation rules to carry out simple arithmetic operations of real numbers. (rationalization of denominator is not required)

(iii) Algebraic Expression

(a) Within realistic contexts, move a step further to familiarize with the meaning of using a letter to represent a number.

(b) Able to analyze quantitative relationships of simple problems, and to use algebraic expressions for their representation. [Please refer to example 3 & 4]

(c) Able to explain the practical background and geometrical meanings of some simple algebraic expressions. [Please refer to example 5]

(d) Able to evaluate algebraic expressions. Able to seek information in accordance with the specified problem; find the necessary formulae, and to substitute concrete values for its evaluation.

(iv) Integral and Fraction Expression

(a) Familiarize with the meaning and basic properties of integral index power. Able to use scientific notation to represent a number. (Including display with calculators)

(b) Familiarize with the concept of integral expression. Able to carry out simple addition and subtraction operations of integral expression. Able to carry out simple multiplication operation of integral expression. (Multiplication of polynomials mentioned is merely multiplication of monomials)

(c) Able to derive the multiplication formulae: $(a + b)(a - b) = a^2 - b^2$; $(a + b)^2 = a^2 + 2ab + b^2$. Familiarize with the geometrical background of the formulae, and able to carry out simple computations.

(d) Able to use common factor extraction method and formula method (direct use of formulae cannot be more than twice) to carry out factorization. (Index is a positive whole number)

(e) Familiarize with the concept of fraction expression. Able to use the basic properties of fraction expression to carry out reduction, as well as reduction of fractions to a common denominator. Able to carry out simple addition, subtraction, multiplication and division operations of fraction expression. [Please refer to example 6]

ii Equation and Inequality

(i) Equation and System of Equations

(a) Able to construct an equation in accordance with the quantitative relationships of concrete problems. Experience that equation as a mathematical model is effective to depict the realistic world.

(b) Involve in the processes of observing, drawing and using the calculator to estimate the solution of an equation.

(c) Able to solve equation of first degree in one unknown, simple system of equations of first degree in two unknowns, as well as to simplify these into fraction equations of first degree in one unknown. (Number of fraction expressions in equation cannot exceed two)

(d) Understand method of completing squares. Able to use methods of factorization, formula, and completing squares to solve simple numerical coefficient equation of second degree in one unknown.

(e) Able to examine whether the result obtained is reasonable or not in accordance with the practical meaning of the concrete problem.

(ii) Inequality and System of Inequalities

(a) Able to familiarize with the meaning of inequalities in accordance with the magnitude relationships of concrete problems, and to explore the basic properties of

inequalities.

(b) Able to solve simple inequality of first degree in one unknown, and to indicate the solution set on the number line. Able to solve system of inequalities involving two inequalities of first degree in one unknown, and to use a number line to ascertain the solution set.

(c) Able to construct an inequality of first degree in one unknown and system of inequalities of first degree in one unknown in accordance with the quantitative relationships of concrete problems, and to solve simple problems.

iii Function

(i) Explore the quantitative relationships and variation patterns of concrete problems. [Please refer to example 8]

(ii) Function

(a) Through simple concrete examples, familiarize with the meanings of constant and variable.

(b) Able to make use of concrete examples to familiarize with concepts of function and the three methods of representation. Able to give concrete examples of a function.

(c) Able to make use of graphs to carry out analyses on the function relationships of simple practical problems. [Please refer to example 9]

(d) Able to determine the domain of independent variable of simple integral expression, fraction, and simple practical problem, as well as to evaluate the function value.

(e) Able to use appropriate function representation methods to depict the relationships of variables of some practical problems. [Please refer to example 9]

(f) Able to make use of analyses of function relationships to attempt an initial prediction of pattern variations of variables.

(iii) First Degree Function

(a) Coupled with concrete situations, realize the meaning of first degree function. Able to ascertain function expression in accordance with known conditions.

(b) Able to draw the graph of first degree function. Explore and understand the property of $y = kx + b$ ($k \neq 0$) based on explication of this expression and graph of first degree function. (i.e. variation of graph when $k > 0$ or $k < 0$)

(c) Understand direct proportion function.

(d) Able to find the approximate solution of equation of first degree in two unknowns in accordance with the graph of first degree functions.

(e) Able to use first degree function to solve practical problems.

(iv) Inverse Proportion Function

(a) Coupled with concrete situations and contexts, realize the meaning of inverse proportion function. Able to ascertain the expression of inverse proportion function in accordance with given conditions.

(b) Able to draw the graph of inverse proportion function. Explore and understand the property of $y = k/x$ ($k \neq 0$) based on explication of this expression and graph of first degree function. (i.e. variation of graph when $k>0$ or $k<0$)

(c) Able to use inverse proportion function to solve practical problems.

(v) Second Degree Function

(a) Ascertain the expression of second degree function through analyzing contexts of practical problems, and experience the meaning of second degree function.

(b) Able to use the method of plotting points on a plane to draw the graph of a second degree function. Able to know the properties of second degree function by means of its graph.

(c) Able to ascertain the vertex, direction of the opening of the curve, and axis of symmetry in accordance with formula (neither memorization nor derivation of formulae are needed), and to solve simple practical problems.

(d) Able to use the graph of second degree function to find the approximate solution of equation of second degree in one unknown.

(2) Exemplary Cases

Example 1 Approximately 200 thousands people's everyday living are affected after a flood disaster. This situation is expected to last one month. Please assert: How many tents should be organized? How many tons of food are needed?

Remark Suppose a family consists of 4 members, and 200 thousands people shall need 50 thousand tents for shelter. Suppose a person on average needs 0.5 kilogram of food, one day will need to consume 100 thousands kilogram of food ……

Example 2 Estimate <insert maths term> and 0.5, which is larger?

Example 3 It is found that crickets in some place produce sound with frequency which varies with temperature approximately as follows: Record the number of times a cricket sounds per minute, multiply this frequency by 7, then add 3 to obtain the temperature at that time. The relationship between temperature (℃) and number of times a cricket sounds is as follows:

Temperature = Number of times a cricket sounds per minute ÷ 7 + 3

Please use letters to represent this relationship.

Example 4 Observe the following figure and fill in the blanks in the table:

<Insert figure and table> Number of trapeziums, perimeter

Example 5 Explain algebraic expression $3a$.

Remark For example, the price of grapes is \$3 per kilogram. We need \$ $3a$ to buy a kilograms of grapes. Or, the length of an equilateral triangle is a, the perimeter of this triangle is $3a$.

Example 6 Simplify: (a) <insert maths equation>; (b) <insert maths equation>.

Example 7 Estimate the solution of the following equations:

(a) $x^3 - 9 = 0$; (b) $x^2 + 2x - 10 = 0$.

Example 8 Five students participate in a ping-pong tournament. How many contests are needed so as to arrange for each of two persons to compete with each other? How about ten students?

Remark May use methods such as listing or drawing graphs.

Example 9 Michael's parents go out for a walk. They walk for 20 minutes to arrive at a newspaper booth 900 metres away from their home. Michael's mother immediately turns back home with the same velocity. Michael's father reads newspaper for 10 minutes and then spends 15 minutes to return home. Which of the following graph represents the relationship of distance and time after leaving home for Michael's father journey? Which of the following graph represents the relationship of distance and time after leaving home for the journey of Michael's mother?

<Insert graphs> distance/metre, time/minute

Example 10 A book's selling price is set at \$8. If one buys more than 10 books, then there will be 20% discount for those extra books bought. Analyze and establish the relationship between the number of books bought and the amount paid for the purchase.

Example 11 Fill in the blanks and observe the variations of the following functions.

<insert table>

(a) Draw the graphs of the above functions on the same rectangular coordinates system. Compare the differences between them.

(b) When x begins to increase from 1, predict which function value will reach 100 first.

2. Space and Figures

At this stage of schooling, students shall explore basic properties and interrelationships of basic figures (rectilinear figures, circles). They move a step further to enrich themselves on the knowledge and feelings of space and figures. They learn basic properties of translation, rotation and symmetry, appreciate and experience the widespread application of transformation in realistic everyday living, learn to deploy methods of coordinates system to ascertain the position of objects, and develop space concepts.

Studies on inference and argument may start in the following directions: During exploring the properties of figures and the processes of engaging in cooperative exchanges with others, students develop plausible reasoning, move a step further to learn to think and express systematically. After accumulating certain activity experiences and fundamentals of properties of figures, students start from some basic facts to prove basic properties related to triangles and quadrilaterals. This is done so as to experience the necessity of proof, understand basic processes of proving, master the format of synthetic proof method, and begin to have a feel on the idea of axiomatization.

During teaching, teachers should pay attention to the connection between realistic everyday living and contents learned, enable students to experience, observe, manipulate, deduce, imagine, as well as to engage in other exploratory processes. Students should emphasize understanding of the proof itself, and do not aim at quantity and skills of the proofs. Requirements on proofs are prescribed by the scope as set in the *Standards*.

(1) Concrete Objectives

i. Knowing Figures

(i) Point, Line, Plane

Through a variety of real examples, move a step further to know point, line and

plane (e.g. use of dots to represent cities in traffic diagrams, and pictures on screens are composed by dots).

(ii) Angle

(a) Through a variety of real examples, move a step further to know angle.

(b) Able to compare the size of the angles. Able to estimate the degree of an angle. Able to compute addition and subtraction of the degree of angles. Know degree, minute and second. Able to carry out simple conversion.

(c) Familiarize with angle bisector and its properties. [1]

[1] Points lying along the angle bisector are equidistant to the two sides of an angle. Points inside an angle that are equidistant to the two sides of an angle are lying on the angle bisector.

(iii) Intersecting Lines and Parallel Lines

(a) Familiarize with supplementary angles, complementary angles and vertically opposite angles. Know that the complementary angles of equaled angles are equal, as well as the supplementary angles and vertically opposite angles of equaled angles are equal.

(b) Familiarize with concepts of perpendicular and its line segment. Familiarize with the property that this perpendicular line segment is shortest. Realize the meaning of distance of a point to a straight line.

(c) Know that through a given point there is one and only one straight line perpendicular to the given straight line. Able to use set squares or a protractor to construct a perpendicular from a point to a straight line.

(d) Familiarize with perpendicular bisector and its properties. [1]

(e) Know that the corresponding angles of straight line crossing parallel lines are equal, so as to move a step further to explore the property of parallel lines.

(f) Know that through a given point outside a straight line there exists one and only one straight line parallel to the given straight line. Able to use set squares and a straight ruler to construct a straight line passing through a given point and parallel to a given straight line.

(g) Realize the meaning of the distance between two parallel lines. Able to measure the distance between two parallel lines.

(iv) Triangle

(a) Familiarize with the related concepts of a triangle (interior angle, exterior angle, median, altitude and angle bisector). Able to construct the angle bisector, median and altitude of any triangle. Familiarize with the stable characteristics of a

triangle.

(b) Explore and master the properties of median of a triangle.

(c) Familiarize with concepts of congruent triangles. Explore and master the conditions under which two triangles are congruent.

(d) Familiarize with the related concepts of isosceles triangle. Explore and master the property of isosceles triangle [2] and conditions under which a triangle is an isosceles triangle [3]. Familiarize with concepts of equilateral triangle and explore its properties.

(e) Familiarize with concepts of right-angled triangle. Explore and master the properties of right-angled triangle [4] and conditions under which a triangle is a right-angled triangle. [5]

(f) Experience the exploratory process of Gou Gu Theorem (i.e. Pythagoras' Theorem). Able to use Gou Gu Theorem to solve simple problems. Able to use the converse of Gou Gu Theorem to determine that a triangle is right-angled.

[1] Points lying on the perpendicular bisector of a line segment are equidistant to the two end points of that line segment. Points equidistant to the two end points of a line segment are lying on the perpendicular bisector of that line segment.

[2] The two base angles of an isosceles triangle are equal. The altitude and median of the base of a triangle and angle bisector of the vertical angle of a triangle meet at the same point.

[3] A triangle having two angles equaled is an isosceles triangle.

[4] The two acute angles of a right-angled triangle are complementary to each other. The median of the hypotenuse is half of the length of the hypotenuse.

[5] A triangle with two angles complementary to each other is a right-angled triangle.

(v) Quadrilateral

(a) Explore and familiarize with the formulae of sum of interior angles and sum of exterior angles of a polygon. Familiarize with concepts of regular polygon.

(b) Master concepts and properties of quadrilateral, rectangle, rhombus, square and trapezium, and familiarize with the interrelationships. Familiarize with the instability of quadrilateral.

(c) Explore and master related properties of a parallelogram [1] and conditions under which a quadrilateral is a parallelogram. [2]

(d) Explore and master related properties of rectangle, rhombus and square [3] and conditions under which a rectangle, rhombus and square is a quadrilateral. [4]

(e) Explore and familiarize with related properties of isosceles trapezium [5] and conditions under which a quadrilateral is an isosceles trapezium. [6]

(f) Explore and familiarize with the centroid and physical meanings of line

segment, rectangle, parallelogram and triangle (e.g. centroid of a uniform rod or a piece of uniform rectangular board).

(g) Through exploring tessellation of plane figures, know that any triangle, square, or regular hexagon can tessellate a plane surface. Able to use these types of figures to carry out tessellated designs.

(vi) Circle

(a) Understand circle and its related concepts. Familiarize with the relationships of arc, chord and angle at the centre. Explore and familiarize with the positional relationships between a point and a circle, a straight line and a circle, and two circles.

(b) Explore the properties of a circle. Familiarize with the relationship between angle at the centre and angle at the circumference, as well as the characteristics of the diameter's angle at the circumference (i.e. angle in a semi-circle).

(c) Familiarize with the in-centre and circum-centre (ex-centre) of a triangle.

(d) Familiarize with concept of a tangent line. Explore the relationship between the tangent line and radius that cuts at the tangent point. Able to determine whether a straight line is tangent to a circle or not. Able to draw a tangent line at a point on a circle.

(e) Able to calculate arc length and area of a sector. Able to calculate the lateral area and total area of a cone.

[1] The opposite sides and opposite angles of a quadrilateral are equal. The diagonals bisect each other.

[2] A quadrilateral, of which a pair of opposite sides is parallel and equal, or two pairs of opposite sides are respectively equal, or diagonals bisect each other, is a parallelogram.

[3] The four angles of a rectangle are all right-angled. Diagonals of a rectangle are equal in length. The four sides of a rhombus are equal, and the two diagonals are perpendicular and bisecting each other.

[4] A quadrilateral with three angles right-angled, or a parallelogram with diagonals equal to each other, is a rectangle. A quadrilateral with four sides equal, or a parallelogram with diagonals perpendicular to each other, is a rhombus.

[5] The two base angles standing on the same base of a trapezium are equal, as well as the diagonals are equal to each other.

[6] A trapezium, of which the base angles of the same base are equal, is an isosceles trapezium.

(vii) Construction of Geometrical Figures Using Compasses and Straight Edges without Markings.

(a) Complete the following figure constructions: Construct a line segment equal in length to a given line segment; Construct an angle equal in size to a given angle; Construct the angle bisector of an angle; Construct the perpendicular bisector of a line segment.

(b) Use basic figure construction methods to construct a triangle: Construct a triangle given the lengths of its three sides; Construct a triangle given the size of two angles and the included edge; Construct an isosceles triangle given the base and the altitude standing on the base of a triangle.

(c) Explore how to draw circles that pass through one point, two points, and three points not lying on the same straight line.

(d) Familiarize with the steps of construction of figures using compasses and straight edges without markings. Given construction problems, able to write down the givens, the required constructions and its methods. (No proof is required)

(viii) Views and Projection

(a) Able to draw the three-view drawing (front view, left side-view, plan/top view) of basic geometrical objects (right prism, cylinder, cone, sphere). Able to judge the three views of simple objects. Able to describe basic geometrical objects or concrete prototypes in accordance with the three views of the objects.

(b) Familiarize with the lateral face of the net of right prism and cone. Able to judge and make three-dimensional models in accordance with the nets of the objects.

(c) Familiarize with the relationships between basic geometrical objects and their three-views/nets (excluding spheres). Through typical real examples, know the application of these relationships in realistic everyday living (e.g. packaging of objects)

(d) Observe pictures related to realistic everyday living (e.g. photographs, simple model diagrams, plan, and maps) so as to familiarize with and appreciate some interesting figures (e.g. snowflake curve, Mobius strip).

(e) Through concrete examples with rich backgrounds, know how shadows of objects are formed, and able to discriminate the shadows of real objects in accordance with the direction of the rays (e.g. observe shadows of hand or human body under a sun or lamp).

(f) Familiarize with the meanings of visual point, visual angle and blind region, and able to represent them on simple two-dimensional and three-dimensional figures.

(g) Familiarize with central projection and parallel projection through concrete examples.

ii. Figures and their Transformation

(i) Axisymmetry (i.e. line symmetry) of figures

(a) Through concrete real examples know axisymmetry, and explore its basic properties. Understand the property that line segments joining corresponding points are perpendicularly bisected by the axis of symmetry.

(b) Able to meet specified requirements to start with a simple plane figure and have it undergone axisymmetric transformations once or twice to arrive at a new figure. Explore the axisymmetric interrelationships of these simple figures, and to indicate the axes of symmetry. [Please refer to example 1]

(c) Explore axisymmetry and its related properties of basic figures (isosceles triangle, rectangle, rhombus, isosceles trapezium, regular polygon, and circle).

(d) Appreciate axisymmetric figures in realistic everyday living. Coupled with typical real examples in everyday living, familiarize with and appreciate reflectional symmetry of objects, as well as able to use axisymmetry to engage in pattern design.

(ii) Translation of a Figure

(a) Through concrete real examples, know translation and explore its basic properties. Understand the property that segments formed by connecting points to corresponding points are parallel and equal.

(b) Able to meet specified requirements to start with a simple plane figure and have it undergone translation to arrive at a new figure.

(c) Use translation to carry out pattern design. Know and appreciate the application of translation in realistic everyday living.

(iii) Rotation of a Figure

(a) Through concrete real examples, know rotation and explore its basic properties. Understand the properties that the distance between a given point to the centre of rotation is the same as that of the corresponding point after rotation, as well as angles formed by segments connecting points to centre of rotation are equal to that of the corresponding points after rotation.

(b) Familiarize with the fact that parallelogram and circle are figures of central symmetry.

(c) Able to meet specified requirements to start with a simple plane figure and have it undergone rotation to arrive at a new figure.

(d) Appreciate the application of rotation in realistic everyday living.

(e) Explore the transformational relationships amongst the figures (axisymmetry, translation, rotation, and their combinations). [Please refer to example 2 & 3]

(f) Flexibly apply axisymmetry, translation, rotation and their combinations to carry out pattern design.

(iv) Similarity of Figures

(a) Familiarize with proportion and its basic properties. Familiarize with ratio of line segments and proportional segments. Through concrete examples of architecture and art familiarize oneself with golden section.

(b) Through concrete real examples, know similarity of figures and explore its basic properties. Know the properties that the corresponding angles of similar polygons are equal, corresponding sides of similar polygons are proportional, and the area ratio is equal to the square of the length ratio of the corresponding sides.

(c) Familiarize with the concept of similarity of two triangles. Explore the conditions under which two triangles are similar.

(d) Familiarize with homothety of figures. Able to use homothety to magnify or reduce the size of a figure.

(e) Through typical concrete examples to observe and know the similarity of figures in realistic everyday living, use similarity of figures to solve some practical problems (e.g. use similarity to measure the height of a flag-staff).

(f) Through concrete examples know trigonometric functions of acute angles (*sinA*, *cosA*, *tanA*), know the trigonometric function values of 30°, 45°, 60° angle. Able to use a calculator to compute the trigonometric function values of given acute angles, and vice versa.

(g) Use trigonometric functions to solve simple practical problems related to right-angled triangles.

iii. Figures and their Coordinates

(i) Know and able to draw the rectangular coordinates system on a plane. Given a rectangular coordinates system, able to plot the position of points, and based on the positions of the points to write down their coordinates. [Please refer to example 4]

(ii) Able to develop appropriate rectangular coordinates system on a piece of grid paper, and describe the positions of objects. [Please refer to example 5]

(iii) Within the same rectangular coordinates system, feel the variation of the coordinates after figure transformations. [Please refer to example 6]

(iv) Flexibly use a variety of methods to ascertain the positions of objects [Please refer to example 7]

iv. Figures and Proofs

(i) Familiarize with the meaning of a proof.

(a) Understand the necessity of a proof.

(b) Through concrete examples, familiarize with the meanings of definition,

proposition, and theorem. Able to discriminate conditions (i.e. assumptions) and conclusion of a proposition.

(c) Coupled with concrete examples, familiarize with the concept of converse proposition. Able to identify two mutually converse propositions and know that the converse of a proposition may not be established even though the original proposition is a valid one.

(d) Through concrete examples understand the function of counter examples. Know that counter examples may be used to prove that a proposition is false.

(f) Through real examples, realize the meaning of proof by contradiction.

(g) Master the format of synthetic proof method, experience that each step in the proving process requires a reason.

(ii) Master the basic facts below so as to use them in proofs.

(a) The corresponding angles obtained by a transversal cutting two parallel lines are equal.

(b) Two straight lines are cut by a third straight line. If the corresponding angles are equal, then the two straight lines are parallel.

(c) If the two sides and the included angle (or two angles and their included edge, or three sides) are respectively equal, then the two triangles are congruent.

(d) The corresponding sides and corresponding angles of congruent triangles are respectively equal.

(iii) Use the basic facts listed in (ii) to prove the following propositions. [1]

(a) Theorem on property of parallel lines (the alternate interior angles are equal, the interior angles on the same side of the transversal are equal) and theorem on conditions under which two lines are equal.

(b) Theorem on angle sum of interior angles of a triangle and its corollary (the exterior angle of a triangle is equal to the angle sum of two non-adjacent interior angles, the exterior angle of a triangle is greater than any one of the non-adjacent interior angle).

(c) Theorem on deciding conditions under which right-angled triangles are congruent.

(d) Theorem on property of angle bisector and its converse. The three angle bisectors of a triangle intersect at one point (in-centre).

(e) Theorem on property of perpendicular bisector and its converse. The three perpendicular bisectors of a triangle intersect at one point (circum-centre or ex-centre).

(f) Theorem on the medians of a triangle

(g) Theorem on properties of isosceles triangle, equilateral triangle, right-angled triangle and their deciding conditions.

(h) Theorem on properties of quadrilateral, rectangle, rhombus, square and isosceles trapezium and their deciding conditions.

(iv) Through introduction of Euclid's Elements, feel the value of geometry deductive system on mathematics development and human civilization.

[1] The difficulty of questions related to proofs in exercises and examinations should match with the difficulty of proving the propositions listed below.

(2) Exemplary Cases

Example 1 Using the tree trunk as the axis of symmetry, draw the other half of the tree.

<insert picture>

Example 2 Please state how Tree *B* can be transformed from Tree *A*?

<insert picture>

Example 3 Observe the pattern below, what pattern can be chosen and what transformations can be applied in order to obtain the desired pattern?

<insert picture>

Example 4 Plot the following points on a coordinates system, and connect the points in each group consecutively.

(a) (2,0), (4,0), (6,2), (6,6), (5,8), (4,6), (2,6), (1,8), (0,6), (0,2), (2,0);

(b) (1,3), (2,2), (4,2), (5,3);

(c) (1,4), (2,4), (2,5), (1,5), (1,4);

(d) (4,4), (5,4), (5,5), (4,5), (4,4);

(e) (3,3)

Observe this figure, what does it look like?

Example 5 The diagram shown below is a tour map of scenic spots of a city. Try to establish a rectangular coordinates system, and to use coordinates to represent the position of each scenic spot in the map.

<insert picture>

Example 6 As shown in the diagram, under the given coordinates system, the pattern "*A*" in figure 1 has been transformed into corresponding patterns in figure 2-6 (original pattern in dotted lines). Try to write down the coordinates of the vertices in

figure 1-6. Explore what variations are discernible before and after the transformations, as well as what relationships can be observed amongst the coordinates of the corresponding points.

<insert picture> Figure 1, Figure 2 …..

Example 7 Mr. Zhang saw an illustrative diagram of the plan of a zoo in front of its entrance. Try to make use of the scale and a protractor to solve the following problem:

<insert picture> Illustrative Diagram of the Plan of Zoo, Scale 1:10 000

Panda Gallery, Hundred Birds Aviary, Entrance, Monkey Hill, Camel Hill

(a) Establish an appropriate rectangular coordinates system, use coordinates to represent the positions of Monkey Hill, Camel Hill, and Hundred Birds Aviary;

(b) Fill in the blanks:

The Hundred Birds Aviary is situated North ____degrees East of the Entrance. Its distance from the entrance on the map is approximately _____ *cm*.

The Panda Gallery is situated North _____ degrees _____ of the entrance. Its distance from the entrance on the map is approximately _____ *cm*.

The Camel Hill is situated South _____ degrees _____ of the entrance. Its distance from the entrance on the map is approximately _____ *cm*.

Remark Apart from using rectangular coordinates to describe the positions of objects, this question seeks to allow students to realize selecting some reference objects and particular directions so as to use distance and angles to depict the position of an object.

3. Statistics and Probability

At this stage of schooling, students shall realize the necessity of sampling, as well as the idea of using the sample to estimate the population. They move a step further to learn methods of describing data, and realize the meaning of probability. They are able to compute the probability of a simple event happened.

During teaching, teachers should pay attention to the connection between the contents learned and everyday living, nature, society, science and technology areas. Teachers should enable students to realize the important function of statistics and probability for decision-making. Teachers should pay attention to allow students to engage in the complete process of data processing, and make reasonable judgments in accordance with statistical outcomes. Students should realize the meaning of

probability within concrete contexts and situations. Connection between statistics and probability should be strengthened. Teachers should avoid treating this part of learning to become numerical computation exercises. There is no need to demand rigorous explication of related terminologies.

(1) Concrete Objectives

i. Statistics

(i) Engage in activities of data collection, organization, description and analyses. Able to use calculators to process relatively complicated statistical data.

(ii) Through a variety of real examples, feel the necessity of sampling. Able to identify population, individual and sample. Realize that different type of sampling may result in different outcomes. [Please refer to example 1]

(iii) Able to use pie chart to represent data.

(iv) Understand and is able to compute weighted mean in concrete contexts and situations. Able to choose appropriate statistics to represent the central tendency of data in accordance with concrete problems.

(v) Explore how to represent the dispersion of a group of data. Able to compute range and variance, and to use these statistics to represent the dispersion of data. [Please refer to example 2]

(vi) Through real examples, understand concepts of frequency and rate. Familiarize with the meaning of frequency distribution and its functions. Able to tabulate a frequency distribution table, draw frequency distribution histogram and broken-line graph, as well as to solve simple practical problems.

(vii) Through real examples, realize the idea of using the sample to estimate the population. Able to use the mean and variance of a sample to estimate the mean and variance of the population.

(viii) Able to make reasonable judgment and prediction in accordance with statistical results. Realize the function of statistics on decision-making. Able to express one's viewpoints relatively clearly, and engage in exchange of ideas.

(ix) Able to locate relevant materials to acquire data information in accordance with given problems. Able to present one's viewpoints on some data encountered in everyday living.

(x) Know the application of statistics in social life and science domain areas, and is able to solve some simple practical problems. [Please refer to example 3]

ii. Probability

(i) Familiarize with the meaning of probability in concrete contexts and situations. Deploy enumeration (including tabulation, drawing tree diagram) to calculate the

probability of simple event happened. [Please refer to example 4 & 5]

(ii) Through experiments, obtain the probability of an event happened. Know that rate can be used as an estimated value to indicate the probability of an event happened when a large number of experiments are replicated. [Please refer to example 6]

(iii) Through real examples move a step further to enrich one's knowledge on probability, and is able to solve some practical problems. [Please refer to example 7]

(2) Exemplary Cases

Example 1 A television broadcast company needs to survey the ratings of TV programs in the city. Is it necessary to ask every person who has viewed the programs before? Can we use the survey results of students of a university as an indication of the ratings? Do you think the survey results corresponding to different communities, age levels, and cultural backgrounds are the same or not?

Example 2 What is shown below is the sales statistics of two fruit groceries from January to June (unit: Kilogram). Compare the stability of the quantities sold of the two groceries.

<insert table> Grocery A, Grocery B, January, February, March, May, June

Example 3 Compute sales statistics of commodities of a shop during a certain month, recommend how this shop should replenish its commodities.

Example 4 A bag contains two yellow balls and two red balls. Randomly draw one ball and have it returned to the bag. Another ball is randomly drawn again. What is the probability of drawing two red balls?

Example 5 Turn the disc as shown in the picture, what is the probability that the pointer will stay at the shadow parts after the disc stops its rotation?

<insert figure>

Example 6 Through experiments obtain the probability of having the tip of the drawing pin touching the ground when it is thrown from a certain height onto the ground?

<insert figure> Cap touches ground, Tip touches ground

Example 7 The probability of winning a lottery game is 1%. If one buys 100 lotteries, will one definitively win the game?

4. Thematic Projects

During this stage of schooling, students shall explore some challenging research topics, develop awareness and abilities in applying mathematical knowledge to solve problems. At the same time, they move a step further to enhance understanding of related mathematical knowledge, know the connections amongst different branches of mathematical knowledge.

Based on the foundation built up in the previous two stages of schooling, teachers should guide students during teaching to propose topics that tie in with their everyday living. Students actively think about the problem encountered, express clearly their viewpoints and solve some of the problems encountered.

(1) Concrete Objectives

i. Involve in the basic processes of "Problem Context/Construct Model/Find Solution/Explain/Apply".

ii. Experience the internal connections amongst the various branches of mathematical knowledge. Begin to form a view of knowing mathematics in a holistic manner.

iii. Acquire some methods and experiences of research problems. Develop reasoning abilities and increase understanding on related mathematical knowledge.

iv. Through cases of successful experiences and involvement in overcoming difficulties, increase confidence in applying mathematics.

(2) Exemplary Case

Example Use a sheet of square paper to construct a cuboid without a lid. How to construct so that the volume is relatively bigger?

Remark This is a problem of comprehensive nature. Students may think along the following directions: (a) What does the net of the cuboid look like? What basic manipulations and steps are required to use a piece of square paper to form a cuboid without a lid? How to express the volume of the cuboid after it is made? Under what conditions will the volume of the cuboid without a lid be relatively bigger? How to use a piece of square paper to construct a cuboid having a lid? What are the steps of construction? What are the major difficulties involved?

Through this thematic study, students enrich themselves further on space concepts, realize the application of ideas of function and symbol representation to

solve practical problems. Students move a step further to experience the abstraction of mathematical problems from practical problems, construct mathematical models, synthesize application of knowledge learned to solve problems, therein increase understanding of related knowledge, and develop their reasoning abilities.

Section Four –Curriculum Implementation

Recommendations

First Stage of Schooling (Grade 1~3)

3. Teaching Recommendations

Mathematics teaching entails teaching of mathematical activities. It is an interacting exchange and joint developmental process between teachers and students, as well as amongst students.

Mathematics teaching should tie in intimately with students' practical everyday living. Teachers should start from students' everyday experiences and knowledge already acquired, create animating and interesting contexts and situations, guide students to explore, manipulate, conjecture, infer and engage in exchange of ideas. Through mathematical activities, students master basic mathematical knowledge and skills, begin to learn to observe objects from a mathematical point of view, think about problems, stimulate interests in mathematics, as well as to aspire to learn mathematics well.

Teachers are organizers, guides, and collaborators of student mathematical activities. In accordance with students' concrete situations, teachers rework the teaching materials and design teaching processes creatively. Teachers need to take into account students' individual differences, and practice individually configured education so that every student can develop further from their own original foundation. Teachers should allow students to experience success, and to establish confidence in learning mathematics well.

(1) Allow students to learn mathematics in an interesting and concrete context.

At this stage of schooling, teachers should make full use of students' daily experiences to design animating, interesting, and intuitive mathematical teaching activities. Examples are story telling, games, intuitive demonstrations, and imitative performances. Such activities can stimulate students' interests in learning, allowing

them to understand and familiarize with mathematical knowledge in interesting situations and concrete contexts. For instance, teachers can lead students to carry out the following games and activities:

Example 1 *Two students form a group to play number guessing.*

A: I have a number in my mind. Can you guess what is it?

B: Is this number larger than 50?

A: Right.

B: Smaller than 70?

A: Right.

B: Larger than 60?

A: Wrong.

B: Larger than 56?

……

Teachers can use this game to lead students to start some interesting mathematical activities. At the time when students are realizing the magnitude of two numbers, they are learning some effective strategies for solving problems, including primitive idea of "successive approximability of nested intervals".

(2) Guide students to engage in independent thinking and cooperative exchange of ideas.

Hand practical work, autonomous exploration and cooperative exchange of ideas are important formats of students' mathematics learning. At this stage of schooling, teachers should allow students to carry out concrete manipulative activities to engage in independent thinking, encourage students to express their ideas, and exchange ideas with peers. Teachers should provide appropriate assistance and guidance, good at selecting valuable questions and opinions from students, and lead students to start discussion so as to find out answers to questions.

Example 2 *Look at the rotating disk (see diagram on the right). The pointer stops rotation at the shaded region, or stops at the white region. Which possibility is greater?*

<Insert Diagram to the right>

During teaching, teachers can start the activity by dividing the students into groups. Each student should guess where the pointer would stop before rotating the disk. Through direct participation in the rotation of the disk, students realize the

followings: Before the disk stops, it is not certain whether the pointer stops at the shaded or the white region. After rotating the disk a number of times, students begin to realize that the frequency for the pointer to stop at the shaded region is different from that of the white region. The frequency of the white region is greater than that of the shaded region, indicating that the possibility that the pointer would stop at the white region is larger than that of the shaded region. Based on the foundation of such student hand practical work, teachers can lead students to start discussion, and exchange feelings amongst themselves.

For some parts of teaching in Space and Figures, teachers should design rich and multifarious activities, such as observation, measurement, folding and stacking, and discussion, so that students can move a step further to familiarize with space in everyday living, and to know some common geometrical objects and plane figures. For instance, during the teaching of cuboids, cubes and cylinders, teachers should select content materials from real objects familiar to the students (e.g. basket ball, ping-pong ball, soft drink bottle, kaleidoscope, chalk box, tooth paste box, and globe), encourage students to engage in activities such as observation, touching and classification so that their intuitive feeling of related geometrical objects can be formed. Other examples of mathematical activities include: Allow four students to sit facing four different directions to observe the same object (e.g. kettle, cup). Students draw what they see first, and then exchange ideas with peers. They can guess who has drawn which picture according to the positions sit. Through observation, comparison and imagination, students realize that the view seen at different directions are different, and space conceptions are developed progressively.

(3) Strengthen estimation; encourage multifarious use of algorithms.

Estimation has a great deal of applications in everyday living. At this stage of schooling, teachers should act promptly to inculcate estimation consciousness and initial estimation skill.

Example 3 *Michael's family income from raising ducks is 243 dollars, and that from raising pigs is 479 dollars. Estimate the total income of these two items.*

Different students may have different estimation strategies. Some students consider: "200 plus 400 equals to 600. 43 plus 79 is greater than 100, therefore the sum of these two numbers is a little bit greater than 700". Some students' estimation method may be: "243 is less than 250, 479 less than 500, Therefore, the sum of these

two numbers is less than 750". Some students may say that, "This number is greater than 200 + 400, less than 300 + 500". All these estimation are correct. Teachers should organize students to exchange ideas amongst themselves on methods of estimation, compare their estimation results, as well as develop progressively students' estimation consciousness and strategies.

Because of the different everyday living background and perspectives of the students, the methods used are necessarily multifarious. Teachers should respect the viewpoints of the students, encourage students to engage in independent thinking, and promote multifarious computation methods. For example, concerning the problem of 34+27, students can adopt a number of methods. The following methods need to be encouraged.

<insert equations here>

Teacher should not be eager to evaluate the algorithms. They should guide students to select methods appropriate to them by comparing the characteristics of different algorithms.

As an example, to solve the problem "During a parents gathering, each bench can be occupied by 5 persons at most. How many benches are needed for 33 parents?", students' thinking methods are multifarious. Some students may make use of small sticks to represent benches, use circular stickers to represent parents, and through manipulations they can arrive at the answer that they need 7 benches. Some students through computing 33/5 judge that at least 7 benches are needed. Some students may use multiplications, 5 x 7 = 35, 35 > 33, since 5 x 6 = 30, 30 < 33, therefore a minimum of 7 benches are needed. Regarding all these methods, teachers should encourage students and provide opportunities for exchanges of ideas, so that students can improve continuously their methods in these cooperative exchanges. In this way not only teachers can familiarize with the learning characteristics of the students, but also foster students' development of personal characters. At the same time, teachers should require students to think about these questions frequently: How do you think about it? How do you do it just now? If …… what happens? What errors occur? Which method do you think is better?……Through these questions teachers can guide students to think and to engage in exchanges on methods of solving problems.

(4) Develop students' initial application consciousness and problem solving abilities.

At this stage of schooling, teachers should make full use of students' everyday experiences acquired, lead students promptly to apply the mathematical knowledge learned in their everyday living, solve mathematical problems encountered, familiarize with the function of mathematics in realistic everyday living, and to realize the importance of learning mathematics.

Example 4 *27 persons are transported to a place somewhere. Two types of plans are available for renting vehicles. One type of vehicle can take 8 persons, another type can take 4 persons.*

(i) Provide at least three plans for renting vehicles.

(ii) *The cost for renting the first type of vehicle per day is 300 dollars. The second type is 200 dollars per day. Which plan is most economical?*

Practical activities are important means to nourish students to carry out autonomous explorations and cooperative exchanges. At this stage of schooling, teachers should organize students to start animating and interesting activities, so that they can involve in processes such as observation, manipulation, inferences and exchange of ideas.

4. Evaluation Recommendations

The purpose of evaluation is to familiarize oneself comprehensively with students' learning situations, inspire students' passion for learning, and foster total development in students. Evaluation is also a powerful means to engage teachers in reflection and teaching improvement.

Concerning evaluation of students' mathematics learning, teachers should not only pay attention to comprehension and mastery of knowledge and skill, but also formation and development of feelings and attitudes. Likewise, teachers should not only pay attention to students' learning outcomes, but also to the variation and development of students' learning processes. There should be a multitude of forms and strategies of evaluation, emphasizing mainly on process evaluation. Regarding description of evaluation results, teachers should use encouraging languages so as to foster the aspiring function of evaluation. Evaluation should pay attention to individual differences, as well as protect the esteem and confidence of the students. Teachers should make good use of the large amount of evaluation information to

regulate and improve the teaching processes promptly.

(1) Pay attention to evaluation of student mathematics learning processes.

Regarding evaluation of student learning processes at this stage of schooling, teachers should examine whether students are participating mathematics learning activities actively and autonomously, whether students like to engage in cooperative exchanges with their peers, and whether students possess interests in mathematics learning. Teachers should pay attention to familiarize themselves with students' thinking processes, and allow students to talk about their thinking processes during problem solving.

Example 1 *Try to measure. How far can you throw a solid ball?*

In this activity, we should examine first the degree of participation of the students, try to know whether students can propose a measurement plan independently, whether they are able to work together with peers cooperatively to solve problems, and whether they are willing to engage in exchanges with others on one's methods and problem solving processes. At the same time, teachers should familiarize with the processes of how students deploy knowledge to solve problems and how they engage in mathematical thinking during activities. Students may have the following types of performances: (a) Carrying out measurements as instructed by the teacher; (b) Think of one's own measurement methods (e.g. using footsteps as a measure, measuring using a rope, measuring using a meter-ruler, measuring using a roll of tape ruler); (c) Through group work, explore different ways to carry out measurements, and exchange ideas on the different measurement methods; (d) Use a number of methods to measure, and explain the reasonableness of the method of measurements in simple terms. For example, if a student can throw a ball out exceeding 3m away, the use of a meter-ruler to measure this distance will result in certain measurement error. One reason is that one has not measured the distance along a straight line, and that the use of a roll tape ruler will be more precise. Teachers can analyze and evaluate in accordance with students' performances in activities mentioned above.

While evaluating students' learning processes, teachers can adopt the use of portfolios to reflect the progress of students' mathematics learning so as to enhance their confidence in mathematics learning. Teachers can guide students to document important information in these portfolios that may reflect their learning progress, e.g. one's most satisfactory assignment, one's most favorite small production, impressive

problem solving processes, reflective experiences on reading mathematics materials etc..

(2) Evaluate students' fundamental knowledge with understanding and basic skills with mastery appropriately.

Concerning evaluation of fundamental knowledge and basic skills at his stage of schooling, teachers should follow the basic principles stated in *Standards*. Based on the prescribed standard of objectives related to knowledge and skills at this stage of schooling, teachers should examine the degree of understanding of fundamental knowledge and degree of mastery of basic skills.

What needs to be emphasized is that stage objectives are objectives to be accomplished by students at the end of each stage of schooling. Teachers should allow some students to make efforts within a period of time to accomplish the prescribed standards through accumulation of knowledge and skills. For example, requirements on computation tabulated below are not indicative of that all students should achieve immediately after learning the contents concerned. They refer to the objectives needed accomplishment at the end of this stage of schooling. The scale should be grasped during evaluation.

Contents of Learning	Speed Requirements
Addition and subtraction of numbers below 10, Mental computations of multiplication and division according to tables.	8~10 questions per minute
Addition and subtraction of numbers of three-digits or less.	2~3 questions per minute
Multiplication of two two-digits numbers.	1~2 questions per minute
Division such that divisor is one-digit, and the dividend is a number of three-digits or less.	1~2 questions per minute

During this stage of schooling, students generally need to use concrete objects or real models to accomplish learning tasks. Therefore, during student evaluation, teachers should give priority to examine students' understanding concerning synthesis of concrete materials together with the practical meaning of the contents of learning materials.

Concerning evaluation of Numbers and Algebra, teachers should couple with concrete situations to examine students' understanding of the meaning of numbers. For example, understanding of the meaning of fractions can be examined using the

following contexts.

Example 2 *(i) In the diagram below, what fraction of the whole region of the figure does the shaded region occupies?*

<insert diagram here>

(ii) Please use a diagram to represent 1/4.

Concerning evaluation of Space and Figures, teachers should couple with intuitive materials and everyday living situations to evaluate students' knowledge of figures. As an example, teachers can use the question below to examine space concepts of students.

Example 3 A small car is depicted below.

<insert picture here>

Helen views this car in the air from the top. Which diagram below shows what Helen sees?

<insert picture here>

Concerning evaluation of Statistics and Probability, teachers should couple with everyday living situations to examine students' initial statistical consciousness and abilities of solving simple problems. As an example, during preparation of class activities, in order to ascertain the types and quantities of fruits bought, teachers can allow students to conduct a survey of the fruits liked most to eat by the students, and count the corresponding number of students. During evaluation, teachers in the main can examine the following aspects: Under the guidance of teachers, whether students can use appropriate methods to collect information on the number of students who are fond of eating each type of fruits; Based on the foundation of data collection, whether students can carry out data classification, organization and description (e.g. able to speak, "In my class, the number of students who like to eat apples is greatest. The number of students who like to eat pears is less than half of the number of students who like to eat apples); and whether students can ascertain their buying plans.

(3) Emphasize evaluation of students' problem finding and problem solving abilities.

Concerning evaluation of students' abilities of discovering problems and solving problems, teachers should pay attention to examine whether under the guidance of teachers students can discover and propose mathematical problems in everyday living; whether students can select appropriate methods to solve a problem; whether students are willing to cooperate with peers to solve problems; and whether students are able to

express generally processes and outcomes of problem solving. For example, teachers can allow students to raise all sorts of questions from their everyday living: Who gets most pencils? Who is the tallest? Whose home is nearest to the school? ……Teachers can furnish qualitative evaluation based on the quantity and quality of the questions raised by the students.

(4) Students are evaluated in multifarious ways.

Students at this stage of schooling have just entered schools. What they feel about mathematics is key to whether they will like mathematics in the future, and whether they can learn mathematics well or not. Therefore, teachers' evaluation on students should as far as possible start from the positive side so as to guide them, as well as ascertain what they have already known and what they have already mastered. When undertaking student evaluation, teachers can combine teacher evaluation with peer and parent evaluation. Concerning evaluation of student learning, teachers should pay attention to combine different forms of evaluation, such as adopting classroom observation, after-class interviews, task analysis, manipulation and practical activities.

Each type of evaluation has its own characteristics. During evaluation teachers can select methods associated with contents of evaluation and learning characteristics of the students. For example, teachers can select classroom observation format to examine students along four dimensions: degree of seriousness in learning mathematics, mastery of fundamental knowledge and basic skills, problem solving and cooperative exchanges. Through learning activities, teachers can also familiarize with students' learning attitudes and consciousness of cooperative exchanges. Through daily home assignments, teachers can familiarize with students' mastery of computational skills. Through portfolios, teachers can familiarize with students' development of abilities of posing and solving problems.

(5) Evaluation results are displayed qualitatively.

Because of the characteristics of students at this stage of schooling, evaluation results should be displayed qualitatively. Teachers should use encouraging languages to describe situations of student learning.

What follows is an example of such evaluation statements: "During this semester, Helen can complete all her assignments seriously, participates actively in small group discussion, and is willing to listen to her peers. Helen is willing to raise questions, and

very often can think of ways that is different from that of other students to solve problems. However, her correctness in computation needs further elevation." Such kind of remark is mainly encouraging in nature, and points out directions for further advancement and efforts.

5. Teaching Materials Editing Recommendations

Teaching materials provide basic clues of what student learning activities should entail. It realizes curriculum objectives, and is important resources for implementing teaching. In principle, editing of teaching materials should be based on Standards. Teaching materials should provide sufficient interesting and personally relevant materials related to children's everyday living. Topics covered should vary and their presentation format should be rich and multifarious. Editing of teaching materials should be conducive to the establishment of the central role of students in the teaching process, arouse students' interests in learning, guide students to acquire good affective experiences during active thinking and cooperative exchanges, and construct one's mathematical knowledge. Editing of teaching materials should also be facilitative of the mobilization of teachers' dynamism, and renders teachers to conduct teaching creatively.

Considering the diversity of individual differences, teaching materials should exemplify its characteristics after fulfilling all the basic prerequisites. Also, it should possess a certain degree of flexibility as well. When editing teaching materials, teachers should consider sufficiently how it can tie in with the exploitation and deployment of other curriculum resources.

(1) Choose animating and interesting materials related to students' everyday living.

Concerning editing of teaching materials at this stage of schooling, teachers should try as far as possible to start from fairy tales and everyday living contexts familiar to the students, select materials personally related and feel interested by the students, and propose related mathematical problems, so as to stimulate students' interests and motivation, and begin to feel the intimate connection of mathematics and everyday living.

Contents selected as teaching materials should enable students to locate easily

corresponding real objects or models. For example, concerning learning of statistical contents, teachers can select stationary, toys, food, fruits, events and objects commonly found in the school campus as statistical targets. Concerning learning of probability contents, teachers can put one small red stick and five small white sticks into a pocket and ask the students to predict which color will have a higher chance being drawn. Such design not only facilitates teachers to organize their teaching, but also allows student to undertake manipulations.

Selection of contents for practical activities should match students' age characteristics and everyday living experiences. Teachers should provide concrete, interesting, and inspiring (to some degree) activities (e.g. mathematical games) to allow students to involve in processes of applying mathematical knowledge to analyze and solve problems, as well as accumulate experiences of mathematical activities. For example, during the New Year Eve, the class needs to prepare for the gala party and buy some fruits. Ask the students to think about what kind of fruits should be bought. This is a problem intimately related to students' everyday living. In order to solve this problem, students need to conduct a survey of the whole class to find out the kind of fruits liked by the students, carry out data analyses that is based on statistical data, and proceed to make reasonable decisions.

(2) Allow students ample room for exploration and cooperative exchange of ideas.

Editing of teaching materials should facilitate students to carry out observation, experimentation, manipulation, inferences and exchange of ideas. Irrespective of the introduction of new topics, or expansion of teaching contents, teachers should strive to create inspiring problem contexts so that students can realize the process of formation of knowledge. Items such as "Try to Observe", "Try to Do", "Try to Think", "Try to Say", and "Try to Read" can be set up within the teaching materials, so as to guide students to undertake autonomous learning activities. Teachers can provide appropriate open-ended questions and opportunities for exchange of ideas to extend students' scope of exploration. For example, when knowing addition, teachers can adopt the following materials to allow students to discover quantitative relationships themselves, and to explain the practical meaning of the addition algorithms.
<insert diagram>

Concerning the problem context of the example mentioned above, three children are playing and there are three trees in the background. One can classify the three

children into two groups according to the roles they play or their gender. The three trees can also be classified into two groups according to their positions or their sizes. In this regard, the context affords rich mathematical information so that students can perceive the computation expression 2 + 1 = 3 from different angles so as to explain its practical meanings. Through providing such explorations and exchanges, students are able to realize the meaning of addition better than before.

Another example concerns knowing the East, South, West and North directions, teaching materials can comprise of the following practical activities:

Example 1 *Teachers and students identify the East, South, West and North directions together on the playground.*

(i) Try to see what are available in the East, South, West and North directions.

(ii) Record down what you see.

(iii) Stick your documentation record on the blackboard. Try to see what is different from the others.

(iv) According to what you record complete the following diagram.

<insert diagram> playground

(3) Format of presentation should be rich and multifarious.

Students at this stage of schooling mainly use iconic mode of thinking. When editing teaching materials, teachers should deploy a variety of formats (e.g. pictures, games, cartoon, tables, and text). Intuitive images and pictures at a glance may be used to present contents in an animating and interesting way, so as to increase students' learning interests and satisfy a variety of learning needs.

Example 2 *A kangaroo runs a shop that sells 16 pencils. A rabbit buys 9 pencils. How many pencils are left behind?*

This example can make use of a series of cartoon pictures to present the problem context and various computation methods of 16–9. For example, A mouse deducts one by one; a rabbit divides 16 into 10 and 6, 10 – 9 = 1, 1 + 6 = 7; a goat divides 9 into 6 and 3, 16 – 6 = 10, 10 – 3 = 7; Mr. Dog said, "I can compute in this way, 9 + 7 = 16, 16 – 9 = 7".

Example 3 *A class organizes a recitation contest, and the recitation time allocated to each student is 3 minutes. A student selects a passage of 930 words. During a*

rehearsal he spends 6 minutes, what to do?

This example can adopt a conversation format to represent the thinking processes of problem solving.

Multifarious ways of content presentation facilitate students to undertake learning activities, foster independent thinking, as well as to engage in small group cooperative exchanges.
<insert picture here> It's OK if I speak faster. Delete a few words.
Too fast speed will affect the performance. You speak ___ words per minute. You spend ____ minutes more than the allowed time. You must delete ____ words.

(4) Important mathematical concepts and ideas should progress from the easy to the difficult.

In accordance with students' past experiences, rules of psychological development, and characteristics of contents learned, some important mathematics concepts and mathematics thinking methods should adopt a presentation format which progressively diffuse, deepen, and spiral upward, so as to realize the learning objectives at this stage of schooling. All contents organized in this format should pay attention to the inheritance relationships, and to void unnecessary duplication.

An example related to learning the contents of Knowing Figures is that students start by "observing an object from different positions". By designing to observe a small car and a kettle from different positions such as from the top and sides, students realize that the shapes seen from different positions are not the same. Later, teachers can guide students to observe figures (e.g. cubes, cuboids) from the top, front, and the sides. Furthermore, teachers can engage students in an activity of stacking and arranging cubes. For example, a three-dimensional object formed by stacking and arranging three cubes if viewed from the top its shape looks like <insert figure>, what is the configuration of this three-dimensional object? Try to manipulate the arrangement of the cubes, how many ways can be arranged? If this figure is what can be seen from the front, what is the configuration of this three-dimensional object? Teachers can move a step further to provide more problems, such as: A three-dimensional object is stacked and arranged using 4 cubes. The shape as seen from the top resembles <insert figure>, can you visualize what this configuration looks like? At the second and third stages of schooling, more blocks may be used.

(5) Design of content should demonstrate some degree of flexibility.

Objectives listed in *Standards* are basic requirements needed to be achieved by all students. Editing of teaching materials must clarify these basic requirements. Do not raise levels at one' own will so as to ascertain that these basic requirements can be realized. On the other hand, because of the differences existed amongst regions, schools and students, editing of teaching materials should demonstrate some degree of flexibility. This is to satisfy the diverse learning needs of the students so that every student can secure developments corresponding to their needs.

One concrete design format is to propose different levels of a problem of the same context. For example, teachers can design some open-ended problems with realistic backgrounds, or problems needed exploration of their patterns, so that every student is given opportunities to express their views to some questions and the successful experiences. Teaching materials may be designed to include some animating and interesting materials chosen for reading by the students. For example, after learning multiplication computations, teachers can arrange the following activities: A flower shop gets some flowers labeled for sale. Allow students to propose different kinds of questions. Facing such materials, different student will raise different questions. For instance, how much does it cost to buy 5 roses? Which kind of flowers can be labeled $10 for sale?

Teachers should design practical activities in such a way that all students are able to participate, and different students acquire different experiences and developments. For example, Tangram is one such kind of activity that students like to participate. The patterns designed all have different meanings, different degree of authenticity, respectively reflecting students' imagination and creativity.

(6) Introduce students to mathematical background knowledge.

At appropriate places in the teaching materials teachers can introduce stories related to mathematicians, interesting accounts and historical mathematics materials. This enables students to know that the formation and development of mathematical knowledge stem from people's needs in everyday living, experience the contribution of mathematics to human historical development, as well as stimulate students' interests in learning mathematics. This part of learning may be presented in the form of reading materials. Introduction of concrete materials should commence according to the age characteristics of the students, and it should be easy and concrete,

animating and interesting.

The following historical materials may be included in the teaching materials: Introduce concepts of numbers and their genesis so that students experience that numbers stem from counting and quantity stems from measuring; Introduce the ancient methods of denoting numbers (using knots and scratches to denote numbers); Through historical materials enable students to experience the dual functions of "0" – as an empty space in the place value notation and as an independent number on its own; Through geometrical objects and figures on stoneware and pottery made by ancient societies, introduce ancient people's knowledge on simple patterns and figures so as to enable students to feel that everyday living is full of figures.

Second Stage of Schooling (Grade 4~6)

1. Teaching Recommendations

Mathematics teaching entails teaching of mathematical activities. It is an interacting exchange and joint developmental process between teachers and students, as well as amongst students.

Mathematics teaching should tie in intimately with students' practical everyday living. Teachers should start from students' experiences and knowledge already acquired, create contexts and situations conducive to students' autonomous learning and cooperative exchanges. Through exploration, manipulation, induction, analogy, conjecture, communication and reflection, students master basic mathematical knowledge and skills, move a step further to develop thinking abilities, stimulate interests in learning, as well as to aspire to learn mathematics well.

Teachers are organizers, guides, and collaborators of student mathematical activities. Teachers should deploy actively all sorts of teaching resources, use teaching materials creatively, and design teaching processes appropriate for students' development. Teachers need to take into account students' individual differences, allow students to experience success in learning and achieve suitable developments. Wherever resources and time permit, teachers should use modern teaching strategies reasonably and effectively so as to raise teaching efficiency.

(1) Allow students to experience and understand mathematics in realistic

contexts and situations.

Teaching at this stage of schooling needs to create learning contexts that are intimately related to students' everyday living, background knowledge, and are of interests to the students. Through observation, manipulation, guessing, and such activities as exchange of ideas and reflection, students realize progressively the processes concerning the germination, formation and development of mathematical knowledge, acquire constructive affective experiences, feel the power of mathematics, and at the same time master necessary fundamental knowledge and basic skills.

For example, computational mathematics should pay attention to the connection with students' everyday living, so as to enable students to feel that through computation they can solve a number of practical problems. We can allow students to estimate which of the following answers is closest to one's age ((i) 500 minutes, (ii) 500 weeks, (iii) 500 hours, (iii) 500 months). Students may use one of the following methods to guess the answer. At this moment, teachers should move a step further to guide students to find out themselves whether their answers are accurate or relatively accurate or not. In order to answer this question, students shall undertake necessary computations so as to realize the necessity of computation. During the concrete computation process, teachers can encourage students to use calculators.

Concerning teaching of Space and Figures, teachers should make full use of events and objects in students' everyday living, guide students to explore the characteristics of figures, enrich experiences of space and figures, and begin to establish space concepts. During teaching teachers can group students into small teams and bring them to the playground. Each team selects a building and observes the chosen building from different angles. In this way, students experience the variations of the forms and shapes of an object seen from different angles, and are able to draw figures and shapes observed. Likewise, students can allow students to draw an illustrative diagram on a grid paper according to the following requirements: Suppose Science and Technology Center is situated 500*m* East of the school, Helen's home is situated North 30 degrees West and 300*m* away from the school, the Hospital is situated South and 1000*m* away from the school, whereas the Car Terminal is situated South 30 degrees West and 400*m* away from the school. Based on the information, students can decide on a piece of grid paper appropriate distance units, and label the corresponding positions accordingly. Afterwards, teachers should grasp the opportunity to organize students to engage in exchange of ideas, so that students can progressively develop space concepts.

(2) Encourage students to engage in independent thinking. Guide students in autonomous exploration and cooperative exchange of ideas.

Mathematics learning processes are full of exploratory and challenging activities such as observation, experimentation, modeling and inferences. Teachers should alter teaching formats which use mainly examples, demonstrations, and explication. Instead, they should guide students to involve in activities such as exploration and exchange of ideas.

Example *Fill in numbers in the blanks below so that the sequence formed possesses some form of pattern and regularity. Explain this pattern and regularity.*

3, 5, 7, ____, _____, _____.

Teachers can start to encourage students to engage in independent thinking, explore from different angles the hidden patterns, and engage the class in exchange of ideas. During problem solving, if a student can put forward an answer and can explain reasonably, endorsement should be given. What follows are some possible answers given by the students:

(i) Fill in 9, 11, 13 into the blanks to form an odd number sequence.

(ii) Fill in 11, 17, 27 into the blanks to form a sequence such that starting from the third term onward what follow equal to the sum of the two preceding terms minus 1.

(iii) Fill in 27, 181, 4879 into the blanks to form a sequence such that starting from the third term onward what follow equal to the product of the two preceding terms minus 8.

This kind of teaching is facilitative of inculcating students' independent thinking and abilities such as exchange of ideas. It is also facilitative of inculcating students to find the patterns and regularities of numbers. Compared with simply doing a few computational problems, this kind of teaching is more challenging and interesting.

In order to enable students to engage better in independent thinking and cooperative exchanges, teachers should encourage students to discover problems, propose problems, dare to query, and are willing to engage in cooperative exchanges. It is necessary to avoid students' cooperative exchanges becoming routine. Teachers should emphasize that cooperative exchange is premised upon individual's independent thinking, as well as expansion of thoughts through cooperative

exchanges.

(3) Strengthen estimation skills and encourage multifarious strategies of problem solving.

Estimation has broad applications in everyday living and mathematics learning. Inculcation of students' estimation consciousness and development of students' estimation abilities so that students possess good number sense are important and worthwhile for their developments.

For example, a book costs 12 dollars. There are 48 students in a class. How much does it cost if everyone in the class buys one? During teaching, teachers should encourage students to exchange ideas on methods of estimation. It can be 10 x 50 = 500, so considers around 500 dollars; it can also be 12 x 50 = 600, less than 600 dollars; also can be 10 x 48 = 480, definitely more than 480 dollars. Different students may have their own method of estimation and teachers should provide opportunities to enable them to engage in exchange of ideas.

Teaching should respect every student's personal characteristics, allow different students to know problems from different angles, adopt different ways to express one's thinking, and to use different knowledge and methods to solve problems. Encouraging multifarious strategies of problem solving is an effective means to undertake individually configured education so as to foster each student's personal development. For example, when learning multiplication of two two-digits numbers, teachers can encourage students to use their own background knowledge, explore and find results of computation. Teachers should not start with demonstration, then proceed to explain the vertical format of hand computation and its rules and rationale, and restrict students' thinking. Instead, teachers can raise problems associated with real object diagrams: There are 24 bottles of soft drinks in a carton, how many bottles in 18 cartons? First let students make estimation, and then try to compute the answer. Students can use the following algorithms:

24 x 10 + 24 x 8 = 432 24 x 20 – 24 x 2 = 432
20 x 18 + 4 x 18 = 432 24 x 2 x 9 = 432
24 x 3 x 6 = 432 18 x 4 x 6 = 432
18 x 3 x 8 = 432

Some students may use the vertical format to compute an answer. Under the foundation of engaging students in independent thinking to solve a problem, students

engage in small group exchanges. Every student can express their viewpoints, listen to peers' solutions, feel that there are multifarious and flexible strategies in problem solving, and compare the characteristics of different methods. Premised on securing each student's basic computational skills, different students can have their own different developments. Some students may master a variety of methods, and are able to express their problem solving solution paths.

For example, in a farm, there are altogether 22 hens and rabbits. The total number of feet is 58. What is the number of hens and what is the number of rabbits?

Teachers should encourage students to use multiple strategies in order to solve the problem:

(i) Trial & error and verification: Can allow students to guess the number of hens and rabbits. Suppose after making several guesses students can arrive at the correct answer, teachers can invite students to review the process of guessing so as to obtain some useful problem solving experiences.

(ii) Listing: Can guide students to use a table to list the number of feet of all possible cases running from "1 hen, 21 rabbits" to "21 hens, 1 rabbit", and solve the problem accordingly.

(iii) Finding patterns and regularities: Can allow students to list down some cases and guide students to find patterns in the table in order to solve the problem.

(4) Emphasize inculcation of students' consciousness and abilities of mathematics applications.

At this stage of schooling, students' knowledge, abilities, affection and attitudes develop further when compared with students at the first stage of schooling. Teachers should make full use of students' past experiences, guide students to apply mathematical knowledge acquired to realistic everyday living so as to realize the value of application of mathematics in realistic everyday living. Synthetic applications are important means to inculcate students' autonomous exploration and cooperative learning. Teachers can make use of the teaching processes of the following example to inculcate students' consciousness of mathematics application and abilities of synthesizing knowledge acquired to solve problems.

Teaching objectives: Through statistical survey of plastic bags discarded by households, students involve in processes of data collection, organization, description

and analyses, deepen understanding of the meaning of statistical quantities, as well as in activities that synthesize deployment of knowledge and skills acquired, feel the pollution caused to nature due to the misconduct of discarding plastic bags so as to awaken students' consciousness of environmental protection.

Teaching processes are detailed below (*T* stands for Teacher, *A*, *B*, *C*…are students):

T: Student *A* please comes forward to fill in the table with statistical data collected from the whole class (Last week Teacher *T* set a home assignment asking the students to tally the number of plastic bags discarded within one whole week. *T* also did similar tallying – as a result *T* diffuses herself naturally and harmoniously into the whole class).

<insert table here> Number of plastic bags, teacher, student

T: Who can make use of this set of data to describe the situation of plastic bags discarded by families of our class?

B: Families of our class within a week discard a total of : 17 + 18 + 12 + 27 + … + 19 + 18 +17 = 761 plastic bags.

C: Teacher, we can use multiplication. Total number of plastic bags discarded is: 18 x 14 + 17 x 14 + 27 x 2 + 19 x 4 + 12 x 2 + 16 x 5 + 10 x 2 + 9 + 8 = 761.

T: Very good. They use different computational methods to arrive at the same result. Are there any other ideas? (*Teachers should promptly guide students to express their understanding of the problem, and are not eager to evaluate the strengths and weaknesses of different methods. This would facilitate students to express their viewpoints autonomously. Actually, students can evaluate themselves and are able to make decision themselves.*)

D: The average number of plastic bags discarded by each family in the class is: <insert equation>.

T: Can you explain the meaning of this result? (*Promptly allow students one more time to comprehend the meaning of a mean*).

A: Each family should on average discard approximately 17 plastic bags.

E: Also, the median of this set of data is 17, whereas the mode is 18.

F: 17 is also a mode.

T: Do you all agree? (*Promptly guide students to think*)

(*Students are quiet for a while*)

C: I think so. Because 17 and 18 both occur 14 times. They occur most.

T: Very Good. The number which occurs most is a mode. What does it imply when 17 and 18 are both modes?

E: The majority of families in our class discard 17 or 18 plastic bags a week

(*pause for a while*). It seems that not the majority, …

T: Student *E* encountered some troubles now. He is not sure whether it is a majority or not. Who can help him?

A: It seems that it is a majority. In reality there are a total of 28 families that have discarded 17 or 18 plastic bags. This number exceeds half of the number of families already.

T: If number of families discarded 17 plastic bags is same as that of 18 plastic bags, both equal to 12, is this still a majority?

A: It looks like it is not a majority any more. Not reaching even half, but still it is the most. Should be that families discarded 17 or 18 plastic bags are the most.

T: Very good. The median is 17. What does this mean?

G: This means to order the family of each classmate according to the number of plastic bags discarded, the one family in the middle discarded 17 plastic bags.

T: Is this the same as a mean? (*Allow students one more time to feel the differences of different statistical quantities*) If somebody asks how many plastic bags are discarded by a family of the class in general, what is the answer? (*Emphasize the use of different statistical quantities to represent the different aspects of the same problem*)

Students engaged in discussion…..

T: Let's consider now how serious these plastic bags pollute the environment. (*Relate issue to environmental protection*)

M: We need to know how large a space is occupied by a plastic bag. (*The problem solving solution path is very clear, possesses estimation consciousness.*) We can assume that it resembles a rectangle; approximately 30 *cm* long, 20 *cm* wide, making a total of 600 cm^2. 761 plastic bags occupy a total of 761 x 600, i.e. 456 600 cm^2, or 45.66 m^3. Really big!

T: Let's think about this. If continued in this way, how big a piece of land would be polluted by us? Considering all families of students in our school, how big a piece of land would be polluted in a year? (*Bring computation into problem solving contexts*) Students, do you know that the campus area is approximately 30 000 m^2, please do some calculations. According to this speed, how long would it take for all families of students of our school to pollute an area same size as our school? (*Use problem of interest to students as a natural extension of classroom teaching*).

2. Evaluation Recommendations

The purpose of evaluation is to examine comprehensively students' learning

situations, inspire students' passion for learning, and foster total development in students. Evaluation is also a powerful means to engage teachers in reflection and teaching improvement.

Concerning evaluation of students' mathematics learning, Teachers should not only pay attention to comprehension and mastery of knowledge and skills, but also formation and development of feelings and attitudes. Likewise, teachers should not only pay attention to students' learning outcomes, but also to the variation and development of students' learning processes. There should be a multitude of forms and strategies of evaluation, emphasizing mainly on process evaluation and qualitative descriptions. Evaluation should pay full attention to individual differences, foster the aspiring function of evaluation, and secure the esteem and confidence of the students. Teachers should make good use of the large amount of evaluation information to regulate and improve the teaching processes promptly.

(1) Pay attention to evaluation of student mathematics learning processes.

During evaluation of learning processes, teachers should pay attention to students' level of participation, consciousness of cooperative exchanges, and development of affection and attitudes. At the same time, students' mathematics thinking processes should be emphasized. Regarding evaluation of level of participation, teachers should examine aspects such as whether students are able to participate in mathematics learning activities autonomously. Concerning evaluation of students' consciousness of cooperative exchanges, teachers should examine aspects such as whether students are willing to cooperate autonomously with peers, whether students realize the roles they play in the group, and whether the students are willing to exchange ideas with their peers. Concerning evaluation of affection and attitudes, teachers should couple with concrete teaching processes and problem contexts to familiarize with the autonomous learning characteristics, confidence and interests in learning mathematics of each student. Concerning evaluation of processes of thinking, through everyday observations and familiarization with the reasonableness and flexibility of students' thinking, teachers can examine whether students can use mathematical language to explicate their viewpoints clearly.

Building portfolios is an important way for the students to conduct self-evaluation, and it can reflect the developmental and growth processes of the students. Contents collected in the portfolios should be selected autonomously by the students, and decided jointly with their teachers. For example, when evaluating the part on synthetic

applications, students can utilize portfolios to collect the following information so as to reflect the progress they make during their exploration.

(i) Discover mathematical problems during everyday living;
(ii) Collect related information;
(iii) Proposal for solving the problem and its processes;
(iv) Activity reports and small mathematics papers;
(v) Reflection on problem solving.

(2) Evaluate students' fundamental knowledge and basic skills appropriately.

At this stage of schooling, evaluation of fundamental knowledge and basic skills should obey the basic rationale of *Standards*. Based on the standards of the knowledge and skills objectives, teachers should examine students' degree of understanding of fundamental knowledge and degree of mastery of basic skills. What needs to be emphasized is that stage objectives are objectives that each student ought to achieve at the end of that stage of schooling. It allows some students to make efforts in a considerable period of time to accumulate knowledge and skills to achieve the designated objectives. In this regard, teachers can choose to delay their judgments. If students are not satisfied with their test performance, teachers should encourage them to apply for re-sit, giving them a second chance to answer again the questions. When students can correct their test mistakes through hard work, teachers can evaluate students based on the re-sit, and give encouraging remarks as well. Delayed judgment dilutes the discrimination function of evaluation, making visible the longitudinal developments of the students. Especially for those students with learning difficulties, delayed judgment enables them to visualize their progress, feel the joy of success, resulting in stimulating the dynamics of another new wave of learning.

Evaluation should be carried out coupling practical background with processes of problem solving. Evaluation of concepts, formulae and rules should pay more attention to understanding of the meanings of the knowledge itself, as well as the associated applications.

Concerning evaluation of Numbers and Algebra, teachers should examine primarily students' understanding and application of meanings of numbers and their operations. The following aspects may be included: Whether students can use knowledge of numbers and computation to describe and solve practical problems; Whether students are able to use appropriate computational strategies to carry out operations accurately; Whether students form the habits to undertake estimation and

verification; Whether students are able to use a calculator to explore patterns and regularities effectively.

Example 1 *Investigators A, B, C observe a house from three directions; the views seen by investigators A and C have been displayed respectively. How about the view seen by B?*
<insert diagram>

Concerning evaluation of learning of Statistics and Probability, the main points should be placed on examining whether students can understand the characteristics of different statistical figures and tables, as well as the meanings of statistical quantities; whether students are able to select appropriate statistical figures and tables, and statistical quantities to represent data; whether students can realize the meanings of the possibility of events happened with varied chances. Those pure computational problems, such as computation of the mean of a given set of data should not be the main contents of evaluation.

Concerning evaluation of Synthetic Applications, it may be difficult to accomplish within a single written examination. Therefore, teachers should emphasize the processes of student participation in activities. They should not include such activities or problems into the scope of the examinations (or tests).

(3) Emphasize on evaluation of students' abilities of discovering and solving problems.

Concerning the issue of evaluation of students' abilities of discovering and solving problems, teachers can examine the followings: whether students can discover and pose mathematical problems in realistic everyday living; whether students can explore and find out effective means of solving problems, and attempt to find out other means; whether students can cooperate with others; whether students can express the processes of solving problems, and attempt to explain the results obtained; whether students possess the consciousness of reviewing and analyzing the problem solving processes. For example, teachers can design the following problem to examine the problem solving ability of the students.

Example 2 *Use a thin string approximately 50 cm long to enclose a rectangle. How to do this such that the area enclosed is a maximum?*

Concerning this problem, teachers should examine first whether students are able to enclose different rectangles and then order these rectangles according to some rules and patterns; whether students are able to discover the relationships between area and the length/width, so as to move a step further to guess that when the enclosed figure is a square the area enclosed is a maximum.

(4) Evaluate students in multiple ways and from different perspectives.

Compared with the first stage of schooling, students are more autonomous and independent at this stage of schooling. Therefore, when evaluating student learning, opportunities should be given to allow students to engage in self-evaluation and peer evaluation. Teachers should not rely only on teacher evaluation and should take the initiative to allow parents and the communities to participate in the evaluating processes as well. There should be multifarious ways of evaluating students. Not only written examination, oral examination and activity reports, but also classroom observation, after-class interview, task analysis, and setting up of the portfolio assessment can be used as well.

Each evaluation method has its own characteristics. Teachers should couple contents of evaluation with learning characteristics of students, select appropriate ways of evaluation, so as to examine the learning realities of students, and to reflect the progress process of the students. Teachers can examine from four aspects: mastery of fundamental knowledge; seriousness in doing assignments; development of problem solving abilities; skills of engaging in exchange of ideas. For example, teachers can familiarize with situations of students' mastery of computational skills in their assignments; through classroom observations familiarize with students' attitudes; through portfolios familiarize with students' consciousness and abilities of posing problems and solving problems; through small-group discussions familiarize with students' consciousness and skills of cooperative exchanges.

(5) Evaluation results are qualitatively and quantitatively integrated for presentation, but paying more attention to the first.

When presenting results of evaluation, teachers should integrate qualitative and quantitative approaches, but paying more attention to the qualitative one. Quantitative evaluation can adopt the grading format. Qualitative descriptions can utilize remarks, paying more attention to what the students have mastered, what progress has been made, and what kinds of abilities possessed. In this way, the evaluation results will

facilitate students to build up their confidence in mathematics learning, increase students' interests in mathematics learning, and foster students' development.

Shown below is an example of a remark. "This semester we have learned how to collect, organize and express data. Through Michael's efforts and hard work, he is able to collect and record data, knows how to compute mean, and familiarize with characteristics of statistical diagrams. The statistical diagram constructed by him is outstanding. As far as this is concerned, he is the best in the class. However, he has some difficulty in using language to explain statistical results. Carry on, Michael! Grading B"

Qualitative remarks read by students are practically exchanges of affection between students and teachers. Students acquire experiences of success, establish confidence in learning mathematics well, and know which aspects he should spend more efforts.

3. Teaching Materials Editing Recommendations

Teaching materials provide basic clues of what student learning activities should entail. It realizes curriculum objectives, and is important resources for implementing teaching. In principle, editing of teaching materials should be based on Standards. Teaching materials provided should tie in intimately with practical life, allow students to experience the function of mathematics in everyday living. Topics covered should vary and their presentation format should be rich and multifarious. Editing of teaching materials should be facilitative of stimulating students' motivation in learning, guide students to start from the knowledge and experiences already acquired, experience the formation and developmental processes of knowledge through independent thinking and cooperative exchanges. Also, editing of teaching materials should be facilitative of mobilizing teachers' autonomy and self-initiation, and encouraging teachers to undertake creative teaching. Important mathematical concepts and thoughts should display the principle of an ascending spiral, step by step enhancing students' understanding of knowledge and methods of mathematics.

Considering the diversity of individual differences, teaching materials should exemplify its characteristics after fulfilling all the basic prerequisites. It should possess a certain degree of flexibility as well. When editing teaching materials, teachers should consider sufficiently how it can tie in with the exploitation and

deployment of other curriculum resources.

(1) Choose learning materials that is realistic and interesting.

Relative to the first stage of schooling, students' everyday living experiences and background knowledge at this stage of schooling are richer. They pay more attention to the people and events around them, move a step further to familiarize with the realistic world, and aspire to solve practical problems. Therefore, contents should be intimately related to students' realistic living. Teachers deploy real examples that are of concern and interest to the students as background of the knowing processes, stimulate students' quest for knowledge, as well as enable students to feel that mathematics is around them and the intimate connection with the realistic world.

For example, at this stage of schooling, students' knowledge of numbers extends from within ten thousands to within 100 millions, but students still lack direct feelings of large numbers. Therefore, choice of learning materials at this stage of schooling should facilitate students' feelings of large numbers and should be selected from familiar objects and events around students so as to enable students to move gradually from grasping small numbers to large numbers.

Example 1 *Measure the frequency of heart beats per minute. Furthermore, deduce the number of heart beats per hour, and per day. How many days are needed in order for the heart to beat one million times?*

Concrete models and figures should be selected as teaching materials. Start with these models and figures to know the related contents. For example, concerning contents standards such as "Identify the shapes and forms seen from the front, the side and the top", teachers can use different ways such as real objects and models for display. Regarding another example on pattern design, teachers can start directly with the various figures already acquired by the students, and discuss with students through what ways these figures can be used to design pretty patterns. Concerning contents of figures and transformation, and figures and coordinates, teaching materials should include real examples close to the students, such as rectilinear motion of an object, erecting stems during tree planting, and schematic diagram of the layout of the main school building.

Teaching materials may include appropriate contents from different media, such as newspaper, magazine, broadcast and television. Teachers can have them presented to

students in suitable ways so as to stimulate students' passion for learning and spirits of autonomous exploration, encourage students to cooperate with others, and to exchange ideas with classmates.

Example 2 From the census annual report, census data (5 years interval) of a place from 1949 to 1995 can be tabulated as follows:
<insert table here> year, population/ten thousands

Teaching materials can guide students to undertake analyses on this set of data, so that students can familiarize with the changes of the population, as well as to allow teachers to diffuse ideas of function.

(2) Allow students ample room for exploration and cooperative exchange of ideas.

Teaching materials should leave enough room for students to engage in exploration and exchange of ideas so as to facilitate changes of learning methods of the students. Editing of teaching materials should demonstrate processes of how knowledge is formed, during which students explore and understand related contents. Setting of questions should be inspiring, and its presentation should facilitate activities such as observation, experimentation, manipulation, inferences and exchanges. Through items such as "Try to Observe", "Try to Do", "Try to Think" and "Try to Discuss" teachers can guide students to undertake explorations and exchanges.

Example 3 *On a five-columns and five-rows chessboard a dice is rolled over its edge. Rolling of a dice on a square of the chessboard is limited to the left, right, forward and backward positions of this square. When the game starts, the dice is at 3C as shown in the diagram below.*
<insert diagram here>

(i) *When dice is rolled from 3C to 3B, configuration of dice is as follows.*
<insert dice here>

(ii) *Then roll the dice from 3B to 2B, configuration of dice becomes <insert dice here>.*

(iii) *Continue to roll the dice from 2B to 2A, the face of the dice facing upward is* _____.

(iv) *At last, roll the dice from 2A to 1A, the number of dots facing upward is* _____.

Try to Think

Starting from 3C, if we would like <insert figure> to face upward, how should we roll the dice? What is the position of the dice then.

Try to Do.

Try to exchange your ideas within your group, and then practically manipulate the dice yourself. Does the outcome matches with what you imagine?

This type of problem is very exploratory and open-ended in nature. It is facilitative to use it to develop students' space concepts. At the same time, students can solve the problem through manipulation, imagination, or a combination of the two methods. Therefore, students can have different developments by doing this exploration.

(3) Presentation formats should be rich and multifarious.

Compared with the first stage of schooling, teaching contents at this stage of schooling comprise of certain amount of texts and symbols. While texts and diagrams are deployed simultaneously in the presentation of teaching materials, the weight on the use of mathematical language should be increased progressively. Teachers can use formats of interest to the students, such as pictures, games, tables and texts, so as to display the contents of the teaching materials intuitively. An example of such material is how to estimate the quantity of a pile of pins. Teachers can use a series of pictures to visualize the scene of student activities so that different pictures display different kinds of activities. Teachers can also use a series of cartoon pictures to display the scene, along with descriptive texts to stimulate students' interests in learning.

(4) Design of content should possess some degree of flexibility.

Premised on the prerequisites of grasping basic requirements of *Standards*, teaching materials can have a certain degree of flexibility. One concrete design format is to propose problems of different levels or open-ended problems having the same contexts and situations so that different students can have different developments. Teaching materials should make available some optional contents or reading materials, diffuse some important mathematical thinking and methods so that those students who can manage can have more room for learning and development. On the other hand, selection of contents of teaching materials should consider the issue of regional differences. For those regions with better conditions, some modern tools can be included in the teaching materials, e.g. the use of calculators to process mathematical

problems. In this way students can relieve themselves from the chores of complex computations, and save their energies to concentrate on understanding of concepts and methods, as well as aspects related to exploratory activities.

Example 4 *Shown below is the results of four opposing basketball matches of two teams in a Peasant Sports Event. (Units: points)*
<insert table here> First Match, Second Match, Third Match, Fourth Match, Team 1/Point, Team 2/Points

Think about what kind of statistical graph is suitable to analyze and compare this data set. Answer the following questions:

(i) How do you design such statistical diagrams?

(ii) Can you read the results of each match of a basketball team intuitively from the statistical diagram?

(iii) Are there special functions related to the use of each type of statistical diagrams?

(iv) How do you evaluate the two teams? Exchange what you think with your peers?

Teaching materials can contain questions asking students to use computers to draw different statistical diagrams of the above-mentioned example. Teachers can guide students to alter some data so as to display dynamically the changes incurred, and increase students' interests in learning.

(5) Important mathematical concepts and ideas should exhibit the principle of an ascending spiral.

The objectives listed in *Standards* are stage objectives. Some important mathematics concepts and mathematics thinking methods should be based on students' psychological characteristics, knowledge background and characteristics of knowledge acquired. There is a need to adopt the ascending spiral approach, but repetition is to be avoided.

Ascending spiral design approach can cut across the different stages of schooling. One example is about knowing the meanings of fractions. At the first stage of schooling, teachers can design "Preliminary knowledge of fractions", whereas at the second stage design "Knowledge of fractions". "Knowledge of possibilities" is another example that can be designed as levels in an ascending spiral manner. At the

first stage of schooling, students primarily experience the phenomenon of uncertainty, whereas at the second stage students mainly attempt to engage in quantitative depiction regarding the magnitude of the possibility that a simple event happened.

(6) Pay attention to relationships and synthesis of various parts of contents.

Mathematics knowledge is an organic whole. Teaching materials should reflect the relationships and synthesis amongst the various parts of contents, facilitating students' knowledge of mathematics as a whole.

For example, Space and Figures at the second stage of schooling comprises of four parts of contents: Knowing Figures, Measurement, Figures and Transformation, and Figures and Positions. These contents are mutually independent of each other, but they are also intimately related. When editing the teaching materials, teachers should arrange the contents appropriately after considering the connection amongst them. Contents on Knowing Figures and Measurement can be intertwined, appropriately handling the relationships of related knowledge on these two parts of contents. However, contents on Figures and Transformation and Figures and Positions are relatively independent of each other. Teachers can arrange contents at different levels of schooling according to the difficulty levels of the contents.

When editing teaching materials, teachers should add some open-ended contents, or contents that afford synthetic applications. This would facilitate students to engage in autonomous explorations and cooperative exchanges, creating loose environments conducive to the development of the students. Regarding contents on synthetic application at this stage of schooling, through introduction of concrete problem contexts, students engage autonomously in observation, manipulation, inferences and exchanges, progressively form holistic knowledge of mathematics, as well as acquire abilities of synthesizing knowledge and methods of mathematics to solve concrete problems.

(7) Introduction of mathematics background knowledge to the students.

Teaching materials should pay attention to demonstrate the cultural values of mathematics. Concerning processes of learning some mathematical topics, teachers can insert in appropriate places topics to introduce knowledge related to discoveries in mathematics and history of mathematics. Students are equipped with holistic view of mathematics development that may stimulate students' ensuing studies in the future.

Concerning the part on Numbers and Algebra, teachers can adopt a historical approach to introduce various kinds of numeration system so that students can realize the superiority of decimal numeration. Through an introduction of the comparison of numeration systems of the large numbers in ancient Egypt, Greece and China, students realize the superiority of modern enumeration of large numbers. Teachers can introduce historically different kinds of computational tools, enabling students to know the influences of different kind of computational tools on mathematics and everyday living.

Concerning the part on Space and Figures, teachers can introduce Tangram and its related history. In particular the patterns constructed by the ancient people should be introduced to enable students feel the beauty of geometrical constructions and appreciate the wisdom of our ancestors. Teachers can introduce historical materials on compasses and set squares (in the form of two straight edges right-angled to each other), so that students can realize their functions in geometrical constructions and measurements in ancient China. Teachers can introduce results and the realistic background of computation of areas and volumes of simple geometrical figures in ancient Egypt, Babylon, India and China so as to enable students to move a step further to experience the intimate relationships between geometry and human living experiences, and between geometry and practical needs.

Concerning the part on statistics and probability, teachers can introduce weather forecast, insurance and related materials. This enables students to familiarize with realistic sources of probability problems and statistical work historically, experience the practical backgrounds of statistical thinking and methods.

Third Stage of Schooling (Grade 7~9)

1. Teaching Recommendations

Mathematics teaching entails teaching of mathematical activities. It is an interacting exchange and joint developmental process between teachers and students, as well as amongst students.

Mathematics teaching should start from students' practical experiences, and create problem contexts and situations conducive to students' autonomous learning.

Through practical work, thinking, exploration and communication, students master basic mathematical knowledge, form skills, develop thinking, and learn how to learn. Under the guidance of the teacher, students are able to learn actively, autonomously and with individuality.

During teaching, teachers should promote democracy in teaching, so as to become organizers, guides, and collaborators of student mathematical activities. Teachers should be good at inspiring students' learning potentials, encourage students to innovate and practice boldly. They should deploy teaching materials creatively, exploit actively, and use all sorts of teaching materials, so as to provide students rich and multifarious learning materials. They should pay attention to students' individual differences and practice individualized teaching effectively, so that students receive sufficient developments. Application of modern educational technologies to teaching should be emphasized. For those areas where conditions permit, teachers should try hard to use computers and related software reasonably and effectively to raise teaching efficiency.

(1) Allow students to experience how mathematical knowledge is formed and applied.

Teaching at this stage of schooling should couple with concrete mathematics contents and adopt a "Problem Context – Model Construction – Explanation, Application and Extension" model, enabling students to involve in processes of knowledge formation and application, so as to understand better the meanings of mathematical knowledge, master necessary fundamental knowledge and basic skills, develop consciousness and abilities of application of mathematical knowledge, as well as enhance aspirations and confidence in learning mathematics better.

Teaching of abstract mathematical concepts should pay attention to the practical background and formation process of concepts, and adopt a new way of learning to assist students to overcome mechanical memorization of concepts. As an example, concepts of functions should not only be focused on discussion of their expressions, domains and ranges. Instead, concrete real examples should be chosen so that students are able to realize that function can be used to reflect patterns of variation of practical objects and events.

Example 1 *It is known that temperature expressed in Celsius degrees (°C) is related to Fahrenheit degrees (°F) as follows:*

<insert table here> Celsius Temperature, Fahrenheit Temperature

Plot points on a rectangular coordinates system to observe the distribution, establish an expression of a function that satisfies the above-mentioned relationships.

During teaching, teachers can guide students to engage in the following activities:

(i) Plotting points: According to the data in the table plot corresponding points on a rectangular coordinates plane.

(ii) Judgment: Judge whether the points are located on the same straight line. (Can use a straight ruler to test, or students can join the points consecutively, and observe whether all the points lie on the same straight line or not).

(iii) Finding solutions: After judging that the points do lie on the same straight line, select the coordinates of two points, and derive the expression of a function of first degree.

(iv) Verification: Verify coordinates of other points whether they satisfy the expression of this first degree function.

During application of knowledge and methods of mathematics teachers should guide students to realize the values of mathematics, and strengthen their mathematics consciousness. Teachers may guide students to use a transformational viewpoint to explain phenomena related to figures in the realistic world, appreciate the beauty of symmetry of some architectures, and allow students to use knowledge acquired to design patterns.

Another example is that teachers can guide students to use knowledge of statistics and probability to discuss the problem below:

Example 2 *One advertisement states: "75% of people use this company's product". What do you think after hearing this advertisement?*

Through discussion of this problem, students are able to know that they need to apply statistical concepts to analyze data of 75% in the advertisement. For instance, how to select the sample and how big is the sample? If a company has interviewed four persons and three of them use the product, then say "75% of people use this company's product". Clearly this is not convincing at all. Therefore, we should question about the authenticity and reliability of this data.

(2) Encourage students to engage in autonomous explorations and

cooperative exchanges.

Effective mathematics learning cannot simply rely on imitation and memorization. Teachers should guide students to engage autonomously in mathematical activities such as observation, experimentation, guessing, verification, inferences and exchange of ideas, so as to enable students to form their own understanding of mathematical knowledge and effective learning strategies.

Contents of Numbers and Algebra at this stage of schooling are abundant with all sorts of models that may be used to represent relationships, such as algebraic expressions, equations, functions and inequalities. Therefore, during the teaching processes teachers should allow students ample opportunities to involve in processes such as exploration of quantitative relationships of events and objects, as well as patterns of variations.

Example 3 Complete the following computations:

$1 + 3 = ?$

$1 + 3 + 5 = ?$

$1 + 3 + 5 + 7 = ?$

$1 + 3 + 5 + 7 + 9 = ?$

According to the results obtained, explore the patterns and regularities.

During teaching, teachers should start to allow students to think about: What can you discover from the above expressions? Students are allowed to involve in processes such as observing (characteristics of each expression and its result), comparing (differences and similarities amongst the expressions), inducing (those possible patterns and regularities), and proposing a hypothesis. During teaching, teachers should not only pay attention to whether students can locate the patterns and regularities, but also whether students have engaged in thinking or not. If students cannot discover the patterns and regularities independently for the time being, teachers should encourage the students to engage in cooperative exchanges and move a step further to explore. Teachers may provide some assistance as well. As an example, teachers can present the following array of dots to the students so as to discover patterns from the connections between numbers and figures:

<insert figures and expressions here>

Teachers can encourage students to predict $1 + 3 + 5 + 7 + 9 + \ldots + 19 = 10^2$.

Afterward, teachers can generalize the result of this problem in accordance with students' practical situations, and obtain $1 + 3 + 5 + 7 + \ldots + (2n-1) = n^2$. Students should know that correctness of this conclusion needed further proof in the future.

Concerning teaching of contents on Space and Figures (e.g. appreciation and design of patterns, basic properties of figures, three-view drawings), teachers can organize students to engage in activities such as observation, manipulation, guessing and inferences, and engage them in exchanges of activity experiences. Teachers can help students accumulate experiences of mathematical activities and develop space concepts and orderly thinking.

Example 4 *Organize students to engage in the following activity:*

(i) *Use cardboard to make an angle.*

(ii) *Put this angle on a piece of white paper, draw ∠AOB (see diagram);*

(iii) *Turn cardboard 180 degrees around O, draw ∠A'OB';*

(iv) *Explore what conclusion you may get during these processes.*

<insert figure>

Through manipulation and observation, each student may discover some of the following conclusions: OA and OA', OB and OB' are straight lines; ∠AOB and ∠A'OB' are vertically opposite angles, the magnitude of ∠AOB and ∠A'OB' are equal. Students may also discover: ∠BOA' and ∠B'OA are also vertically opposite angles, and they are equal as well; ∠AOB and ∠A'OB are complementary, ……

During such activities, students not only can acquire knowledge autonomously, but also enrich their mathematical experiences continuously, learn how to explore and how to learn as well

(3) Respect students' individual differences and satisfy multifarious learning needs of the students.

Students' individual differences can be perceived as differences of cognition and thinking strategies, as well as differences in cognitive levels and learning abilities. Teachers should promptly familiarize with and respect students' individual differences so as to satisfy the multifarious learning needs of the students.

During teaching teachers should encourage and promote multifarious problem solving strategies, and respect students' different performance levels as revealed in

the problem solving processes. Design of problem contexts, implementation of teaching processes, and arrangement of exercises should as far as possible enable students to participate autonomously, so that they can propose their own problem solving strategies. Teachers should guide students to select appropriate strategies when they are engaged in exchange of ideas with others, enrich students' experiences in mathematical activities, and promote levels of thinking.

Concerning those students with learning difficulties, teachers should provide assistance and care to them promptly, encourage them to participate autonomously in mathematical learning activities, attempt to use one's methods to solve problems, and to express one's viewpoints. Teachers should acknowledge promptly any small progress made by the students, guide them patiently to analyze reasonably the causes of mistakes made by them, and encourage them to correct mistakes so as to promote their interests and confidence in mathematics learning. For those students that can afford to further themselves and like mathematics very much, teachers should provide them sufficient materials, guide them on the reading of these materials, as well as develop their mathematical abilities.

(4) Pay attention to the necessity of proofs, their basic processes and methods.

Teaching of proofs should pay attention to the understanding of the necessity of proofs, to the experiences of the basic methods of proofs and their processes, and not simply to quest for the quantity of propositions to be proved, nor the proving skills. Concretely speaking, this would entail the following aspects:

In teaching of propositions, teachers should explicate through concrete examples what a proposition is about in mathematics and everyday living. Students can discriminate the truth and falsehood of a simple proposition, and are able to use counterexamples to confirm that a proposition is false. Concerning basic propositions in geometry, teachers should ask students to draw out the corresponding figures and move on progressively to learn how to use symbols to represent a proposition.

In teaching of proofs, through everyday living and concrete examples in algebra and geometry, teachers should start to enable students to realize that some propositions can be acknowledged by all of us through observation and experimentation, whereas some propositions may not be acknowledged simply through observation and experimentation, so as to realize the necessity of proofs. Second, teachers should enable students to understand the basic requirements of

proofs, explicate systematically one's own ideas, know that deduction requires evidences, and presentation of the proving processes must be clear and systematic.

Proof by contradiction is also an important method of proof. During teaching, through real everyday living and simple mathematical examples, students realize the ideas of proof by contradiction. However, it is not necessary to furnish format of proof by contradiction during obligatory stage of schooling.

During teaching, teachers should treat proofs as natural extension and necessary development of exploratory activities, guide students to start from problems, deploy induction and analogy to arrive at hypotheses in accordance with results of observation and experimentation, and then carry out proving. This will facilitate students' comprehensive understanding of proofs. Teachers should enable students to use the relatively more prescribed mathematical languages to express the process of argumentation, so that students can express their viewpoints clearly and systematically and understand other people's ideas. Teachers can organize students to explore the considerations of proofs, and to engage in appropriate comparison and discussion, facilitating the widening of the perspectives of the students. Teachers should provide some propositions with realistic backgrounds to increase the interests of the proofs. This would help stimulate students' interests in mathematical proofs and master synthetic proofs with confidence.

(5) Pay attention to the connection amongst different branches of mathematical knowledge; enhance the problem solving abilities of the students.

During teaching, teachers should consciously and systematically design teaching activities to guide students to realize the connection amongst the different branches of mathematics, feel the holistic nature of mathematics, enrich oneself continuously on strategies of problem solving, and to enhance problem solving abilities.

Example 5 Prepare a number of square and rectangular cards (see diagram below)

<insert diagram>

(i) Teachers write down randomly a quadratic expression involving a and b, and this quadratic expression should be factorable as a product of two linear expressions, of which all the coefficients are positive integers, e.g. $a^2 + 2ab + b^2$, $a^2 + 4ab + 4b^2$, $2a^2 + 5ab + 2b^2$.

(ii) According to the quadratic expression given by the teacher, students choose

corresponding types and number of cards, and attempt to use the cards to make up a rectangle.

(iii) Discuss the algebraic meaning of that rectangle.

(iv) Ask students to choose randomly appropriate number and types of cards to make up rectangles of different sizes, and reply the algebraic expressions used to represent the rectangles.

During this activity, students shall realize the connection between algebra and geometry.

At this stage of schooling, through contents of thematic studies, students can involve in "Problem Context – Model Construction – Explanation, Application and Extension" to solve problems, develop one's cognitive abilities, and acquire some experiences and methods for researching problems.

Example 6 *Conduct a survey of students' extracurricular activities.*

Regarding this relatively complex topic of study, teachers should give sufficient time and space to students to carry out exploration and exchange of ideas.

Questions that students should discuss first are what kind of data is required to depict the situations of extracurricular activities: Is it that time used for extracurricular activities, or is it that kind of extracurricular activities or number of participants, or is it that there are other criteria needed to be considered? Through group discussion, students can select one or more criteria to depict the situations.

Then, students shall discuss how to conduct the survey and collect data. During the discussion process, students may have different opinions: Some students propose to conduct a survey to all students in the school, whereas some consider all one needs is to sample a group of students and then "Use the sample to make inferences to the population". If there are students who insist to conduct a survey to all students in the school, teachers can give such examples as "Know the life span of a batch of light bulbs" to illustrate the necessity of sampling. Another way is to allow students to manipulate in practical terms so as to realize that collecting data from all the students in the school is a relatively difficult undertaking.

The next question is "Who are the targets of the survey?" Regarding this, students may have a lot of ideas: Conduct a survey of their classes; Sample those students that

are playing balls on the playgrounds; Randomly select some students at the main entrance of the school; Draw a sample of boys and girls according to the sex ratio in each school grade; Draw a number of students randomly from each class's student name list. Teachers should not be eager to endorse nor to negate these ideas. They should let students engage in practical manipulation and sufficient discussion, so as to know that different samples may produce different results. Discussion may ensue accordingly: What inferences may be made as a result of these explanations? Can we use some methods to confirm or refute the conclusion drawn from the survey data? According to the characteristics at this stage of schooling, the main point of teaching should be placed on the feeling of sample representation, as well as the influences of sampling on the survey results. Regarding how to arrive at a random sample, and how to ascertain the sample size, these should not be set as teaching requirements.

This is an open-ended problem. Students need to go out of the classrooms to conduct a survey. Those students who feel interested not only can survey all students in the school, but also through information searching and other means to obtain information of extracurricular activities of other students in the whole city, country, or even other countries. Students can also survey on other topics, providing evidences for the schools to make informed decisions.

(6) Deploy modern information technology sufficiently.

After the students have secured a foundation to understand and apply formulae and rules to carry out computation correctly, teachers can guide students to use calculators to undertake complex computation. During classroom teaching, home assignments, practical activities and examinations, teachers should allow students to use calculators, and encourage students to use calculators to carry out exploration of patterns and regularities.

For those regions with better conditions, teachers should enable students to use function calculators, computers and related software as far as possible. Such modern educational strategies and technology shall change teaching approaches effectively, and increase teaching efficiency. Examples are: the use of computers to display the images of functions, geometrical figures and their transformational processes in order to research their properties; the use of database to locate information, and to draw different kinds of tables and figures on the same set of data so that students are able to choose appropriate images to describe data; the use of computers to produce sufficient modeling results so as to help students realize the meaning of probability of an event

happened.

2. Evaluation Recommendations

The purpose of evaluation is to examine comprehensively students' learning situations, inspire students' passion for learning, and foster total development in students. Evaluation is also a powerful means to engage teachers in reflection and teaching improvement.

Concerning evaluation of students' mathematics learning, teachers should not only pay attention to comprehension and mastery of knowledge and skills, but also formation and development of feelings and attitudes. Likewise, teachers should not only pay attention to students' learning outcomes, but also to the variation and development of students' learning processes. There should be a multitude of forms and strategies of evaluation, emphasizing on the combination of outcome and process evaluation, as well as combination of quantitative and qualitative descriptions. Evaluation should pay full attention to individual differences, foster the aspiring function of evaluation, and protect the esteem and confidence of the students. Teachers should make good use of the large amount of evaluation information to regulate and improve the teaching processes promptly.

(1) Pay attention to evaluation of student's processes of mathematics learning.

Evaluation of students' mathematics learning processes includes: degrees of participation in mathematical activities, confidence, consciousness of cooperative exchanges, as well as habits of independent thinking, aspects of developmental levels of mathematical thinking. Examples are:

- ☺ Whether participate autonomously in learning activities;
- ☺ Whether possess confidence in learning mathematics well, able to face difficulties encountered;
- ☺ Whether feel happy to cooperate with others, willing to exchange ideas with others;
- ☺ Whether able to obtain solution paths of a problem through independent thinking;
- ☺ Whether able to find effective ways to solve problems, attempt to consider problems from different perspectives;

- ☺ Whether able to use mathematical languages and express one's thinking processes systematically;
- ☺ Whether understand others' ways of thinking, and to benefit from cooperative exchanges with peers;
- ☺ Whether possess consciousness in reflecting one's thinking processes;

……

Through development of portfolios, students can reflect their mathematics learning situations and growth trajectories. Contents of portfolios include:

- ☺ Student's particular problem solving methods;
- ☺ Learning experiences most impressive to the students;
- ☺ Most satisfactory assignments;
- ☺ Documentation of exploratory activities;
- ☺ Summary of knowledge units;
- ☺ Proposals of challenging problems;
- ☺ The book liked most by the student;
- ☺ Self-evaluation and peer evaluation;

……

Students should select autonomously the contents of the portfolios themselves. The contents chosen must be authentic and renewed periodically. According to the characteristics of students at this stage of schooling, teachers should provide explanations for any contents selected or renewed. For example, when students put into the portfolios new assignments to replace the old ones, they must give their reasons. If they think that the new assignments are done better than the previous ones, they need to explain the reasons of the learning progress. Teachers should guide students to reflect promptly their developmental situations, such as what learning objectives have been achieved, what progress has been made, characteristics of one's masterpieces, strategies of problem solving, areas needed further hard work, as well as to organize students to engage in display and exchanges.

Development of growth records of student mathematics learning enables students to familiarize comprehensively with their learning processes. In particular, students feel that they are progressing continuously and this is beneficial to nourish students' confidence. Likewise, teachers can familiarize comprehensively with the learning situations of the students, providing pertinent evidences to improve teaching and to practice individually configured education.

(2) Evaluate students' fundamental knowledge and basic skills appropriately.

Concerning evaluation of fundamental knowledge and basic skills at this stage of schooling, teachers should follow the basic rationale in *Standards*, use knowledge and skills objectives at this stage of schooling as criteria, and examine the degree of understanding and mastery of fundamental knowledge and basic skills. What needs to be emphasized is that stage objectives are basic requirements that students need to attain at the end of that stage of schooling. Therefore, if students are not satisfied with the examination results, teachers should create opportunities to let students retake examinations. This kind of "delayed judgment" dilutes the discrimination function of evaluation, respects individual differences amongst students, and creates conditions for different students' development. In particular, "delayed judgment" enables students with learning difficulties to perceive their learning progress, acquire the joy of success, so as to stimulate new learning momentum.

Evaluation of fundamental knowledge and basic skills should couple with practical backgrounds and processes of problem solving, paying more attention to the understanding of meaning of knowledge itself and the application of knowledge with understanding.

Concerning evaluation of Numbers and Algebra, teachers should examine primarily students' understanding of concepts, rules, operations, and the level of operations. Teachers should not simply examine memorization of knowledge, and evaluation of operations should not demand skills excessively.

Concerning evaluation of Space and Figures, teachers should examine primarily students' understanding of basic geometrical facts, concepts of space, abilities of plausible reasoning, as well as acquisition of preliminary deductive abilities. Concerning evaluation of the part on proofs, teachers should pay attention to students' understanding of the meaning of proofs, as well as that each step of the proving process is based on evidences.

Concerning evaluation of Statistics and Probability, the main points should be placed on whether students possess the knowledge and skills to apply statistics and probability in activities with realistic backgrounds, as well as whether students possess concepts of statistics.

At this stage of schooling, weighting of written examination is higher than that in the earlier two stages. During evaluation, teachers should tie written examination organically with other forms of evaluation. When conducting written examination,

teachers should follow the requirements of *Standards*, avoid setting questions that are uncommon, queer, or demand sheer memorization. Teachers should design problems that couple with realistic background to examine students' understanding of mathematical knowledge, and abilities in the use acquired knowledge to solve problems. Teachers should limit the proportion of usage of objective question format, and should design some exploratory and open-ended problems so as to expose students' thinking processes to a greater extent, and to permit students to use longer time to respond.

Example 1 *A family of 3 adults and 4 children go for a tour. The fees standards set by tour agent A are: For any four tickets bought, the rest shall receive 50% discount. The fees standards set by tour agent B are: There will be 25% discount for family group ticket. For both tour agents, the original tour fee is 100 dollars per person. Which tour agent should be chosen so that the fee paid by this family is a minimum? Which tour agent's offer would be more competitive when the number of children of the family varies?*

This problem examines primarily contents of solutions of linear functions and inequalities. The main focus is neither placed on the memorization of concepts nor the imitation of skills. Instead, it provides a problem context connected with realistic everyday living in order to examine students' understanding of related knowledge and deployment of acquired knowledge for problem solving. At the same time, the problem provides ample space to students to think about various solution paths. Through evaluation of students' problem solving processes, teachers not only can examine situations about students' mastery of related knowledge and skills, but also familiarize with students' cognitive characteristics.

(3) Emphasize on evaluation of students' abilities of discovering and solving problems.

Evaluation of abilities of students' problem finding and problem solving at this stage of schooling includes:

- ☺ Whether students can couple with concrete contexts and situations to discover and propose mathematical problems;
- ☺ Whether students are able to attempt to analyze and solve problems from different viewpoints;
- ☺ Whether students realize the importance of cooperating with others to solve problems;

- ☺ Whether students are able to use words, letters, figures and tables to explicate clearly the problem solving processes, and are able to use different ways of communication;
- ☺ Whether students are able to explain the results obtained are reasonable or not;
- ☺ Whether students are able to reflect on the problem solving processes, and acquire problem solving experiences.

Example 2 *Shown below is a calendar of a certain month:*

<insert figure here>

(i) What is the relationship between the sum of the nine numbers in the shaded parts and the number situated at the middle of the shaded part?

(ii) Is this relationship applicable to other shaded parts? Can you use an algebraic expression to formulate this relationship?

(iii) Is this relationship applicable to any month of the year? Why?

(iv) What other questions can you raise?

In this example, when students raise their questions, teachers should pay attention to whether the students are enthusiastic or not. Another is to pay attention to the depth and scope of the questions raised by the students. Some students may ask whether there exist relationships in the monthly calendar for other sets of numbers. When evaluating students' problem solving processes, the emphases should be on whether students engage actively in thinking, and attempt to discover patterns and regularities from the calendar; whether students possess the consciousness to verify the patterns and regularities discovered; whether students can clearly and systematically communicate with others and benefit from these cooperative exchanges; whether students consciously reflect their problem solving processes. Teachers should give encouragement and guidance and record down observations when students raise questions and suggest ways of handling the problems.

(4) Evaluate students in multiple ways and from different perspectives.

Students' self-evaluation, peer evaluation, teacher evaluation, parent evaluation and evaluations conducted by others should be combined. Ways of evaluation should be multifarious. Apart from using written examination, oral examination, and task analysis, teachers should consider classroom observation, after-class interview, big project, setting up of portfolio, analysis of small assignment, and activity report as well.

Each form of evaluation has its own characteristics, and it should be chosen coupled with contents of evaluation and learning characteristics of the students. If teachers would like to examine situations of students' fundamental knowledge and basic skills, they can use written examinations. If teachers would like to examine students' cognitive insight and situation of cooperative exchanges, they can use project assignment extended over a long period of time. If teachers would like to examine students' progress over a period of time, they can develop portfolio for each of the students. Irrespective of the method used, the choice should be based on whether students' learning can be stimulated and whether students' development can be enhanced.

During everyday teaching, teachers should emphasize observation of students. The main aspects are: mastery of fundamental knowledge and basic skills; whether students are autonomous during the learning processes; independent thinking and degree of seriousness in learning; problem solving abilities; situations of cooperative exchanges with others.

(5) Evaluation results are qualitatively and quantitatively integrated for presentation.

When displaying evaluation results, teachers should emphasize the function of qualitative evaluation, and adopt methods that are qualitatively and quantitatively integrated.

Teachers can use system of grades or percentiles to report the evaluation results promptly to the students, but ranking of students is not encouraged. Teachers should be alerted that although the "Rank List" may instill some dynamics to a small group of students, it may cause anxiety, fear and hard-blow to a lot of students as well.

Remarks may be adopted in qualitative evaluation. Teachers should use encouraging remarks that can describe the learning situations objectively and systematically, ascertain students' progress and development sufficiently, pay attention to what have been mastered by the students, progress made, abilities and potentials possessed, as well as assist students to clarify directions needed further hard work and efforts. Evaluation results should build up students' confidence, and increase students' interests in mathematics learning so as to promote further development.

Students' characteristics at the third stage of schooling are even more prominent. Evaluation should fully consider these differences, and try hard to enable students to experience the joy of success. In this regard, teachers can design open-ended problems to reflect the different learning characteristics of the students. During evaluation teachers should follow the requirements of *Standards* and the response situations of the students to decide the passing criteria. Students' original thoughts or conclusion should receive teachers' encouragement during evaluation as well.

3. Teaching Materials Editing Recommendations

Teaching materials provides basic clues of what student learning activities should entail. It realizes curriculum objectives, and is important resources for implementing teaching. In principle, editing of teaching materials should be based on Standards.

Teaching materials provided should stem from nature, society, phenomena in science and practical problems. It should reflect a certain degree of mathematical values, and display the interrelationships of different content areas. Arrangement of contents and style of presentation should emphasize the process of the formation and application of knowledge. Teachers should guide students to start from the knowledge and experiences already acquired, engage in autonomous exploration and cooperative exchanges, and learn step by step how to learn during the learning processes. Humanism for students' well-being should be inculcated as well. Furthermore, editing of teaching materials should be facilitative of mobilizing teachers' autonomy and self-initiation, encouraging teachers to undertake creative teaching. Important mathematical concepts and thoughts should display the principle of an ascending spiral, step by step enhancing students' understanding of knowledge and methods of mathematics.

Considering the diversity of individual differences, teaching materials should exemplify its styles and characteristics after fulfilling all the basic prerequisites. It should possess a certain degree of flexibility as well. When editing teaching materials, teachers should consider sufficiently how it can tie in with the exploitation and deployment of other curriculum resources.

(1) Choose learning materials from nature, society and other subjects.

Scope of students' activities at this stage of schooling, compared with those at the first and second stages, has expanded considerably. Problems that are of interest to the students extend to the many aspects of the objective world. Students are increasingly paying attention to problems stemmed from nature, society, as well as diverse phenomena from other subjects. They express greater interest to those problems with challenging contents. Hence, contents selected as teaching materials should as far as possible stem from phenomena and problems of nature, society and the sciences, and reflect a certain degree of mathematical values.

For example, concerning contents of Statistics and Probability, editing of teaching materials should provide real examples of modern society living. Teachers can select problems from newspapers and magazines, television and broadcast, as well as computer networks. Also, teachers can select problems that are of interest to the students, particularly those related to everyday living, e.g. survey of traffic conditions of roads in the schools' environment (traffic load, number of vehicles, traffic jam, traffic accidents, etc.), elimination of the highest and lowest score in singing contest, and sale of lotteries. These contents would guide students to pay more attention to exploring practical problems, understand the practical meaning of concepts, and to know the realistic world better in the light of learning mathematics.

Example 1 *Conduct a survey of the flow of pedestrians at a passageway near your school. Can you suggest opinions for improvement regarding the safety and convenience of this passageway? Design a survey plan and divide the students into groups to conduct the survey. Exchange views on each group's survey reports.*

(2) Allow students ample room for exploration and cooperative exchange of ideas.

At this stage of schooling, students' independent thinking and aspiration and abilities of exploration have been considerably increased. Students are able to form one's viewpoints during the processes of exploration, and to perfect one's ideas through listening to others' opinions. Editing of teaching materials should pay attention to make visible these characteristics. Teachers should provide ample room for exploration and exchange of ideas so that students can move a step further to carry out activities such as observation, experimentation, guessing, inferences, exchange of ideas and reflection.

Teaching materials can comprise of problems with challenging contexts and

situations so as to stimulate students to engage in thinking. Teachers can propose a series of problems to guide students to carry out autonomous exploration, encourage students to engage in exchange of ideas by remarks such as, ”Communicate what you think with your peers.”, provide students with open-ended problems (i.e. problems should possess some degree of openness regarding problem conditions, conclusion, problem solving strategies, etc.), so as to enable students to understand knowledge acquired during the exploration processes. Teachers should provide appropriate problem solving activities that may be used by students to engage in cooperative exchanges (e.g., exploratory projects, social surveys, etc.) so as to enable students to involve in knowing problems from different angles, representing problems in a variety of formats, thinking problems using multiple strategies, attempting to explain in what ways the different answers are reasonable, and subsequently developing consciousness in creativity and practical abilities. Teachers can also raise questions to guide students to monitor and reflect their own learning processes.

Example 2 *Explore patterns and regularities.*

(i) Compute and observe the following sets of expressions:

<insert equations here>

(ii) Given 25 x 25 = 625, then 24 x 26 = _____.

(iii) Can you give similar examples?

(iv) From above, what patterns and regularities have you discovered? Can you use words to state these patterns and regularities? Can you use algebraic expressions to express these patterns and regularities?

Through a series of questions, students involve in important processes of mathematical exploration: induction in accordance with special examples, establish a hypothesis, use mathematical symbols to represent and prove the hypothesis.

Cultivation of students’ space conceptions, development of inferential abilities, appreciation of the beauty of figures are all contingent upon the foundation that students involve in observation, manipulation, guessing, inferences and exchange of ideas, and teaching materials should sufficiently make visible these processes. For example, when arranging axisymmetric contents, logos, maple leaves, snowflakes and other patterns and figures can be made visible in the teaching materials for students’ observation. Other arrangements may include: explore some figures to find out axisymmetric relationships implicit in them; provide activities that afford to use axisymmetry to design patterns and figures; through reading provided materials, introduce related scientific principles (e.g. symmetry of planes and ships enable them

to navigate in a balanced manner, whereas symmetry in architecture is mostly due to concepts of beauty, but may also due to usage convenience and balance of forces exerted onto them); and to use axisymmetry to solve some interesting problems.

Example 3 *The license-plate of a car is reflected on the water surface. Can you ascertain the license number based on its reflection?*

Regarding proving of the basic properties of basic figures, teachers should design materials to enable students to know the necessity of proofs, explore the thinking paths of proofs, experience that each step of the proving processes requires evidences.

(3) Making visible processes of formation and application of mathematical knowledge.

Teaching materials at this stage of schooling should make visible how to abstract mathematical problems from concrete problem contexts, use a variety of mathematical languages to express problems, establish mathematical expressions, acquire reasonable answers, understand and master meaningful learning processes of corresponding mathematical knowledge and skills. Display of contents of learning materials should make visible as far as possible the model of "Problem context – Development of mathematical models – Explanation, application and extension". Based on the main themes of study, teachers should select contents that are realistic, challenging to the students, representative of the meaning of mathematics, and beneficial to the development of general abilities of the students. This would enable students to establish and solve theme-based mathematical models during autonomous exploration and cooperative exchanges, judge whether the solutions are reasonable or not and apply the themes learned to other situations, and proceed further to acquire corresponding mathematical knowledge, method and skills, as well as to assist students who need additional means to familiarize with the themes further. Through the above-mentioned processes, students shall progressively master basic mathematical knowledge and methods, form good habits of mathematical thinking and consciousness of application, enhance one's abilities of problem solving, feel the joy of creativity in mathematics, increase confidence in learning mathematics better, and acquire relatively comprehensive experience and understanding of mathematics.

For example, in Numbers and Algebra, students shall learn equations, inequalities, functions, etc. These mathematical models are important and can be used to examine quantitative relationships and patterns of variation in the realistic world. They can

help people describe and grasp the realistic world more accurately and clearly from the angle of quantitative relationships. Hence, editing of such contents should make visible the processes of building of a mathematical model. An example is the "ladder problem" commonly encountered in everyday living. Teachers can guide students to engage in discussions, so as to acquire the model of "quadratic equation in one unknown" and its approximate solutions.

Example 4 *A ladder 10 m long is slanted against a wall. The vertical distance of the upper end of the ladder to the ground is 8 m. If the upper end of the ladder slips down 1 m, then*

(i) Make a guess. Does the bottom also move 1 m ?

(ii) Formulate the equation that satisfies the movement made by the bottom end of the ladder.

(iii) Can you try to obtain the approximate solutions of this equation? Is the movement of the bottom end more than 1, or less than 1? Exchange your ideas with your classmates.

Teaching materials can provide more quantitative relationships of concrete problems, so as to enable students to formulate the quadratic equations in one unknown, and to involve in processes of finding the roots that satisfy the equations, and proceed further to form aspiration to obtain the general solutions of equations. After learning the general solution of quadratic equation in one unknown, apart from reviewing the above-mentioned "ladder problem", teachers can also assign the following open-ended problem to the students:

Example 5 *Construct a garden on a piece of land of rectangular shape with length 50 m and breadth 30 m. It is required that the plants cultivated should be half of the total area of the land. Please display your design.*

This is a problem that invites strong participation. Every student can utilize fully their imagination and design different figures and patterns in accordance with one's design rules. Students should try their best to quantify their designs, and through this process realize the function of quadratic equation in one unknown in handling quantitative relationships, know that computation of quadratic equation in one known is not a mechanical process and that the result obtained should be meaningful with regard to the concrete situation, as well as choose and verify solutions appropriately.

(4) Presentation formats should be rich and multifarious.

Students at this stage of schooling need to borrow letters, figures, texts and other related materials to engage in mathematical activities. Presentation formats of teaching materials should be multifarious. Teachers can combine usage of photographs of real objects, sketches, texts, forms, figures, letters and other forms to enable students to engage actively and autonomously in the whole learning process, as well as deepen students' understanding of the meaning of mathematical knowledge learned. As an example, teachers can use layout and photographs of real objects to display the problem context and situation, arrange some interesting materials and a variety of activities (e.g. manipulation, experimentation, survey) so that mathematics learning is intimately related to the realistic world. The learning materials should comprise of rich mathematical ideas so as to enable students to discover the implicit meaning during the learning processes. For example, if it is desired to deepen students' understanding of *power*, teaching materials can include a real example of cell fission in biology. Teachers can use a cell fission diagram to display the processes involved: Each cell each time divides into two, two cells then divide into four, in this way a sequence 1, 2, 4, 8, … is generated. This would increase students' interests in mathematics learning, enable students to familiarize with the function of mathematics on other subjects, and deepen students' understanding of the knowledge acquired.

Figures that are rich and multifarious are important Space and Figures learning materials. Teaching materials should combine pictures with heuristic problems, pictures with necessary texts, computation and inferences, as well as numbers and shapes, so as to demonstrate the intuitive function of figures. Teaching materials should be illustrated using both texts and pictures and they should be inspiring as well.

Functions are important contents of Numbers and Algebra. Functions may be expressed in a number of formats (e.g. tables, graphs, expressions and languages). Teaching materials should provide a number of examples to express functions in multiple formats so as to learn the meaning of linear, quadratic and inverse proportion functions from multiple perspectives, as well as to deepen students' understanding of ideas of functions.

(5) Design of content should possess some degree of flexibility.

On one hand, teaching materials should follow the requirements stipulated in *Standards* to guarantee that students acquire fundamental knowledge and basic skills

as well as certain degree of training. On the other hand, considering the individual differences of students' development and imbalance in regional development, teaching materials after guaranteeing the basic prerequisites should exhibit some degree of flexibility to satisfy different requirements of students so that all students are able to achieve corresponding developments and teachers can demonstrate their creativity. Concrete design formats are multifarious: Teachers can raise problems at different levels, or design open-ended problems having the same problem contexts and situations, so as to enable different students to have different developments; Teachers can provide a certain quantity of reading materials for students to select for study; Selection of home assignment and their arrangement should emphasize the underpinning hierarchy, such as setting up of consolidation exercise, extension exercise and exploratory problem; When designing theme-based studies, the theme selected should allow all students to participate so that after satisfying the prerequisites that all students secure developments necessary to them different students can receive different forms of experiences; Teaching materials can include optional contents in order to extend the knowledge base, but attention should be paid on the mathematical thinking methods, students' development, facilitation of students' knowledge of the nature and function of mathematics, enhancement of students' interests in mathematics learning, and not quest for difficulty, techniques and speed.

Through design of concrete themes and reading materials, teaching materials can introduce students who are competent enough to deploy learning technologies such as computers and function calculators, so as to allow students to invest more energy in meaningful exploratory activities. Activities may include: Exploration of some quantitative relationships, properties of functions, and characteristics of figures; Transformation of figures using axisymmetry, translation and rotation; Construction of figures using coordinates, as well as projects on pattern design; Display of rich, multifarious geometrical figures so as to allow students to explore the pattern of variations; Data collection, processing and modeling of probability experiments.

(6) Important mathematical concepts and ideas should exhibit the principle of an ascending spiral.

Stipulated in *Standards* are terminal objectives needed to be achieved by all students at the end of the third stage of schooling. When editing teaching materials, teachers should not only base on students' age characteristics, patterns and regularities of cognition, particulars of instructional knowledge, but also need to make sure that

learning of important mathematical concepts and ideas follow the principle of an ascending spiral progressing upward step by step, but repetition of contents is to be avoided.

For example, ideas of functions have been diffused in teaching materials of the previous two stages of schooling, and it is time to show students concepts of functions at this stage of schooling. Students' understanding of function concepts follows a process of progressive development, and hence teaching materials with regard to content arrangement should make visible the principle of an ascending spiral, a process that is ever evolving and deepening. It is not desirable to let students learn all the contents at one time, thereby facilitating students to deepen their understanding of the idea of functions. At each stage of schooling and in each domain of study, teachers should design contents on inferences and proofs, with levels progressing from substantiation, explication and upward to the level of proofs.

(7) Pay attention to relationships and synthesis of various parts of contents.

Teaching materials should pay attention to the connection amongst the various parts of mathematical contents, including connection amongst contents of the same domain, as well as selection of some concrete contents to make visible the qualitative relationships between Numbers and Algebra, Space and Figures, and Statistics and Probability, and exhibit the wholeness of mathematics. Teaching materials should pay attention to the connection between mathematics and the realistic world, as well as between mathematics and other subjects.

For example, concerning contents of Statistics and Probability, teaching materials should emphasize diffusion of connection between statistics and probability, through frequency one can estimate the probability of an event happened, and through samples and related data one can estimate possibilities of the population. Teaching materials should link statistics and probability with contents in other domains, provide problem contexts from the statistical and probability point of view, naturally use knowledge and methods in other domains to solve statistics and probability problems, as well as provide opportunities for developing students to deploy knowledge to solve problems synthetically.

Concerning contents of Numbers and Algebra, teaching materials should pay attention to the geometrical background of related contents, deploy intuition in geometry to help students understand and solve related mathematical problems. As an

example, teachers can base on an array of points regularly patterned on a plane to deduce the sum of terms of a corresponding sequence of whole numbers (e.g. 1 + 3 + 5 + 7 + …can be represented as a square array of points). Teachers can also use diagrams to help students understand formulae of identities of perfect squares and square of differences, as well as to use images of functions to understand the tendency of variation of functions.

Theme-based studies at this stage of schooling shall exhibit more the research and exploratory nature of activities, and link more mathematics and social everyday living with knowledge of other subjects, so that students can proceed to realize different kinds of mathematical knowledge and the connection of mathematics with the outside world, begin to learn methods of researching problems and enhance their practical abilities and consciousness of creativity. Contents of theme-based studies need not be completed within the class lesson. Teaching materials can provide some activities to encourage students to use time outside the school hours to collect information and to engage in activities such as surveys.

(8) Introduce students to mathematical background knowledge.

During the process of learning the mathematical contents, teaching materials should include some supplementary materials, such as history, problems for further research, introduction of mathematicians, background information, etc. Teachers can also introduce the vast application of mathematics to modern everyday living (e.g. architecture, computer science, remote sensing, CT technology, weather forecast). Doing these not only enable students to familiarize with mathematics developmental processes, stimulate students' interests in mathematics learning, but also enable students to realize the function and value of mathematics to the history of development of mankind. Supplementary materials can appear in the form of reading materials.

Concerning the part on Numbers and Algebra, teachers can insert history of algebra and its language, display pictures of important figures contributory to the genesis and development of algebra, as well as the related historical relics. Teachers can also introduce history of positive/negative integers and irrational numbers, genesis of some important symbols and their evolutions, materials related to equations and their solutions (e.g. Jiu Zhang Suan Shu, Qin Jiu Shao Method), as well as genesis, development and evolution of concept of functions.

Concerning the part on Space and Figures, teachers can introduce mathematical background knowledge to students through the following lines of thoughts: Introduce Euclid's Elements so as to enable students to begin to feel the value of deductive geometric system on mathematical development and human civilization; Introduce Gou Gu Theorem and some of its famous proofs (e.g. Euclid's Proof, Zhao Shuang's Proof), as well as some famous associated problems, so as to enable students to feel the flexibility, elegance and delicacy of mathematics, to feel the rich cultural inner attributes of Gou Gu Theorem; Introduce related contents of machine proofs and the outstanding contributions of Chinese mathematicians; Briefly introduce the history of π so that students realize the historical inner attributes and modern value of its methods, numerical values, formulae and properties associated with π (precise computation of π has been acknowledged to be amongst the best methods to evaluate the efficiency of a computer); Coupled with related mathematical contents to introduce ancient Greek's and Ancient Chinese Exhaustion Method of Circles so as to enable students to have an initial feeling of approximation in mathematics and the inner attributes of mathematics in different cultural background; As topic for appreciation, teachers can introduce topics such as "compass and straight rule construction" and the three great problems of geometry, golden section, Konigsberg Bridge Problem, etc. so as to enable students to feel mathematical thinking methods, as well as to realize the aesthetic value of mathematical propositions and methods.

Concerning the part on Statistics and Probability, teachers can introduce topics on the genesis of probability theory, tossing coin experiment, history of Buffon Needle Problem and geometrical probability, application of statistics and probability on cryptography. This will enable the students to familiarize with processes of how mankind grasps random phenomena, as well as to stimulate students to proceed to further their studies and developments.

Exploitation and Deployment of Curriculum Resources

Mathematics curricular resources refer to all teaching materials exploited in accordance with the *Standards*. These resources comprise of all sorts of teaching resources, tools and places that may be deployed in the mathematics curriculum, including primarily all sorts of practical activity materials, video tapes, multimedia CD, computer software and network, libraries, as well as newspapers and magazines, television broadcast, youth palaces and museums. Editors of teaching materials, school administrators, teachers and related personnel should consciously and

purposefully exploit and deploy all sorts of resources in accordance with the facilitating conditions. Below are some recommendations on the exploitation and deployment of related resources:

(1) **Practical activities materials**

In order to enable students to participate sufficiently in activities in the classrooms and understand better important mathematical concepts and methods in activities, schools should deploy and exploit real objects/materials and equipments (e.g. calculators, pin-board, three-dimensional models, campus facilities) for students to engage in practical activities.

(2) **Musical information and information technology**

Schools can exploit resources such as videotapes and CDs. For example, schools can videotape some backdrops to serve as corresponding problem context of contents learned by the students; record application of mathematics on science and technology; record the biography and stories of mathematicians, as well as record case studies of teaching for teachers' discussion. It is noteworthy that contents of the videotapes and CDs should not simply repeat teachers' classroom explanations.

All those schools that can afford or create conditions should deploy calculators, multimedia, Internet and other information technology as mathematics curricular resources, and actively organize teachers to exploit courseware. Schools should fully exploit the strengths of information technology, so as to provide students rich and multifarious educational environments and powerful tools for learning; provide opportunities for students to explore complex problems and understand mathematical ideas from multiple viewpoints so as to enrich students' perspectives in mathematical exploration; provide opportunities for some students who need individualized learning so as to facilitate teachers to render assistance to students with special needs; provide mathematical guidance and intellectual resources to students in the remote areas so as to attract and help students engage in mathematics learning effectively. As multimedia technology enables teachers to provide and display all sorts of needed information, including texts, sounds and images, and to retrieve and broadcast them at any time, schools should create and model all sorts of contexts and situations and have them matched correspondingly with the teaching contents. Application of Internet in mathematical activities is increasingly widespread nowadays. The function and value of Internet in retrieving information and exchange of ideas are increasingly prominent,

and so it will become an indispensable curricular resource. Also, using Internet teachers can locate a lot of websites related to mathematics education local and abroad. Through accessing these websites, teachers can collect some learning materials, download contents that have direct bearings with the curriculum and apply them in their teaching. When conditions permit, teachers can recommend good websites for students to select browsing, encourage and guide students to retrieve information via the Internet, and engage in exchange of ideas.

It is noteworthy that we do not recommend the use of computers to model experiments to replace practical activities that can be undertaken by the students (e.g. the use of computers to model "pouring sand experiment", so as to enable students to understand the volumes between a cylinder and a cone of same base and same height); we do not recommend the use of computers to display images to replace students' intuitive imagination, so as to replace them to engage in exploration of patterns and regularities in mathematics. Also, schools need to increase their links with each other, share resources, avoid duplication of low-level courseware, actively bring in advanced educational software from abroad, and to improve them in accordance with the students' characteristics.

(3) **Curricular resources of other subjects**

Schools should link mathematics intimately with other subjects, extract useful resources from other subjects (e.g. natural phenomena, social phenomena, human civilization) to create contexts and situations, and deploy mathematics to solve problems in other subject areas. For example, the cell fission processes can be displayed (1 cell is divided into 2, and then into 4, 8, 16, …) so as to allow students to understand the concept of *power*. Teachers can allow students to collect and analyze data to examine the factors influencing the period of a simple pendulum. Furthermore, teachers can allow students to study environmental problems from the mathematical perspective.

(4) **Curricular activities groups**

Schools can organize mathematical extracurricular activities to stimulate students' interests in learning, guide students to deepen their studies, develop students' practical abilities, and develop students' spirits of individuality and creativity. During extracurricular activities, teachers can supply students reading materials, and the contents can cover application of mathematics in everyday living,

fun of mathematics, history of mathematics, stories of mathematicians, and extended knowledge so as to broaden students' domain of study and stimulate students' interests in mathematics learning.

It is noteworthy that students should be given the opportunity to participate in extracurricular activities voluntarily so as to avoid students to become tools in the competitions. Editing of reading materials should conform to students' cognitive characteristics and everyday living experiences, and these are optional studies for the students.

(5) **Library resources**

School libraries should satisfy the basic reading needs of the students. This would expand students' scope of knowledge, and serve important function in stimulating interests in mathematics learning. At present, apart from the fact that the books in the school libraries are not adequate, one main problem is that the proportion of books used for mathematics counseling and tuition is very high, and this situation should be changed. Schools should make full use of the libraries outside their schools to broaden the horizon of the students and enrich teachers' teaching resources.

(6) **Media such as newspapers and magazines, as well as television broadcast**

Media such as newspapers, magazines, television broadcast provide a lot of meaningful problems. Teachers and editors of teaching materials should extract fully materials appropriate for students' learning. Teachers can also introduce students television programs related to mathematics learning, and organize students to exchange ideas with others on some of the contents presented to them.

(7) **Activity centers such as community centers, youth palaces and museums**

Schools should make full use of activities centers such as community centers, youth palaces and museums. On one end teachers can find appropriate learning materials from these places, such as natural phenomena and social problems that are of interest to the students. On the other hand, teachers can organize students to engage in activities, such as visiting the legacy of human civilization in the museums. This would stimulate students' interests in learning, and develop students' practical abilities.

(8) **Intellectual resources**

Teachers should make full use of school's and society's intellectual resources, such as inviting specialists to give lectures to the students, consulting specialists on some problems, and locating information related to international mathematics education.

Concerning effective exploitation of mathematics curricular resources, there is a need to establish the evaluation standards of mathematics curricular resources, including encouraging students to participate in society, regulating reporting procedures, setting the basic requirements of curricular resources (e.g. heuristic, creative, practical), setting reasonable costs and prices, and encouraging competitions of all those concerned in an orderly manner.