The subject of Mathematics from an international perspective

Mathematics A and B in HTX and STX

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1 Summary

This report presents the results of the evaluation from an international perspective of Mathematics at levels A and B in the upper secondary education programmes HTX (higher technical examination) and STX (the general upper secondary programme) in Denmark. The evaluation was commissioned by the Danish Ministry of Education and can be regarded as a supplement to the national evaluation of Mathematics carried out by EVA at the request of the Danish Ministry of Education. Physics has also been evaluated from an international perspective, and a parallel report presents the results of this evaluation.

Purpose and organisation of the evaluation
In August 2005, a sweeping reform of upper secondary education took effect in Denmark. The reform entailed structural changes to upper secondary education programmes as well as alterations to the individual subjects taught in the programmes. On the basis of a comparison of the Mathematics curricula and examination sets before and after the reform, this evaluation aims to assess the relevance of the development of Mathematics from an international perspective.

An international expert panel has discussed and assessed the development of Mathematics in relation to international developments and trends within Mathematics in upper secondary education and requirements regarding Mathematics in further education. The composition of the expert panel covers different areas of expertise within the field of Mathematics and Mathematics education. The expert panel is responsible for the conclusions in the evaluation, while EVA has been responsible for the organisation of the evaluation and writing of the report.

Comparison with international trends and developments in society
In general, the expert panel finds that the development of Mathematics corresponds well to trends in other countries, as well as trends in the field of Mathematics in research, further education and business. According to the expert panel, there is a continuing need and effort to define what constitutes Mathematics as a subject, as it is becoming still more relevant in society and in an increasing number of professions. With regard to the general educational aspect of Mathematics, the expert panel finds it important to stress the contribution of Mathematics to creating active citizens in a democratic society. Moreover, the expert panel stresses the importance of making Mathematics relevant, accessible and useful.

Generally, the expert panel assesses that the intentions behind the development of Mathematics are innovative and compatible with changes in society. The expert panel observes and appreciates that the applied dimension of mathematics has been strengthened in the new curricula, but at the same time finds it important to note the need for continued focus on the purely mathematical and primarily theoretical dimension of Mathematics. From the panel's point of view, it is important that both dimensions of Mathematics are emphasised. It is not a question of choosing one over the other. Rather, there is a need to establish bridges and connections between the two perspectives. The expert panel is favourably disposed to the new approaches to Mathematics, including the interdisciplinary approach and the experimental approach. Both approaches correspond well to the reality of using Mathematics subsequent to upper secondary education. However, the expert panel emphasises that international experience shows that there is often a need for support and inspiration to teachers in order to exploit the potential of the new approaches. Although the implementation of the reform has not been the focus of the international evaluation, the expert panel found it clear from their investigations that this has entailed, and still entails, many significant challenges for the teachers. Thus, the expert panel finds it important to consider how teachers can be prepared for the task.

Focus on competences is in line with international trends
Overall, the new curricula place more emphasis on describing mathematical aims in terms of the competences that pupils are to achieve than did the former. At the same time, the level of detail in defining the content has been reduced. The expert panel finds that a focus on competence, rather than a content based approach, is in line with international trends and strengthens the focus on pupils' mathematical understanding. According to the expert panel, the Danish system has a strength here, which is reinforced by recent developments in the subject. The expert panel approves overall of the balance between the different competences in the curricula, including modelling, problem solving and communication, as well as a mathematical way of thinking, reasoning, proofs and the use of symbolic language. The panel adds that it is important to ensure that pupils gain relevant skills within the areas of exact calculation and algebra.

Appropriate adaptation to technological developments
In both programmes and at both levels, the use of IT tools/CAS to solve mathematical problems, calculate and substantiate, is required in the new curricula. Internationally, a discussion is taking place about how much technology and IT – in particular CAS – should be used in teaching Mathematics. The expert panel assesses that the Danish Mathematics curricula have been suitably and necessarily reviewed and adapted by introducing CAS and IT programmes in the teaching. The panel acknowledges that the implementation of CAS in the teaching presents initial challenges; that it might require extra resources for the teachers; and that it is important to consider how the teachers can become confident that the pupils' mathematical skills will not be reduced. The panel stresses that the use of technology does not necessarily make classical aspects of Mathematics less important. New technology may be used to ease computations, but must be expected to increase, rather than decrease, the need for mathematical reasoning.

Flexibility versus uniformity of the subject content
In contrast to the former curricula, where the main part of the content was compulsory, the content in the new curricula is divided into core material and extension material. This allows among other things interdisciplinary projects that are suitable in a given specialised study package. The expert panel agrees with the introduction of extension material, as this might increase motivation and creativity in the teaching, as well as contribute to an increased focus on understanding Mathematics in depth. However, the introduction of extension material involves less uniformity, which might constitute a challenge for the universities. On the other hand, the universities are expected to benefit in terms of other competences being improved. The panel finds it important to maintain a substantial amount of core material in the curricula, and the expert panel questions whether the right balance between core and extension material has been achieved and also why the amount of extension material differs between the two programmes.

Satisfactory level in the test sets
The expert panel assesses that the examination types in both programmes have strengths. However, the panel finds that HTX has adopted a more innovative approach by using preparation material in the written examination. The panel finds, however, that the preparation material over time could be used more thoroughly in the written examination. The level in the test sets for both programmes seems, from the panel's point of view, to be more than adequate, and compares well internationally. The expert panel approves the combination of an open and a closed book examination in STX, although the panel finds that the level of the closed book examination has declined with regard to the questions focusing on pupil's conceptual understanding. Considering the comprehensive developments of the new curricula, the expert panel would have expected the test sets to have also developed further in both programmes, but especially in the STX programme. Although the expert panel acknowledges a cautious approach to changing the examinations, the panel emphasises that change is necessary in order to move the teaching forward and benefit from the intentions of the curricula.

While acknowledging the different profiles of the HTX and STX programmes, the expert panel generally wonders why there are such big differences between the subject of Mathematics – and not least the examinations – in the two programmes, and considers it potentially beneficial for the two programmes to allow themselves to be mutually inspired by their respective strengths.

2 Introduction

This report presents the results of the evaluation of Mathematics in the upper secondary education programmes HTX and STX from an international perspective. It discusses and analyses Mathematics at levels A and B with regard to the development of the subject, and can be regarded as a supplement to the national evaluation of Mathematics. This international evaluation was commissioned by the Danish Ministry of Education and was conducted by the Danish Evaluation Institute (EVA) in cooperation with an international panel of experts within the field of Mathematics and Mathematics education.

2.1 Background to the evaluation

In August 2005, a sweeping reform of the Upper Secondary Education took effect. The reform intends to strengthen and renew the quality of the education programmes according to the needs and requirements entailed by societal changes. The reform has entailed major structural changes to the upper secondary educational programmes as well as alterations to the individual subjects taught in the programmes.

At the request of the Danish Ministry of Education's Department of General Upper Secondary Education, the Danish Evaluation Institute (EVA) has during 2008 carried out a number of evaluations of subjects taught in upper secondary education programmes, including evaluations of Physics and Mathematics. These evaluations have been completed according to a specified procedure adopted by EVA in all subject evaluations conducted in connection with the Danish reform of upper secondary education. The procedure requires that the subjects are evaluated according to existing guidelines, i.e. ministerial orders, curricula and guidelines issued in connection with the reform. The subject evaluations focus partly on the teachers' experience of the new guidelines, and partly on the outcome of the teaching. Thus, they illustrate the extent to which the aims in the curricula are achieved and how the pupils benefit from the organisation of the teaching.

In addition to the national evaluations of all subjects in the upper secondary programmes, the Department of General Upper Secondary Education has commissioned EVA to carry out supplementary evaluations of Mathematics at levels A and B, and Physics at levels A and B in HTX (higher technical examination programme) and STX (the Danish *Gymnasium* – general upper secondary programme) from an international perspective. This report presents the evaluation of the subject of Mathematics from an international perspective. There is a parallel report presenting the international evaluation of Physics.

2.2 Purpose and setting of the evaluation

The purpose of this evaluation is to supplement the national subject evaluations with an international perspective on the development within the subject of Mathematics. The relevance of the development in the subject is assessed in terms of general education and the pupils' preparedness for higher education by an independent panel of international experts. The evaluation deals with Mathematics at levels A and B in HTX and STX, thus narrowing the focus from other programmes and levels. The programmes and levels are described in section 3.1.

On the basis of a comparison of the curricula and written test sets before and after the reform, the evaluation assesses the relevance of the development in Mathematics from an international perspective. The analysis of the development in the subject covers the following issues:

- Standing of Mathematics in the programmes;
- Subject aims;
- Subject content;
- Principles of pedagogic organisation, including the requirements of interdisciplinary interaction;
- The form and content of the examinations.

The setting of the evaluation involves certain possibilities and limitations that are decisive for the results presented in this report. In this connection, it is important to stress the main premises for the evaluation-process.

This evaluation focuses on the overall framework for and new approaches to the subject of Mathematics at a central level, rather than the actual local implementation. The core issue is the official framework for the subject of Mathematics as set up by the Ministry of Education, i.e. the formal curricula and selected national examination sets. The local implementation of the changes is undoubtedly decisive for the development of the subject of Mathematics, and thus it is difficult to totally isolate the discussions and assessments in the evaluation from the implications connected with the implementation. However, it is important to note that this evaluation does not include documentation of local implementation.

The reform of 2005 is still in the process of being implemented in the general upper secondary education programmes, and only one year group of pupils have completed the programmes according to the reform. Thus, this international subject evaluation – as well as the national one – has been carried out while the reform is still relatively young, and it is of course too early to assess how the reform has influenced pupils' success in higher education, which should be kept in mind by readers of the report. Thus, the discussions in this report concentrate on the potentials of the new approaches to the subject.

In this evaluation the development of Mathematics as a subject is evaluated *from an international perspective*. Here, this implies that an expert panel consisting of foreign and internationally oriented experts has discussed and assessed developments in the subject as identified by the sources of documentation in relation to international developments and trends within the field. However, an exhaustive mapping of international tendencies has not been carried out. The expert panel's composition covers different areas of expertise within the field of Mathematics and Mathematics education. Thus, the analyses and reflections in this report are based on the professional experience and knowledge of the individual members of the expert panel.

2.3 Project organisation

The expert panel comprises three international experts, and the panel is composed to ensure that the following areas of expertise and knowledge are covered:

- Academic expertise;
- Knowledge about Mathematics at the "user" level, for instance the higher education system;
- Knowledge about the standing of Mathematics in the international educational arena;
- Knowledge about international trends within Mathematics in higher education.

The members of the expert panel are:
- Søren Eilers, Professor and Associate Chair for Education, Department of Mathematical Sciences, University of Copenhagen;
- Ian Forbes, Lecturer in Mathematics Education, Dept of Curriculum Research and Development, Moray House School of Education, University of Edinburgh;
- Anette Jahnke, Licentiate and Project Manager, National Centre for Mathematics Education (NCM), University of Gothenburg.

More detailed information on the expert panel is provided in appendix A.

The expert panel is responsible for the conclusions of the evaluation. The task of the expert panel has been to carry out the subject-related analysis and to evaluate the subject of Mathematics from an international perspective, while EVA has been responsible for the organisation of the evaluation, the methodological aspects and the writing of the report. The expert panel has attended two one-day meetings at EVA during January 2009, which included discussions of the written documentation and interviews with key persons.

The project team at EVA comprises: Evaluation Officer Katrine Strange and Evaluation Assistant Louise Bunnage. In addition, Evaluation Officer Signe Mette Jensen and Evaluation Officer Bo Söderberg have participated in selected parts of the evaluation. Evaluation Officer Rikke Sørup is the project manager of the Subject Evaluations 2008.

2.4 Sources of documentation

The main sources of documentation in this evaluation include former and present curricula and test sets that have all been translated into English. The translation of the documents is provided by the Danish Ministry of Education. In addition, the evaluation includes interviews with key persons within the field and other sources of documentation. These sources are described below.

Former and present curricula

The main reference for the analysis of the development in the subject of Mathematics in this evaluation has been the Mathematics curricula imposed by the Danish Ministry of Education. The curricula are formal documents which all schools and teachers are committed to comply with. In this way, the curricula form the central framework of the subject. By scrutinising and comparing the former and present curricula, the aim has been to identify developments of the guidelines and framework for the subject and to assess the development from an international perspective.

The curricula are relatively brief documents, and the Ministry of Education has formulated more detailed teaching guidelines that go into greater depth concerning the different aspects of the curricula, explaining and giving examples of how to implement the elements. These guidelines are, however, not compulsory. The guidelines in their entirety do not form part of the documentation material for this evaluation. However, the section in the guidelines regarding assessment criteria has been used as background information (See descriptions of other sources of documentation below).

The following curricula have been used in the evaluation:

- Present curricula:
  - level A and B in HTX (executive order 743 of 30.06.2008, appendix 20 and 21)
  - level A and B in STX (executive order 741 of 30.06.2008, appendix 35 and 36)
- Former curricula:
  - levels A and B in HTX (executive order 524 of 15.06.2000, appendix 8)
  - levels A and B in STX (executive order 820 of 04.11.1999, appendix 23)
  - levels A and B in STX (executive order 319 of 19.05.1993, appendix 22)

Test sets used in written examinations

Another main source of documentation which reflects the development in the subject of Mathematics is the test sets used in the written examinations. The test sets show among other things, which competences and skills are tested when the pupils complete the course. The test sets also to some extent reflect the competences and skills that are in focus in the teaching, insofar as the teaching is affected by and dependent on the examination. There is a tradition of using previous years' test sets[1] in the teaching and as assignments to prepare the pupils for the kind of questions they can expect in the examination.

The test sets provide an opportunity to analyse and understand the development in the subject with regard to the level of difficulty, the content and the types of competences and skills that are in focus in the subject. Combined with the exam results, the test sets to some extent indicate

[1] In connection with the reform, guiding test sets have been prepared especially to comply with this tradition.

which competences and skills pupils are expected to possess when leaving the education programme.

The following post-reform test sets are included:
- Written exam at A-level in HTX, summer of 2008[2];
- Written exam at A-level in STX, summer of 2008;
- Written exam at B-level in STX, summer of 2007.

The following test sets prior to the reform are included:
- Written Exam at A-level in HTX, summer of 2007[3];
- Written Exam at A-level in STX, summer of 2000 and 1991[4];
- Written Exam at B-level in STX, summer of 1999 and 1990.

Interviews with key persons
With the aim of complementing the official written documents, the expert panel has carried out a number of interviews with key persons having knowledge of the intentions and ambitions behind the reform of the subject as well as the practical importance of the changes. Thus, the interviews served to clarify uncertainties when reading the documents and have, furthermore, contributed towards a deeper understanding of the development in the subject.

The key persons interviewed by the panel were:
- *The subject advisors for Mathematics*. Each programme has a subject advisor who is employed – often part time – by the Ministry of Education. Subject advisors have overall responsibility for the subject and take part among other things in preparing and updating curricula. Besides their work as subject advisors, they are often part time teachers. They play a key role as the link between the ministry on one side, which is in charge of the official guidelines for the subject, and the schools and teachers on the other, which constitute the implementation level.
- *Representatives of the exam commissions*. For both HTX and STX, there are central exam commissions that compose the national written test sets. The exam commissions are appointed by the Ministry of Education and comprise of a number of highly experienced Mathematics teachers, and sometimes a representative from a university.
- *Chair of the Mathematics association in STX.* The Mathematics teachers in the STX programme are organised in a subject association that collaborates with the Ministry of Education. The association serves as a forum for the teachers to share material, experiences and knowledge. There is at present no association representing the HTX Mathematics teachers.

More detailed information on the persons interviewed is provided in appendix B.

Other sources of documentation
The following sources of documentation have served as background information for the expert panel's assessments:
- Statistical data regarding grades and the standing of Mathematics in the upper secondary programmes, including data on the proportions of pupils at A and B levels;
- Assessment criteria, including examples of when to give the grades 02, 7 and 12 (extracts from the official guidelines);
- Survey data from EVA's national evaluations of Mathematics concerning the knowledge, skills and competences acquired by the students. A questionnaire survey among Danish Mathematics teachers and examiners, carried out in 2008.

[2] *After the reform there is no longer a traditional written examination at level B in HTX. Instead a project examination has been introduced and replaces the former written and oral examination. The project examination consists of an oral examination based on a project report, which the pupils prepare on the basis of a centrally determined topic. Thus, there is no test set available at level B in the HTX programme.*

[3] *The last pupils that completed the program "before the reform" commenced in 2004 and completed the programme in summer of 2007.*

[4] *The selected STX test sets prior to the reform follow the years where the subject curricula have previously been revised.*

2.5 Content of the report

The report contains an executive summary, this introductory chapter, four main chapters and six appendices.

The executive summary in chapter one presents the main conclusions of the evaluation in terms of the expert panel's reflections and assessments regarding developments in the subject of Mathematics. Chapter 2 introduces the background and purpose of the evaluation as well as the relevant methodological aspects of the evaluation.

Chapter 3 presents the structural placement of the subject of Mathematics in the educational context, including a short introduction to the Danish upper secondary education system, particularly the STX and HTX programmes, the 2005 reform and the post-reform standing of Mathematics in the two programmes.

Chapters 4 – 7 present the development in the subject identified by comparing the former and the present curricula and test sets, and the expert panel's reflections and assessment of these developments. Chapter 4 focuses on the overall approaches to the subject and the teaching of Mathematics. Chapters 5 and 6 go into further detail concerning, respectively, the subject aims, the subject content, and the examinations.

3 Danish upper secondary education and Mathematics

This chapter presents the structural placement of the subject of Mathematics in the two upper secondary programmes. Firstly, the chapter places the HTX and STX programmes in relation to the Danish system of upper secondary education, looking into the purpose, focus and scope of the two programmes. Secondly, the chapter gives an account of the main changes brought in by the 2005 reform of upper secondary education in Denmark. Finally, the chapter outlines the standing of Mathematics in the HTX and STX programmes after the reform, including an account of the respective formal levels of the subject in the programmes and the development of pupils' choices of levels.

3.1 Education programmes at upper secondary level

Four different upper secondary education programmes[5] exist in Denmark: the general upper secondary education programme (STX); the higher commercial examination (HHX); the higher technical examination (HTX); and the higher preparatory examination (HF). STX, HHX and HTX take three years to complete and admit young people who have completed nine years of primary and lower secondary school. When enrolled in these programmes, pupils are approximately 15-17 years old. HF takes two years and admits persons who have completed 10 years of basic school. These pupils are often older than 16 when they enrol in the programme. Over 50% of a Danish year group completes an upper secondary education programme.

The STX and HF programmes are general and cover a broad range of subjects in the fields of humanities, natural and social sciences. The HHX and HTX programmes have a more vocational focus: HHX focuses on business and socio-economic disciplines in combination with foreign languages and other general subjects; HTX emphasises technological and scientific subjects in combination with general subjects.

Each of the education programmes has its specific range of compulsory subjects. Additionally, in STX, HHX and HTX, each school offers a number of different specialised study packages (normally containing three subjects) and elective subjects for the pupils to choose from. All subjects are placed in a system of levels, C, B and A, in relation to the subject's scope and depth, A being the highest level. C-level subjects are, normally, allotted 75 lessons of 60 minutes, B-level subjects have, normally, 200 lessons, and subjects at A-level have, normally, 325 lessons. There are, however, a number of exceptions in the individual programmes, in particular regarding levels A and B, which also applies to Mathematics.

This evaluation focuses on Mathematics in STX and HTX. Both programmes aim at preparing pupils for higher education and providing general education (*almen dannelse* in Danish) – as do HHX and HF. The two programmes are, however, based on different traditions and have different areas of focus.

In STX, the pupils are to be generally educated and to obtain study skills within the humanities, natural science and social science, which will enable them to enrol in and complete a programme of higher education. Traditional classical subjects have had a central role in the programme. STX

[5] *In OECD terms, Danish Upper Secondary Education corresponds to ISCED level 3.*

reflects a tradition of several hundred years with roots back to the Middle Ages. The HTX programme on the other hand, is a relatively new programme, established in 1982. The objective for the establishment of HTX was a desire to offer a broader range of education and training possibilities and a desire to create a new and relevant way to access Higher Education within technical areas. Therefore the emphasis in the HTX programme is on technological perspectives. The aim of preparing pupils for academic studies is realised within the areas of technological development, natural science and other general subjects. Furthermore, HTX has a tradition of interdisciplinary teaching and of using projects and topic based tasks in the teaching.

The table below provides an overview of the similarities and differences between HTX and STX.

Table 1
HTX and STX

	HTX	STX
General aim	To provide general education and to prepare the pupils for higher education.	
Focus	• Emphasis on vocational perspectives; • The preparation for further study is oriented towards areas of technology and scientific subjects in combination with general subjects; • The pupils should be able to study in depth and analyse practical issues.	• Emphasis on general education; • The preparation for further study is general, but the academic standard is closely linked to aspects of the academic subjects; • The pupils should achieve general education and study competences within the humanities, natural science and social science.
Scope	• Approximately 2,500 pupils completed the HTX programme in the summer of 2008, which corresponds to 8% of the pupils who completed an upper secondary education in 2008.	• Approximately 19,800 pupils completed the STX programme in the summer of 2008, which corresponds to 59% of the pupils who completed an upper secondary education in 2008.

3.2 The reform of upper secondary education

In 2003, a political agreement was made to reform upper secondary education, and the reform was put into effect in 2005. The overall aim of the reform is to strengthen and renew the quality of the upper secondary education according to needs and requirements brought about by changes in society. It intended to make significant changes in the upper secondary education programmes and serves three overall goals:

- To strengthen the pupils' preparedness for higher education;
- To update and extend the general education function;
- To create a clear profile for each upper secondary education programme and at the same time enhance the equivalence between them and establish similar structures to the programmes.

The reform includes changes to the structure of the programmes as a whole, as well as new curricula and aims at subject level. A central aspect in the new curricula is a transformation from a focus on content listed in a syllabus to a focus on aims and competences. This implies an enhanced focus on the pupils' ability to apply their achieved skills in different contexts. Furthermore, the new curricula regulate pedagogic approaches and interaction between the subjects, and introduce new examination types.

A major structural innovation in the reform is the introduction of an introductory period of one semester for all pupils enrolled in a general upper secondary education programme. The purpose of this introductory period is to give the pupils an opportunity to learn more about the different subjects in order to make a qualified choice of which subjects they wish to focus on in particular.

At the end of the introductory period, each pupil chooses a specialised study programme consisting of a package of, normally, three subjects. This is a significant change from the previous structure which consisted only of compulsory and elective subjects. Prior to the reform, the pupils could choose from a range of electives, thus assembling an individual composition of subjects (within certain limits of combinations). Following the reform, the pupils have a limited choice between predefined subject-packages provided by the schools. These packages are central to the individual programme, which also contains compulsory subjects and a few electives.

The reform emphasises interaction between different subjects. Firstly, there is to be a close interaction between the subjects within the specialised study programmes. Secondly, interdisciplinary courses have been introduced called *general study preparation* in STX (Almen studieforberedelse in Danish) or *the study programme* in HTX (Studieområdet in Danish). These courses consist only of interdisciplinary projects and focus on general education and preparation for higher education.

The new curricula contain less compulsory material than previously. Thus, the freedom for teachers to choose content has increased; although the subject aims need to be achieved. This flexibility is intended to facilitate work with topics that are suitable for interdisciplinary projects, and at the same time to leave more time for going into depth with a particular topic of special interest to the individual class or pupil.

An intention of the reform was to prioritise the natural sciences, Mathematics and technological development. Therefore, more pupils are required to study subjects within the natural sciences and should have the possibility to go into greater depth with these. Moreover, it has been intended to strengthen the natural sciences' contribution to the aim of general education and preparation for further study in the programmes. The natural science subjects thus have to form part of the interdisciplinary coursework of *the study programme* (HTX) and *general study preparation* (STX).

In HTX, the strengthening of natural science has been less significant than in STX, as the scientific subjects already had a high priority in the HTX programme. Thus, both Mathematics and Physics at level B are compulsory in the HTX programme. Nonetheless, the scientific and the technological subjects have gained more equal priorities with regard to forming the core of the education programme.

In STX the intention of strengthening the natural science subjects means that:

- Mathematics and Physics as distinct subjects at level C is now compulsory for all pupils. In addition, all pupils have to study at least two of the other subjects within the natural sciences (Chemistry, Biology or Natural Geography). At least one of the scientific subjects, (Physics, Chemistry, Biology or natural Geography) must, furthermore, be studied at a minimum of level B, while the other can be studied at level C.
- New possibilities and requirements exist for interaction between scientific subjects. 60 hours are allotted for a natural science basic course within the first semester of introduction. The purpose of this course is to give the pupils a grasp of the scientific methodologies, i.e. both the differences and the similarities between the respective subjects within the natural sciences.

3.3 The post-reform standing of the subject of Mathematics

In HTX Mathematics is taught at two levels: A and B, where A is the highest level. Level B is compulsory and consists of 285 teaching lessons of 60 minutes. Many of the specialised study programmes include Mathematics A, which consists of 410 teaching lessons. Pupils, who after having studied Mathematics at level B, want to continue to level A, can do so by adding 125 teaching lessons.

In STX, Mathematics is taught at three levels: A, B and C. Level C is compulsory and consists of 125 teaching lessons. However, pupils can also at the outset choose to study Mathematics at level B or A. Level B consists of 250 teaching lessons in total; and level A consists of 375 teaching lessons in total. Alternatively, pupils starting at level C have the option of continuing to level B or

A, and the pupils starting at level B have the option of continuing to level A, thus adding 125 teaching lessons for each level.

There is a progression from level B to A. Level A has a higher level of abstraction and an extended content area. For example, level A in HTX deals with "calculating, interpreting and using functions for both derivatives and integrals…", whereas level B deals with "calculating, interpreting and using functions for both derivatives and *simple* integrals…". Another example of progression is seen in STX, where differential equations and vectors are part of level A, but not level B.

The choices of levels are relevant for pupils that continue to certain higher education programmes after completing the upper secondary education programme. For example, in order to study Mathematics, the natural sciences, Engineering, Economics and medical sciences in Denmark, pupils need Mathematics at level A, whereas the social sciences require Mathematics at either level A or B.

The pupils' choices of levels

This section presents the results of the pupils' choices of subjects and levels. The structural changes have along with other things (for example changes in the requirements for Higher Education) led to an increase in the number of pupils studying Mathematics in the two programmes. In HTX, the number of pupils studying Mathematics at level A has increased. In STX, the increase is within Mathematics B, whereas there has been a small decrease in the number of pupils studying Mathematics A.

Table 2 below shows the development in the proportion of pupils completing the programmes having Mathematics at levels A, B and C. It shows: firstly, the proportion of pupils leaving the programme in 2005-2007 with the respective levels; secondly, the chosen level among pupils leaving the programmes in 2008; and thirdly, a forecast of the number of pupils that will be taking their final examination in summer 2009 in Mathematics A, B or C.

Table 2
Share of pupils leaving the HTX and STX programmes with Mathematics at levels A, B and C

	2005-07 (average)	2008	2009	Development from 2005-07 to 2009 (percentage points)
HTX				
Mathematics A	70%	74%	79%	+ 9
Mathematics B	30%	26%	21%	- 9
STX				
Mathematics A	44%	40%	41%	- 3
Mathematics B	24%	36%	35%	+11
Mathematics C	32%	25%	23%	- 9

Source: UNI-C Statistics & Analysis

Note: For the years 2005-2007, the average for the three years is shown.

HTX: Number of pupils in total (N) (2005) = 2,115, N (2006) = 2,227, N (2007) = 2,228, N (2008) = 2,403, N (2009) = 2,616 (The number of pupils that are expected to complete the HTX programme in the summer of 2009.)

STX: Number of pupils in total (N) (2005) = 16,993, N (2006) = 17,798, N (2007) = 18,561, N (2008) =18,954, N (2009) = 21,217 (The number of pupils that are expected to complete the STX programme in the summer of 2009.)

In HTX, the proportion of pupils studying Mathematics at level A has increased by nine percentage points. In STX, the total number of pupils studying Mathematics A or B has increased from 68% to 77% due to the increase at level B. In addition there has been an increase in the proportion of pupils in both programmes with a strong natural science profile. In HTX, there has been an increase in the number of pupils studying the combination of Mathematics A and both Physics and Chemistry at least at level B, from 45% in 2006-2007 to 79% in 2009. In STX, there has been an increase from 14% to 25% in the proportion of pupils with this subject combination over the period.

The structural changes and the introduction of the new specialised study programmes have affected the mechanisms of the pupils' choices of subjects. Mathematics forms part of several subject packages. In STX, for instance, all natural science subjects at levels A and B, as well as social science at level A, must be combined with Mathematics at level B in the subject packages. In practice, this means that pupils who want to study social science, for instance, will have to choose Mathematics at level B. However, other factors might have affected the pupils' choices, for example changes in the requirements for Higher Education.

4 Approaches to the subject of Mathematics

This chapter focuses on the development of the subject of Mathematics in HTX and STX with regard to the identity and purpose of the subject and the approaches to teaching. Firstly, the chapter outlines how the subject of Mathematics is defined in the new curricula in the two programmes, and which approaches to the subject can be identified by comparing the curricula before and after the reform. This is to provide an understanding of the fundamental ideas and ambitions connected to the subject development. Secondly, the chapter presents the reflections and assessments of the expert panel with respect to these developments.

This chapter must be seen in relation to the overall innovations and approaches to Mathematics as a subject. The following chapters go into details about the developments with regard to aims, content and examinations.

4.1 Subject definition and teaching approaches in the new curricula

The new curricula place more explicit emphasis on describing the identity and overall purpose of Mathematics as a subject in the respective programmes than did the former. The identity and purpose of Mathematics is defined in relation to a duality: on the one hand, a purely mathematical and primarily theoretical dimension of Mathematics; on the other, the applied dimension with an emphasis on interdisciplinary work. Both the STX and the HTX curricula seek to strike a balance between these two dimensions of the subject.

The first dimension defines Mathematics by its focus on mathematical reasoning, abstraction and logical thinking; the aim of the subject in this respect is to contribute to the pupils' analytical sense, logical thinking and precise use of symbolic language. The other dimension defines Mathematics in connection with its interaction with and contribution to society, industry, technology and political decision making, etc. This perspective is related to teaching aims which are focused on developing the pupils' ability to understand, formulate, analyse and solve problems within different contexts, as well as assessing the use of Mathematics and making decisions.

Furthermore, the identity and purpose of Mathematics is more or less explicitly related to the bipartite overall aim of the upper secondary education programmes: preparing the pupils for higher education on the one hand and providing general education on the other. In addition, the subject has to address the fact that the pupils studying Mathematics at upper secondary level have rather varying purposes for studying Mathematics. Some intend to apply their skills in areas of tertiary education such as social science or medicine, where the emphasis is mainly on statistical issues; others in areas such as Physics, Economy or Computer science where Mathematics underlies all theory and serves as an indispensable vehicle for obtaining insight; and a few are pursuing tertiary education within the subject itself.

Approaches to the teaching of the subject

As described in chapter 3, some general changes to the programmes have involved changes in Mathematics. These include more focus on competences, changes in the pupils' possibilities for choosing subjects at different levels, and the introduction of new interdisciplinary courses in both programmes. Furthermore, the reform has to some extent altered the approaches to the subject of Mathematics.

Thus, in the teaching of Mathematics, an enhanced focus is placed on the pupils' ability to understand mathematical methods and apply them in solving different problems in different contexts. It is assumed that this is what the pupils will need to do with Mathematics in real life – both during higher education and in their professional and private lives. This involves a new way of defining the main competences that the subject aims to develop. The competence of application will be elaborated further in relation to the subject aims considered in chapter 5.

Moreover, the application approach entails new requirements regarding interdisciplinary teaching and projects in Mathematics teaching. The interdisciplinary approach is not, however, entirely new in the HTX programme, where for some time there has been a focus on working with mathematical, technical or technological problems in cooperation between the practical subjects and Mathematics. In STX, however, the change has been considerable. Part of the teaching should have as a main goal the development of the pupils' knowledge of Mathematics' interplay with culture, science and technology. Furthermore, when Mathematics is included in the pupils' specialised study package, an interdisciplinary approach which includes a broader use of Mathematics should be adopted. The interdisciplinary approach also entails new requirements in STX regarding the use of project based work in Mathematics, which is already known and used in the HTX programme.

Another development after the reform is an enhanced focus on experimental approaches to Mathematics. The purpose is to develop the pupils' grasp of mathematical concepts as well as develop their innovative capabilities and to imitate the mathematical scientific processes and situations that the pupils might meet further on. The ambition is that the pupils work with Mathematics through their own investigation of problems as a complement to reproducing given results and proofs. At the same time, the experimental approach aims to strengthen their understanding of the theoretical material. In the former curricula for STX and HTX, the experimental and inductive approaches to Mathematics were not as clear in either HTX or STX as the deductive approach and the elements of proof and reasoning. However, in the former curricula for the HTX programme, relatively more focus was placed on the exploratory approach.

Finally, new technology, e.g. Computer Algebraic Systems (CAS), has been introduced to teaching and examinations with the dual purpose of giving the pupils concrete skills in using IT tools, as well as making use of the pedagogic possibilities that the technological tools offer. The introduction of IT requirements in the curricula constitutes an innovation in the subject of Mathematics that will be discussed thoroughly in chapter 5.

4.2 Reflections and assessments of the expert panel

This section presents the expert panel's reflections and assessments of the development in the subject of Mathematics with regard to how the subject is defined in the two programmes and the approaches to the teaching of Mathematics, which are reflected in the new curricula. This includes the interdisciplinary and the experimental approach.

4.2.1 Defining the identity and purpose of Mathematics

The expert panel finds the discussion and definition of the subject of Mathematics relevant and necessary. Mathematics means different things to different people, in different contexts and in different periods of time. Mathematics is becoming still more relevant in society, and in an increasing number of professions, in line with the increased focus on the application of Mathematics in other subjects and contexts. According to the panel, this development leads to a continuing need and effort to define what constitutes Mathematics as a subject. This effort can be identified in the Danish curricula, but it is, according to the expert panel, at the same time a general international trend.

The expert panel notes that, when defining the subject, it is important to stress the general educational aspect of Mathematics and its contribution to creating active citizens in a democratic society. This aspect could be stressed even more in the Danish curricula. According to the panel it is crucial to define the aims of developing citizens who are able to assess, criticise and make decisions in relation to societal problems and statements involving Mathematics.

The expert panel observes that Mathematics as a tool to be used in different contexts has been strengthened in the new curricula, in both programmes. This implies an increased emphasis on the competence of setting up and solving problems through modelling. From the panel's point of view this dimension of Mathematics at upper secondary level is important, and the panel is in favour of this development. The panel points out, however, that the current development in the subject requires a continued focus on theoretical Mathematics and Mathematics "in its own right". Partly because it is a precondition of being able to use Mathematics in practice, and partly because the increased use of new technology makes theoretical mathematical reasoning and proof necessary. New technology will enable more modelling and applied Mathematics; it will reduce the need for calculation, but it will also at the same time increase the need for mathematical proof. From the panel's point of view, it is important that both of these two dimensions of Mathematics are emphasised. It is not a question of choosing one over the other. Rather there is a strong need to establish bridges and connections between the two perspectives.

4.2.2 Motivation, self confidence and fascination

In a number of other countries around the world (e.g. Sweden, Singapore, Korea, China and Japan and the UK) concepts such as motivation and mathematical self-confidence among the pupils are becoming more prevalent in the Mathematics curricula. Furthermore, a positive attitude to Mathematics is listed in the "Key competences for lifelong learning" provided by the lifelong learning framework of the European Commission DG Education and Culture[6]. Appreciation of Mathematics, confidence and belief in one's own ability are according to the panel, important aspects when defining the identity and purpose of Mathematics. The panel furthermore, finds it important to stress the beauty of Mathematics. In the UK curricula it says:

> *"Mathematics equips pupils with uniquely powerful ways to describe, analyse and change the world. It can stimulate moments of pleasure and wonder for all pupils when they solve a problem for the first time, discover a more elegant solution, or notice hidden connections".*[7]

The ambition of drawing attention to the motivation for Mathematics is connected to the aim of general education. This concerns the ideas that Mathematics should be accessible to more people; that competences for using Mathematics should be more widespread; and that anxiety over Mathematics should be reduced.

The aim of enhancing pupils' motivation for and fascination with Mathematics is to some extent implicitly stated in the Danish curricula, but it could from the panel's point of view be highlighted even more explicitly. The relevance of this aim is increased when the extended group of pupils studying Mathematics is taken into consideration (cf. section 3.3).

4.2.3 Experimental approach and project based work

The expert panel is favourably disposed to the experimental approach in the new curricula. The panel considers this approach to be important for the mathematical methods that the pupils will need further on. Moreover, it is crucial in order to understand the development of mathematical theory and of Mathematics' interplay with society. The experimental approach summons the creativity, intuition and curiosity which the panel considers to be of great importance to Mathematics.

This dimension of Mathematics could be highlighted in the description of the subject identity, and not just as part of the pedagogic principles, as is the case at present. From the point of view of the panel, it is important to stress the exploratory dimension of Mathematics when the subject

[6] *http://ec.europa.eu/dgs/education_culture/publ/pdf/ll-learning/keycomp_en.pdf*

[7] *http://curriculum.qca.org.uk/key-stages-3-and-4/subjects/Mathematics/keystage3/index.aspx?return=/key-stages-3-and-4/subjects/Mathematics/index.aspx*

is defined. Mathematics is a subject that is still developing. It is not primarily a question of learning old theorems, but it is according to the panel a subject that continues to uncover new and unsolved puzzles and mysteries. The experimental approach to Mathematics is part of the curricula in a number of other countries.

The panel considers project work to be a relevant innovation in STX. Project and topic based work is a feasible learning format to concentrate on the deductive, and also the inductive dimensions of Mathematics. Through project work, the pupils are given the opportunity to practice a range of vital skills in a variety of contexts.

The panel suggests, however, that since the experimental approach is a new dimension of the teaching, it is crucial to provide support and training for teachers to fully exploit its potential. This support could focus on ways to integrate experimental approaches in the teaching.

4.2.4 Interaction with other subjects

Overall, the expert panel welcomes the interdisciplinary approach, which involves more interaction between the subjects. Historically, Mathematics has always had a close relationship with Physics. However, the panel notices a trend in research and business life, whereby Mathematics is increasingly engaging in deep interaction with other disciplines. The panel emphasises that it is important that Mathematics is perceived as useful by the pupils. By working with Mathematics in combination with other subjects, the usefulness of Mathematics is stressed. The interviews convey the impression that there was more interaction between the subjects in HTX than in STX before the reform. The subjects in HTX are closely related within the areas of technology and the natural sciences. In STX there was also cooperation between Mathematics and the natural sciences, particularly Physics, but this was not always the case.

It is the panel's impression that most countries have not introduced the interdisciplinary approach to Mathematics in upper secondary education to the same extent as is the case in Denmark. Furthermore, there are examples of the difficulty of implementing such an approach from those countries that did attempt it. Sweden, for instance, changed its upper secondary school in 1994 and increased the level of interaction between subjects. The Swedish experience was, however, that it is important not to underestimate the difficulty for teachers – Mathematics teachers, as well as teachers in other subjects – to see the connections between Mathematics and other subjects, and that it requires extra support for the teachers in order to function.

The panel finds it important to maintain the focus on pupils' actively learning Mathematics through interdisciplinary projects. This might be easier in some subjects (e.g. Physics and technology) than in others (e.g. biology, history and social science). However, if the interdisciplinary approach is to function, this aim must be pursued in the interaction with all of the subjects, which might require a reinforced effort within the different forums that provide inspiration for teachers.

According to the panel, it is important to acknowledge that there is a dual responsibility. The panel would expect that the success of the interdisciplinary work would also depend on the extent to which the other subjects, with which Mathematics is to interact, take responsibility for integrating Mathematics in a meaningful way. This is particularly important, since Mathematics might be seen as a "difficult" subject. On this point, the panel finds it important to consider the advantages of developing the different subject curricula in close collaboration. This would make it possible to introduce the use of Mathematics in other subject curricula. The sources of documentation in this evaluation do not provide information about the scope and content of support and education for teachers. On the basis of international experience, the panel does, however, emphasise the importance of considering how teachers can be prepared for the task.

Internationally – especially in the USA and Europe[8] – the panel has observed a trend that industry shows increasing interest in being involved and cooperating with schools. Thus, the panel considers it worthwhile to consider new possibilities for interdisciplinary approaches where cooperation

[8] *In the UK, there is an organisation called Mathematics in Education and Industry. The organisation is committed to improving Mathematics education and cooperates with industry: http://www.mei.org.uk/*

with industry and the universities is introduced. This could highlight the usefulness of interdisciplinary approaches in real life.

Approaches to the subject – key findings:

- The panel finds it important to strike a balance between emphasising the dimension of pure Mathematics and mathematical application in different contexts. Neither of the two can stand alone. Thus the panel welcomes the enhanced emphasis on mathematical application, but still finds it important to maintain focus on mathematical reasoning and proof.

- The panel finds it important that the relevance of Mathematics is made apparent to pupils and suggests that this ambition might be considered part of the subject's identity and purpose.

- According to the panel, independent exploration of Mathematics and the use of technology motivate pupils, and encourage them to think and strengthen their understanding of Mathematics. Thus, the panel is favourably disposed to the experimental approach.

- The panel welcomes the interdisciplinary approach. However, international experience shows that it is important not to underestimate the challenges of making this function in practice, and the resources needed.

5 Subject aims

This chapter focuses on the mathematical aims described in the Mathematics curricula. The first part of the chapter focuses on the development identified when comparing the Danish curricula before and after the reform. In the second part of the chapter, the panel's reflections and assessments regarding the mathematical aims are presented.

5.1 Mathematical aims in the new curricula

Overall, the new curricula place more emphasis on describing the mathematical aims in terms of the competences that pupils are to achieve than did the former. At the same time, the level of detail in defining the content is reduced. Rather than being the defining core of the curricula, content is now more of a means to achieving the goals. Broadly speaking, this entails both a development from management by content towards management by objectives, and a development in focus from the pupils' knowledge over to their skills and competences, i.e. what the pupils should be capable of doing with their knowledge. Skills and competences are listed as the mathematical aims and are – the majority of them – generic, which means that they are not tied to specific topics within Mathematics. At the same time, there has been a development regarding the kind of competences the pupils are expected to gain.

It is important to be aware that the mathematical aims must be seen as ideal competences which the teaching aims at providing. This means that not all pupils are expected to completely fulfil the aims.

The mathematical aims are supported in the curricula by guidelines for the pedagogic principles and learning formats. This reflects the idea that the new competence aims cannot be fulfilled through only defining the content, but depend on the approaches adopted in the teaching.

There are differences in the mathematical aims listed in the HTX and STX curricula. However, they involve similarities regarding the kind of mathematical competences that are highlighted. Below is a condensed presentation of the mathematical aims across the two programmes. A detailed overview of the aims at levels A and B in HTX and STX, respectively, is provided in the subject curricula in appendices C-F.

Communication: working with formulae and using the symbolic language of Mathematics
Both programmes contain mathematical aims involving the pupils' ability to work with formulae and to translate between the symbolic language of Mathematics and normal language. The aims connected to the communicative aspect of the subject are supported in the description of the learning formats. In both HTX and STX, the written dimension is emphasised, i.e. the pupils work with assignments which include solving exercises and writing project reports. In addition, the teaching includes an oral dimension where the pupils' oral presentation skills are emphasised, including the independent study and presentation of a given mathematical text. Furthermore, part of the course is carried out in groups, with the intention of developing the pupils' mathematical competences via peer discussions. Thus, the intention is to develop the pupils' ability to express their point of view. In both of the programmes, the aims regarding communication are identical at levels A and B.

Using mathematical models in other subject areas
In STX, the modelling competence is divided into two mathematical aims: using simple models based on *statistics or probability* theory to describe a given set of data or phenomena; and using *functions and their derivatives* in setting up mathematical models (the derivatives are only included at level A). Both emphasise the use of data or knowledge from other subject areas. In HTX, the modelling competence at level A concerns analysing concrete theoretical and practical problems, primarily within the fields of technology and science. In the level B curriculum for HTX, the use of models is not explicitly stated, but pupils are required to use mathematical theories and methods to analyse and solve practical problems. Thus, the modelling aspect is present in the teaching and it is, furthermore, part of the project examination at level B. In both programmes the pupils should, moreover, be capable of posing questions as a result of the use of such models. They should also have an idea of what answers might be expected, as well as be able to clearly formulate conclusions and have an opinion about the idealisations, ranges, limits and validities of the models.

Application in and interaction with other subjects
In STX, it is an explicit independent mathematical aim to demonstrate knowledge of the application of Mathematics in selected subjects and, furthermore, to demonstrate knowledge of Mathematics' evolution in interaction with society's social, cultural and historical development. This aim applies to both levels A and B.

Mathematical way of thinking
Both programmes focus on developing the pupils' mathematical way of thinking and their ability to explain mathematical reasoning and proofs. In the B levels, however, the reasoning and proofs are relatively simple.

Working with derivatives and integrals
In STX at level A, an aim is to be able to use different interpretations of integrals and different methods of solving differential equations. At level B, differential equations are omitted, and the integrals have to be for simple functions. In HTX, the pupils should be able to calculate, interpret and use functions for both derivatives and integrals (simple integrals at level B), hereunder different interpretations of definite and indefinite integrals.

Working with geometry and vectors
Creating geometrical models and solving geometrical problems form part of the mathematical aims in both programmes. In HTX level A, several individual mathematical aims focus on geometry and vectors, including using both classical Euclidean and coordinate geometry, using vectors in 2 and 3 dimensions, and investigating and explaining vectors for a single variable, including movement in one plane. In STX level A, the focus is on using trigonometry and analytical descriptions of geometric shapes in coordinate systems. Using vectors in 2 and 3 dimensions is mentioned in the subject content. In both programmes, there is a focus on using the geometry to answer theoretical and practical questions. The B levels in both programmes also include aims within geometry, albeit at a lower level. In HTX at level B for instance, using vectors in one dimension is included.

Using IT/CAS
In both programmes and at both levels, the use of IT tools/CAS to solve mathematical problems, calculate and substantiate, is an independent mathematical aim. The traditions of using IT in HTX and STX differ, since IT-tools were also used prior to the reform in HTX.

When the new mathematical aims are compared with the aims identified in the former curricula, there are some differences. A stronger emphasis is now placed on the pupils' use of Mathematics in different contexts as well as the pupils' generic analytical sense. Thus, dealing with problems from other subjects and applying mathematical modelling is essential in both programmes. At the same time, a stronger explicit emphasis is placed on the development of the pupils' mathematical or logical thinking and the use of mathematical language. In this respect, the new competence aims reflect each of the two dimensions described in the chapter dealing with subject identity.

Furthermore, the mathematical aims indicate a move toward more generic competences, and not just specific skills within different areas of the syllabus, e.g. calculating percentage. This is the general trend. There are, however, still competence aims which are based on specific content areas, e.g. derivatives and integrals, and geometry and vectors.

5.2 Reflections and assessments of the expert panel

This section presents the panel's assessments with respect to the mathematical aims provided in the new curricula, and the panel's reflections on mathematical aims at upper secondary level.

5.2.1 The focus on aims and competences

The move towards defining mathematical aims in terms of competences is in line with curricula development in a number of other countries. There is a prevalent transition from presenting a long list of nouns in curricula – areas of content and specific formulae, etc. – to curricula that also include a description of aims, i.e. what the pupils can actually do with this content. This trend appears in many different European as well as Asian countries, e.g. Sweden[9] and Singapore[10].

In general, the expert panel is optimistic about the development towards focusing on competences, as well as the specific competences in focus in the Danish curricula. Furthermore, the panel approves of the inherent intention of the curricula to cover all competence aims in the assessment of the pupils. The panel stresses that the examinations and the competences tested in the exam situation to a large extent will define the focus of the teaching. If the intention is to move from a list of content to a definition of competences, this must be reflected in the way pupils are evaluated.

According to the panel, when teaching Mathematics it is crucial that the pupils' are able to transfer their understanding of Mathematics and the specific content that is taught into other contexts. The focus of the teaching should not primarily be the pupils' repetition of examples and problems presented by the teacher, but rather the pupils' understanding of Mathematics[11]. The goal is clearly that the pupils are competent users of mathematical skills in differing contexts. This presupposes that they understand the mathematical method and the possibilities and limits that are related to using it. With this as a crucial point of view, the panel is content with the fact that the new curricula seem to stress a number of competences that support this perspective. Particularly the competences of setting up, using and assessing mathematical models in relation to problem solving in other subject areas, and the knowledge of the application of Mathematics in selected subjects is considered as crucial. Also the focus on Mathematics' evolution in interaction with society's social, cultural and historical development is important. This aim is only stated in the STX curricula, which the panel finds surprising.

With respect to modelling, the panel wonders why this is not defined as a general competence. In the STX curricula, modelling is connected to specific mathematical content – that is statistics or probability, and functions and their derivatives. This means that the competences and the content are not clearly separated, as the mathematical aims contain some specific content. From the point of view of the panel, this might create an ambiguity in the curricula and possibly make it difficult for teachers and pupils to establish the differences between aims and content.

The panel agrees that if you want to evaluate the pupils with respect to all of the competences, there might be a need to delimit and define the content area in relation to the description of the competences. Regarding the specific competence of being able to set up and use mathematical

[9] http://www3.skolverket.se/ki03/front.aspx?sprak=EN&ar=0809&infotyp=8&skolform=21&id=MA&extraId=

[10] http://www.moe.gov.sg/education/syllabuses/sciences/files/maths-secondary.pdf

[11] *The panel points to the fact that the importance of focusing more on the pupils' mathematical understanding is highlighted in an international context. For example, by a survey based on inspections of Mathematics in 192 maintained schools in England. Ofsted, 2008: Mathematics: Understanding the score. (http://www.ofsted.gov.uk/Ofsted-home/Publications-and-research/Browse-all-by/Documents-by-type/Thematic-reports/Mathematics-understanding-the-score)*

models, the panel would suggest that the curricula contain a more precise definition of this competence at generic level. In the aims, the pupils' ability to pose questions is mentioned, and the panel subscribes to this as a step in the right direction, but does also see the possibility of developing this competence even further.

5.2.2 Fundamental mathematical skills

The panel notes that the subject contains important aims related to mathematical ways of thinking. The panel emphasises the importance of paying attention to the competences of mathematical reasoning and proof in order to make sure that the higher order skills in Mathematics are also highlighted. Furthermore, the panel emphasises the importance of stressing fundamental mathematical skills, for instance techniques of calculation, algorithms and procedures. The panel argues that accuracy in doing routine tasks is a crucial mathematical skill. It is important to be able to use methods efficiently and with accuracy, as a precondition of being a good problem solver.

More pupils study Mathematics A in HTX, and B in STX, and the panel has discussed the consequences and possibilities of adjusting the subject to the changed groups of pupils. From the point of view of the panel, it is important that large numbers of pupils study Mathematics, but also to maintain a high level in the teaching of Mathematics. However, if the pupils' background and qualifications for studying Mathematics change, the subject must address this fact. It is necessary to balance the challenge of maintaining an adequate academic level with adjusting to the variations in the pupils' qualifications, and the possible need for repetition of basic skills. Mathematics at upper secondary level needs to assume responsibility for updating fundamental mathematical skills – adding fractions for instance – that the pupils are expected to know from their lower secondary schooling. There should be time in the teaching for this, but this need not necessarily be stated explicitly in the curricula.

The panel notes that the content areas present in the mathematical aims focus more on mathematical analysis than on other important areas of Mathematics such as abstract algebra, geometry and discrete Mathematics. The panel speculates that this is a deliberate choice to facilitate applications in natural or social sciences. The panel sees a need to continually reassess and balance the different mathematical disciplines.

5.2.3 Communicative competences

The expert panel appreciate that the communicative mathematical competences are being emphasised in the mathematical aims. Developing the pupils communicative competences involves both the ability to communicate in "mathematical language", that is to use the proper mathematical terms when communicating with other "mathematicians"; and to communicate about Mathematics when relating mathematical theory and practical results to other subject issues and contexts. Both dimensions are crucial and important to stress in upper secondary education. Another competence which the panel finds crucial in this context is the ability to critically question and interpret mathematical results.

According to the panel it is important to strengthen the Danish mathematical language. In higher education the language is often English, but as a basis for learning this, it is important that the pupils gain a good grasp of the Danish mathematical terms, and in this respect the upper secondary school has an important role to play. That is one of the reasons why the importance of communicating in mathematical language needs to be stressed.

The communicative competences are also emphasised in the pedagogic principles and learning formats in the curricula. The panel notes that both written and oral dimensions are important. Before the reform, the oral examination ensured that the oral dimension was emphasised in the teaching. Now the oral dimension is explicitly stated, and this is, according to the panel, a positive development.

The expert panel concludes from analysing the curricula that the Danish pupils will achieve more than adequate communication skills compared with other countries. In this respect, Mathematics in Danish upper secondary education has a good starting point, and this strength should be appreciated.

5.2.4 Use of technology

Internationally there is a debate regarding how much technology and IT – in particular CAS - should be used in teaching Mathematics. The arguments against emphasise the importance of developing fundamental mathematical skills, which include being able to calculate without using other aids than a pencil and paper. It is argued that if the pupils are allowed to use machines, they will not practise basic mathematical skills and this will eventually lead to a reduction in the academic mathematical level. Due to these concerns, a number of countries are reluctant or even dismissive to the introduction of CAS in Mathematics teaching. The CAS supporters argue that CAS allows pupils to go further with a mathematical problem than if they had to do all the calculations themselves, and it enables them to explore mathematical problems. Finally, familiarity with the use of IT/CAS is required in professional practice and in society as a whole.

The expert panel assesses that the Danish Mathematics curricula, unlike those of a number of other countries, have been suitably reviewed and adapted with regard to the use of CAS. According to the panel, new technology, such as CAS, needs to be integrated in the Mathematics teaching. In this context, the panel argues that in real life situations there is a need to access information in the most appropriate way. In some situations, the use of IT tools will be the best way to access the information, and the pupils should not be denied this possibility. Technological development entails changes that the education system needs to adjust to, but it also presents new possibilities. The use of technology does not necessarily make classical aspects of Mathematics less important. On the contrary, parts of the subject might become more important because technology requires it or allows it.

The panel acknowledges that the implementation of CAS in the teaching presents initial challenges, and that it might require extra resources from the teachers, and that it is important to consider how teachers can be confident that pupils' mathematical skills will not be reduced. Therefore, the panel emphasises that it is important that the teaching contains mathematical problems where the use of IT is not a part of the solution.

In order to fully exploit the possibilities of CAS, it is important that CAS is considered as an integral part of the teaching, rather than something that has to be taught *in addition* to the normal teaching. This might require an increased awareness of supporting and training teachers in using CAS in their teaching in appropriate ways.

The use of CAS is, from the panel's point of view, partly connected to experimental approaches. The panel stresses that pupils can use CAS to independently explore mathematical problems, and not just to solve the problems they are presented with. CAS also allows a combination of proof and exploration. It can provide a good starting point for learning about the theoretical dimensions of Mathematics.

Finally, if CAS is to be an integral part of the teaching, the panel stresses the importance of CAS as an integral part of the examination. Then it would not be possible to avoid CAS in the teaching. Thus, the panel approves of the fact that the use of CAS is mentioned throughout all sections of the new curricula, for both of the programmes and at both levels.

Subject aims – key findings:

- The panel finds that focus on competence rather than a content based approach is cutting edge, and in line with international trends. It is crucial that the pupils understand Mathematics and are able to use it actively, rather than knowing a long list of content.

- The panel is positive towards the introduction of mathematical aims in the curricula and the balance between the different competences. The panel stresses the modelling competence and the competence of meeting, exploring, solving and assessing mathematical problems independently as being crucial.

- Communicative competence in Mathematics is a key skill internationally, and Danish pupils seem to be well equipped in this respect.

- The skills regarding a mathematical way of thinking, reasoning and proof are still highlighted in the mathematical aims, and this is also positively assessed. The panel adds that it is important to assure that the pupils gain skills within the areas of exact calculation and algebra.

- The panel concludes that the Danish Mathematics curricula have been suitably and necessarily reviewed and adapted with respect to technological development by introducing CAS and IT programmes in the teaching. CAS does change the use of Mathematics, but it also enables new and interesting approaches. New technology may be used to avoid computations, but must be expected to increase, rather than decrease, the need for mathematical reasoning.

6 Subject content

The curricula also describe the content to be taught in Mathematics. However, the new curricula entail some changes concerning the way in which the content is described, which also have consequences for the areas of content included. This chapter summarises these changes in the first section, and subsequently presents the reflections and assessments of the expert panel.

6.1 Subject content in the new curricula

In contrast to the former curricula, where the main part of the content was compulsory (with a small elective part at level A in both programmes), the content in the new curricula is divided into core material and extension material, which implies more freedom to choose between different topics and materials. The core material is compulsory, whereas the extension material is elective. This should firstly be seen in relation to the development towards management by objectives rather than management by content. The content is increasingly considered to be a means of achieving the mathematical aims rather than an aim in itself. Secondly, the extension material is needed in order to allow the use of specific content and topics which are suitable for the interdisciplinary projects in the specialised study programmes. In HTX for instance, it is highlighted in the curricula that when Mathematics is part of the pupils' specialised study package, the extension material should be chosen so that the mathematical material supplements the aims of the other subjects in the package. In STX, the extension material should allow interaction with other subjects, also in relation to the evolution and history of Mathematics. Finally, it should allow space in the teaching for working with local interests.

In STX, the extension material should account for approximately one third of the teaching at both levels. In HTX, the extension material should account for 25% of the teaching at level A and 20% at level B.

In the former curricula, the content was divided into headings of main areas. In HTX, the levels B and A together[12] covered the following headings:
- Geometry and trigonometry;
- Functions, equations and inequalities;
- Vectors in a plane and analytical geometry;
- 3-dimensional bodies;
- Differential calculus (expanded if studying level A);
- Integration (expanded if studying level A);
- Vector functions (only level A);
- Vectors in 3 dimensions (only level A);
- Differential equations (only level A).

In STX, the overall topics in levels A and B in the former curricula were:
- Numbers;
- Geometry;
- Functions;
- Differential calculus;
- Statistics and probability;

[12] *Before the reform, HTX pupils first studied the obligatory level B and could then subsequently opt for level A. The former level A curriculum thus builds upon the obligatory level B curriculum.*

- Vectors (only at level A);
- Infinitesimal calculation, including differential equations (only at level A).

In addition, the STX content involved three general aspects: The historical aspect, the modelling aspect and the inner structure of Mathematics.

In contrast to this, the content in the new curricula is no longer grouped under content headings, but is now stated in a long list of content. This does not mean, however, that the substance of the content has changed dramatically from the former to the present curricula in either of the programmes. The main difference is that content is now described in a less specific way, and with fewer details. Furthermore, it is not specified how much of the teaching the different content areas should take up, as was the case in the previous HTX curricula.

The increased amount of elective material has apparently made it necessary to cut down on the core material. The most notable change in the HTX programme is that differential equations, which used to be part of the content at level A, are no longer included in the core material. In STX, the most obvious change is that probability theory is no longer part of the core material. Instead, the extension material should, among other aspects, include the use of at least two types of models based on statistics or probability theory.

6.2 Reflections and assessments of the expert panel

This section presents the expert panel's reflections and assessments of the development in the subject content in relation to the division between core material and extension material.

6.2.1 Division into core and extension material

The expert panel is favourably disposed to the introduction of extension material in the Mathematics curricula. The panel supports the idea of making space for local decisions in the teaching of Mathematics. It is important that the teachers do not become "robots" implementing a long list of predefined content with only little space for creativity. The extension material might – if this purpose is achieved – lead to increased motivation and creativity in the teaching as it becomes possible for teachers and pupils to gain influence over the content of the teaching.

Furthermore, the panel appreciates that the new curricula contain less content, as this might make it possible to secure the pupils' understanding of Mathematics rather than giving them more superficial knowledge of a longer list of topics. This is possible because the extension material can be used to go into greater detail with some of the content areas from the core material. This is, according to the panel, in line with trends in other countries. In Singapore for instance (as with many leading educators), they have an expression: "Teach less, learn more" which has led to a reduction in the content descriptions.

However, the panel is concerned that one of the consequences of having less obligatory core material than before is a reduction in the transparency and uniformity of the pupils' learning. At University, the uniformity of the new students' mathematical knowledge will be diminished. This means that the universities might have to teach some content that most of the students have already been through at upper secondary level, because a few students have not. This problem is likely to be relevant for the pupils who use their upper secondary Mathematics degree to study natural sciences, engineering or economics at University. Students of Mathematics at the universities will cover all of the content at University in any case. In disciplines that require Mathematics, Mathematics itself is often not taught. Instead, Mathematics is considered to be a fundamental tool. This might constitute a challenge for the universities, since they cannot assume that the same fundamental content is known by all students.

On the other hand, the expert panel emphasises that the universities in return will benefit from the pupils' competences in terms of applying, understanding and independently exploring problems involving Mathematics, which the panel thinks will be improved in the future.

Considering the intentions and consequences of the new content description in the Mathematics curricula, the panel assesses that the introduction of the extension material is a positive development. The panel does, however, find it important to keep a substantial amount of core material to secure some uniformity and transparency regarding the subject content taught. The panel is not convinced that the right balance has been found yet, and the panel finds it hard to see the reason why the amount of extension material differs between the two programmes.

6.2.2 The coverage of the core material

Overall, the panel finds the substance of the core material satisfactory in both programmes and at both levels. The panel is, however, slightly concerned with some of the content areas that are now being left out of the curricula. Firstly, with regard to probability theory, which is no longer part of the core material of STX, the panel finds it important to teach this in upper secondary education, more so in a social and cultural context than in terms of applications. After all, modern society to a large extent employs probabilistic ideas and tools such as samples, strategies and games. The interviews carried out by the panel indicate, however, that the decision to move probability theory from the core material to the extension material is based on the assumption that different models of statistics or probability are relevant for different classes, depending on which other subjects the class is studying in the study package. Statistics and probability are considered to be areas of Mathematics teaching which are relevant in interplays with other subjects. The intention is to make it possible to gear Mathematics in the study package towards, for instance, social science or biology. The panel, however, argues that probability is fundamental for teaching statistics and, therefore, wonders why statistics is in the core material when probability theory is left out.

Secondly, the panel is concerned with differential equations being left out of the curricula at level A in HTX (it was never a part of the curricula at level B). Differential equations form part of the core material in STX, and the panel wonders why there is this difference, as it diminishes the uniformity of the students continuing to higher education, and thus requires all tertiary education to teach it from scratch, or leave a few of their students to sort it out on their own. According to the panel, the work with differential equations also represents a certain level of abstraction, which should be present in the curricula.

Besides those content areas which have been left out in order to make more room for extension material, the panel considers the fundamental areas of algebra and arithmetic to be important content areas, also at upper secondary level. It is generally believed that the fundamental skills within those areas should be learned before commencing upper secondary school. However, the panel finds it important that every level of mathematical education assumes responsibility for teaching this. Firstly, some pupils might not have acquired adequate and required skills from the primary school; secondly, the skills need to be updated and developed by using them continuously. Therefore, the panel suggests that consideration is given to taking this aspect into the material stated in the curricula as well.

Subject content – key findings:

- The expert panel agrees with the introduction of extension material, as this might produce motivation and creativity in the teaching as well as contribute to an increased focus on understanding Mathematics in depth rather than superficial learning.

- The introduction of extension material involves less uniformity regarding the content knowledge that the pupils can be assumed to possess when entering higher education. This constitutes a new challenge to the universities. Conversely the universities are expected to benefit in terms of other competences being improved.

- Some important content might be missing in the core material. According to the panel it would be an improvement if probability theory again became part of the core material in STX levels A and B. The panel likewise suggests a reintroduction of differential equations in the core material of HTX level A.

- The panel finds it important to maintain a substantial amount of core material in the curricula, and thus the expert panel considers whether the right balance between core and extension material has been achieved in the two programmes. In this context, the expert panel wonders why the balance is not identical for the two programmes.

7 Examinations

In this chapter, the changes to the examinations in Mathematics for the two programmes are described. Furthermore, the expert panel's discussion and assessment of the innovations are presented. Besides looking at the changes in examinations which emerge from the curricula, the expert panel has compared and discussed selected test sets used before and after the reform in the written examinations.

The panel has considered the level of difficulty and the types of competences and skills that are tested. The analysis and assessment of the test sets are important, as it must be assumed that there is a correlation between the required competences and the level of the test sets and the focus during the teaching. The test sets provide a good indication of the actual developments in the subject and the level and scope of competences that pupils are expected to achieve.

7.1 Examinations after the reform

The evaluation of pupils consists, as was the case before the reform, of a written and an oral examination for level A in both programmes, and at level B in STX. In HTX level B a new project examination has been introduced. While all pupils normally have to take the written examinations – or the project examinations at HTX level B – the Ministry of Education decides, through random choice, whether the oral examination in Mathematics is to form part of the individual pupil's examination.

The written and oral examinations have been adjusted to the approaches and aims in the new curricula. However, the examinations for HTX and STX remain rather different. Below, the different types of examinations and the adjustments made are outlined.

Oral examinations and project examination
The oral examinations have mainly been altered with respect to a greater integration of a project based approach. This approach was, however, already integrated in the HTX oral examinations prior to the reform, and the integration of the project based approach is still, after the reform, more comprehensive in HTX. At level B in HTX, the former written and oral examinations have been replaced by a project examination consisting of, firstly, a project report to be written on the basis of a centrally determined topic within a given period of time, and secondly an oral examination based on the pupils' presentation of their reports. Moreover, the oral examination includes a question related to one of the project reports written during the course. At HTX level A, where the 5 hour written examination has been maintained, the oral examination is – as it was before the reform – based on the project reports written during the course. The pupils randomly draw a question, which addresses a problem from one of the reports. The question may subsequently also deal with a subject area outside the project report.

When the reform was launched, it was possible to choose between two different oral examination types in STX levels A and B: an oral exam with examination questions that were released before the exam, or an oral exam based upon reports written during the course. Now a third oral examination type has been introduced for both levels, and this type will be obligatory in the future. In this new oral examination, the questions for the examination are released before the examination, and it is emphasised that the examination questions should make it possible to involve the project reports written during the course.

In both programmes and at both levels, 30 minutes are allotted to the oral examination, with 30 minutes of preparation time.

Written examinations
All pupils at STX levels A and B and HTX level A conclude Mathematics with a written exam, where the pupils answer a national test set of exercises, which is identical for all pupils in a given year. An exam commission – one for HTX and one for STX – composes the tests sets. 5 hours are allocated for the written examination at level A in both programmes; 4 hours for level B in STX.

The written examinations in the two programmes have traditionally differed with respect to the use of closed book exams, and this difference has been maintained after the reform. In STX the written examination includes both a closed book part and an open book part at levels A and B. The closed book part for STX level A has been reduced from 2 to 1 hour, which is also the time allotted for the closed book part at level B. In HTX, there is only an open book examination, where all types of aids are allowed, with the exception of communication with the outside world.

A major innovation in the written examinations in both programmes is the adoption of CAS and IT tools in the examinations. The questions and exercises of the written exams assume that the pupils have access to CAS programmes, which can perform symbolic manipulation. However, pupils in the HTX programme were also allowed to use CAS before the reform, but only a graphing calculator was required.

In other ways the reform has led to greater differences between the two programmes. In the HTX programme, a new process of preparation for the exam has been introduced. The examination questions are handed out at the start of the examination, but the examination is also based on preparation material which is handed out two days before the exam. This material includes introductions to new topics and concepts. 10 teaching hours, split between two days, are provided as preparation time, and in this period the pupils can receive guidance and supervision from a teacher as required. In the STX written examination, this kind of test has not been introduced.

7.2 Reflections and assessments of the expert panel

When assessing the written exams, the panel compared and discussed the examination types as they are described in the curricula and also studied the test sets for the written examination – one or two from before, and one from after the reform for each of the levels STX A, STX B and HTX A. The panel did not have the opportunity to study examples of questions asked in the oral examinations.

Overall, the examination types used in Mathematics after the reform are, according to the expert panel, comprehensive and suitable for evaluating the competences stated in the curricula. In particular, the expert panel assesses the oral examinations, including the project work perspective, to be a real strength of the Danish system. The oral examination makes it possible to evaluate the pupils' understanding of the material and their ability to use and apply Mathematics independently.

When looking into the written examination types, as well as the written test sets, the differences between the HTX and the STX programmes are striking, according to the panel. Even if the different traditions and focus areas of the two programmes are taken into consideration, the panel does not see the reason for this degree of difference in the evaluation of the pupils in the two programmes. From the interviews carried out, the panel gained the impression that the central organisation and preparation of the STX and HTX exams occurs relatively separately. As the panel considers both programmes to have strengths, the panel suggests that they might benefit from cooperation. Some of the most apparent differences are the use of closed book examination in STX and the use of preparation material in HTX, and these features will be elaborated below.

7.2.1 Open and closed book examinations

According to the panel, it is reasonable to construct the written examination in such a way that the pupils' fundamental calculating and conceptual skills are tested, and thus reduce the concern

– which some might have – that new technology and a focus on new sorts of competences might reduce the level of the pupils' fundamental mathematical skills. According to the expert panel, the closed book part of the examination in STX might serve this purpose. However, when the expert panel compared the post-reform 1 hour closed book exam at level A (summer 2008) with the 2 hour closed book exam at level A prior to the reform (summer 2000), the panel noticed that the level has declined due to the reduction in time allotted. According to the panel, the questions have become less challenging and more routine, which according to the panel jeopardises the purpose of the exam.

The closed book examination according to the panel, serves two overall purposes: to evaluate the pupils' basic operational skills, and to evaluate their conceptual understanding. For the purpose of testing the pupils' fundamental conceptual understanding, the panel wishes to draw attention to one question from the 2000 closed book STX exam at level A, which the panel finds especially useful for this purpose (question 12). The pupils are presented with a figure showing three differentiable functions f, g and h, and they are asked to show which of the functions g and h is the integral of f. This exercise has the advantage of showing whether the pupils understand the concepts – in this example the integral and the connection between the derivative and the integral. From the panel's point of view, the new closed book test set contains mainly problems that test the pupil's abilities to perform fundamental operational tasks, while these kind of conceptual questions are less prevalent. This makes the test less relevant in order to evaluate the fundamental conceptual skills at an adequate and meaningful level, which the panel finds important.

The panel assesses that an adequate closed book examination – or questions where aids are not useful – in combination with an open book examination which is closer to real life problems and situations, provides a good balance when testing a variety of skills and competences. The expert panel wonders why the closed book examinations are only used in the STX programme. However, the panel acknowledges that the fundamental conceptual understanding is evaluated in the open book examination in HTX.

7.2.2 Preparation material

The panel is favourably disposed to the idea of introducing preparation material in the written examination in HTX level A and finds the idea of making the exam forward-looking innovative. When introducing new topics and concepts in the preparation material, the pupils will learn something new in the exam and not just repeat content from the course. Furthermore, it strengthens the evaluation of pupils' ability to grasp new mathematical problems and motivates the pupils to focus on general skills and competences while preparing themselves for the exam. This concept was, according the expert panel, previously tried and was very successful in Scotland. However, the panel wonders why this innovation is limited to the HTX programme.

When discussing the use of the preparation material in the actual examination, the panel noticed, however, that the preparation material could have been used more thoroughly, and that the questions referring to the preparation material could have gone into greater depth. The expert panel recognises that caution must be exercised when introducing the preparation material for the first time, so that the entire examination does not depend on how well the pupils coped during the two days of preparation. In the future, it might be possible to further develop and expand the use of the preparation material.

7.2.3 Academic level in the test sets

Aside from the closed book examinations in STX, the academic level in the test sets appears adequate and fair to the panel and the level seems to equal the level before the reform, although some change of focus has been made. According to the panel, the HTX test sets stand out extremely well from an international perspective, and the STX level seems more than adequate. The HTX written test set at level A is from the panel's points of view more challenging than the STX level A test set. They present the pupils with new problems – based on the preparation material – and the tasks are more coherent, providing a higher level of progression within the tasks. However, the grades also indicate that the HTX set might be more challenging. The documentation shows that 26% of the pupils failed the HTX written exam in Mathematics A in 2008, while for STX level A, this group of pupils constituted 18%[13].

In general the panel wonders why there is this difference between the written exams of the two programmes. This might have the unintended consequence that the pupils from the two programmes are not equally prepared for their next level of education.

7.2.4 Agreement between the aims of the curricula and the written test sets

Considering the comprehensive development of the subject curricula and the introduction of the new subject aims, the panel would have expected the test sets to have changed more than they actually have. However, the panel expects that some of the new competences will be evaluated in the oral examinations as well.

The expert panel identifies that some cautiousness has been exercised with regard to altering the test sets in both of the two programmes. However, the panel assesses that the connection between the aims in the new curricula and the test sets might be slightly clearer in HTX than in STX.

The panel fully understands the concern not to change the exams too drastically. It is necessary to protect the pupils and make sure that they get a fair examination. One concern is that the new approaches might not have been implemented similarly at all places, and that some pupils might not have had teaching that fully corresponds to the new aims. It might take some time to implement the innovations in the teaching, and there might be a well-founded cautiousness not to treat the pupils unfairly at the examinations by asking questions which assume that the teaching has been altered right away. Instead, it might be necessary to alter the test sets in relation to the curricula over a longer period of time. However, the panel argues that if the intention is to change the way of teaching Mathematics, as suggested by the curricula, it is important to make the changes evident, not only in the curricula but also in the test sets. In order to gain from the good intentions in the reform, the test sets must not become static and consist only of a few types of problems. The HTX test sets are, from the panel's point of view closer to the intentions identified in the curricula and might serve as inspiration to the STX test sets.

The integration of CAS in the written examination is an example where the expert panel finds it important that is it fully implemented in the examination, as this will in turn ensure CAS becomes part of the teaching. Thus, the expert panel is positive about the integration of CAS in the examinations in Denmark. The panel finds it essential to test the methodologies for using CAS in the exams, as it is unsatisfactory merely to use CAS as a means to faster computation.

Another central issue that the expert panel has discussed is the importance of coherent and holistic problems or exercises with inner progression in the test sets. More mathematical steps and questions at different levels of abstraction must be included, rather than one or two steps within one problem context and then a shift to a new context. In real life situations, problems are not presented in this way, and in order to keep the problems as realistic as possible it is important to work with coherent problems at multiple levels so pupils can demonstrate their abilities to solve a problem through a progression of arguments. The panel finds that the STX test sets could benefit from more coherent questions within fewer problems.

One of the important competences emphasised in the new curricula is the competence of being able to set up, use and assess mathematical models. The panel has the impression that this competence could be evaluated more thoroughly in the test sets in both programmes[14]. There is a change in approach where the problems are set into context. However, the panel finds it important to be aware that the context in itself does not constitute modelling. It is important that necessary information for setting up the models is included, and adding context for its own sake should be avoided. The panel adds that this is not only a challenge in the Danish upper secondary

[13] The panel recognises that structural matters, i.e. the introduction of a new grade scale, could have affected the grades, along with structural changes regarding which pupils are to take the written examination.

[14] The expert panel acknowledges that this competence might be tested more thoroughly in the oral examinations based on project reports from the teaching.

programmes, but a problem that examiners in many countries have when trying to formulate questions in contexts that are suitable for modelling.

Another important competence in the new curricula of both programmes is to be able to understand how Mathematics is being used in different contexts, and to assess and pose critical questions to the use of mathematical results and models. The panel notes that these competences of considering the use of mathematical representations are evaluated more clearly in the written examination in STX at level B (e.g. 2007, task 15) and could be included in the level A examination as well.

The expert panel notes that the experimental approach emphasised in the new curricula has not become more explicit in the test sets. According to the panel, this is understandable since it constitutes quite a challenge. Moreover, the innovative competence is not directly stated as a mathematical aim to be evaluated, rather as a pedagogic principle. However, there are examples of more experimental approaches in the written test sets in other countries that evaluate the pupils' ability to investigate mathematical problems, e.g. Sweden and Australia, and the expert panel considers that this approach potentially strengthens the emphasis in the teaching.

Examinations – key findings:

- The expert panel considers the oral examinations to be a strength of the Danish system.
- The expert panel assesses that the examination types in both programmes have strengths, but HTX has adopted a more innovative approach.
- The levels in the test sets in both programmes seem more than adequate, from the expert panel's point of view, and compare well internationally. This applies not least to the HTX test set. However, the proportion of pupils that did not pass the test was higher than in the STX programme.
- The combination of closed book exams to protect basic skills and real life open book exams strikes a good balance in the STX programme, though the closed book examination could, from the panel's point of view, have more challenging questions emphasising the pupils conceptual understandings.
- The preparation material introduced for HTX level A is, according to the expert panel, an interesting and innovative element that might be inspiring to the STX programme.
- Considering the comprehensive development of the new curricula, the expert panel would have expected the test sets to have developed further. Although the expert panel acknowledges a cautious approach to changing the examinations, the panel emphasises that it is a necessary step in order to move the teaching forward and benefit from the intentions stated in the curricula.

Appendix A

The expert panel

Professor and Associate Chair for Education, Søren Eilers, University of Copenhagen, Department of Mathematical Sciences. Søren Eilers is an active researcher in the area of non-commutative geometry. He has spent several years in residence at research institutions and universities in North America, and is currently a member of the University of Copenhagen's excellence centre "Symmetry and deformation" which among other aspects aims to create a flourishing international research environment. As Associate Chair for Education, Søren Eilers is responsible for the effectuation and the quality of all courses taught at the Department of Mathematical Sciences, University of Copenhagen. He has in particular been responsible for the design of the introductory courses for students of mathematics, physics and computer science. In this capacity he has worked on significantly strengthening the use of CAS tools for all these student populations.

Lecturer in Mathematics Education, Ian Forbes, University of Edinburgh, Moray House School of Education, Dept of Curriculum Research and Development. Ian Forbes has worked with the University of Edinburgh, training teachers for Primary and Secondary Mathematics teaching, and before that has been teaching mathematics himself. Additionally, he has participated in a leading educational reform arranged by the Scottish Government with particular emphasis on technology, including investigating practices outside Scotland. He has been involved in among other things curriculum development in Mathematics within the Scottish Qualifications Authority. Within Her Majesty's Inspectorate of Education in Scotland, he has evaluated and reported on standards in schools. Finally, he has carried out research and co-authored a book on CAS with teachers from various European countries and from Australia.

Licentiate and Project Manager, Anette Jahnke, University of Gothenburg, National Centre for Mathematics Education (NCM),. Anette Jahnke was assigned during 2005 as a senior specialist by the *Swedish National Agency for Education* for the mathematics curriculum reform work 2007, and is currently an advisory expert in connection with national curriculum development in Sweden. She is an advisor to the *Qualifications and Curriculum Authority* (QCA) in Great Britain in the development of a new curriculum (grades 1 - 6). She has attended international conferences concerning curriculum development arranged by the *Centre for the Study of Mathematics Curriculum*, University of Chicago. At the National Centre for Mathematics education at the University of Gothenburg, she is currently manager of a project concerning the transition from secondary to tertiary mathematics. She was head of mathematics at a Swedish upper secondary school in 2000 – 2008.

Appendix B

Key persons interviewed

Subject advisor for mathematics in HTX, Marit Hvalsøe Schou, the Danish Ministry of Education. Additionally, Marit Hvalsøe Schou is a Senior Lecturer in Mathematics, Physics and Statistics at Odense Tekniske Gymnasium (HTX), Syddansk Erhvervsskole.

Subject advisor for mathematics in STX, Bjørn Grøn, the Danish Ministry of Education. Additionally, Bjørn Grøn is subject advisor for the interdisciplinary coursework *General study preparation (Almen studieforberedelse)* in STX and Senior Lecturer in Mathematics at Vordingborg Gymnasium (STX) and HF.

Chair of the Association of Mathematics Teachers, Marianne Kesselhahn. Additionally, Marianne Kessselhahn is Senior Lecturer in Mathematics and Social Science at Egedal Gymnasium (STX) and HF.

Member of the HTX exam commission, Morten Brøns. Morten Brøns is a Professor in the Department of Mathematics, Technical University of Denmark (DTU).

Member of the STX exam commission, Bodil Bruun. Bodil Bruun is a Senior Lecturer in Mathematics and P.E., Mulernes Legatskole (STX).

Appendix C

Current HTX curriculum - Mathematics A

1. Identity and purpose

1.1 Identity

Mathematics as a subject consists of both theoretical mathematics as well as applied mathematics, including both purely mathematical and interdisciplinary applications.
Mathematics is of great importance in a democratic society, where knowledge of mathematical methodology is a prerequisite for understanding and participating in the political decision making process. The subject's practical dimension is given extra emphasis in this course, via the use of mathematical theories and models to describe analyse and evaluate technical, scientific and social topics and relations.

1.2 Purpose

Using mathematical and practical problems as their starting point, the pupil will gain skills that give the individual a qualification, both on paper and in reality which is comparable to the highest level of senior high school.
The subject aims to contribute to the development of the pupil's personal qualifications as well as their analytical sense, logical thinking and precise use of language.
Working with mathematical material aims to help the pupils to obtain mathematical abilities that will allow an individual pupil to understand, analyse, assess and make decisions in complex systems in a social, professional and educational context.

2. Mathematical aims and content

2.1 Mathematical aims

Pupils should be capable of:

- writing formulae and functions based upon a non-mathematical description of a problem containing related variables, as well as solving these mathematical problems and interpreting the results
- setting up, solving and interpreting geometrical problems using both classical Euclidean and coordinate geometry
- using vectors in 2 and 3 dimensions to solve problems in mathematics and technical and scientific subjects
- calculating, interpreting and using functions for both the derivative and antiderivative, hereunder different interpretations of definite and indefinite integrals
- investigating and explaining vectors for a single variable, including movement in one plane
- being comfortable with a mathematical way of thinking and reasoning
- be able to change between the different ways of representing a single mathematical concept
- analysing concrete theoretical and practical problems, primarily within the fields of technology and science, setting a mathematical model up for a problem, solving the problem and documenting and explaining the solution in a practical way, hereunder give an account of any limitations the model might have and its validity
- using CAS tools and mathematics programmes for calculation and documentation/substantiation
- expressing themselves in, and changing between, the symbolic language of mathematics and everyday written or spoken Danish,

2.2 Core Material

The core material is:
- the order of operations for algebraic expressions, equation reduction, equation solving both analytically, graphically and using IT, factorization, rules for calculating with exponents, roots and natural numbers
- the unit circle with angles expressed in radians and degrees, definition of cosine, sine and tangent
- basic Euclidean geometry and trigonometry, hereunder trigonometry in right angled and other triangles , calculation of the surface area and volume of 3-dimensional forms (prism, cylinder, cone, truncated cone, pyramid, truncated pyramid, sphere, section of a sphere, spherical cap)
- analytical descriptions of 3-dimensional shapes in a coordinate system and the use of analytical calculation techniques
- calculation of vectors in 1 and more than 1 dimensions, hereunder vector coordinates, scalar product, cross product, projecting one vector onto another, breakdown into components, lines, planes, distances, angles, spheres, tangent planes
- the concept of functions, the characteristics of the following type of functions: polynomial, exponential, logarithmic, exponent and trigonometric functions, as well as combinations of these. Description of a graph, identification of a descriptive model, hereunder the use of regression, equation solving and the use of functions in setting up models and solving technical or scientific problems
- description of vector functions in one dimension, hereunder the definition of a vector, tangent, velocity and acceleration vectors, speed, the use of vectors in connection with technical and scientific problems
- definition and interpretation of the derivative generally, and specifically for the afore mentioned functions, as well as the sum rule, difference rule, product and quotient rule in differentiation, the combination of two functions and the inverse function
- monotonic functions, local maxima and minima, optimization and the connection between these and the derivative
- integration (definite and indefinite) of the afore mentioned functions, hereunder the use of integration to calculate area and volume; the sum and difference rule of integration plus integration of a function multiplied by a constant
- the use of IT and mathematics programmes on a computer or calculator for both symbolic and numeric work, simulation and interpretation of results, the use of IT tools to formulate an answer using correct mathematical notation

2.3 Extension Material

The extension material should account for 25% of the total amount of mathematics teaching, and should be selected such that it:
- contributes, along with the core material, to the achievement of the mathematical aims
- involves mathematical theory, which is built upon, and is an extension of, the core material
- supports the pupil in understanding that mathematics can be used in the context of many different subjects. This should be done by choosing areas within the subject that help to fulfil the aims for other subjects that the pupil is taking, either elective or compulsory, and where an interdisciplinary approach is natural, e.g. differentiation in conjunction with biology, chemistry, physics or sociology
- helps strengthen the pupil's education in technical subjects: for example complex numbers in conjunction with studies of electrical and technical subjects and statistics and probability on conjunction with studies of production processes
- puts topics from the core material into perspective and expands the mathematical aims that are based on that material

3. Organisation

3.1 Pedagogic principles

The mathematics course will consist of an approach that switches between theory and application, specifically its use in solving technical or scientific problems.

With the use of both inductive and deductive educational approaches, the pupil will work with a theory that is to be used to solve a given problem. The distinctive feature of mathematics is the creation of mathematical proofs based upon axioms, and therefore an important part of the teaching is helping the pupil to works with mathematical logic and deduction. At the same time, it is important that the pupil, through mathematical activities, learns that an experimental approach to mathematics strengthens their understanding of the theoretical material. The use of CAS tool plays an important role in this.
By increasing the amount of independent study and thought and by working with some of the material at a high level of abstraction, both the pupil's general study skills and mathematical skills will be improved.
The pupil should also think of mathematics as a subject that can be used to solve problems in other subjects, specifically practical problems from technical subjects and more theoretical problems from scientific subjects. Via the use of an indicative approach to their work and problem solving tools taken from mathematics, the pupil should work with analysis, setting up models to obtain solutions and evaluating the achieved results both in mathematics and in other technical and scientific subjects.

3.2 Learning Formats

Both mathematical theory and proofs will be part of the course, along with practical problems, where mathematics will be used as a tool to analyse and mathematize the problem. Teaching will be both topic and project orientated and the pupils will switch between working as an individual and in groups. The proposals for the different projects, and the work on these will be prepared such that there is a progression in the work demanded to solve the task.
Teaching will be organised so that the individual pupil has an opportunity to present central parts of the material orally, with an emphasis being placed on their ability to take a wider view, generalise and understand how they should support their point of view.

Written work

The aim of the written work is:
- to ensure independent work upon mathematical problems and thus contribute towards the pupil's knowledge of the material
- to practise written communication, hereunder the correct use of mathematical language and symbols
- to give the pupil an opportunity to document the in mathematical qualifications
- to provide a basis for the teacher's assessment of the pupil's level of attainment and the pupil's own evaluation of their attainment level.
- to train a systematic approach and give the pupil the opportunity to gain an broader view of the subject

The written work can take the form of tests, group projects or individual work.
The tasks given to the pupils should be formulated such that they can be given to the pupils on several occasions, with the focus being on a different aspect of the task each time.
Furthermore the pupils should complete a number of project reports that, together, cover both the core and extension material. These projects should be larger, less prescribed tasks, where the pupil themselves has to think about some of the prerequisites for the task and its possible content.

3.3 IT

The individual pupil will work with CAS and other mathematics programmes, so that they can become comfortable with the syntax and terminology in, and the use of, at least one mathematics programme.
During the course of the teaching, IT can be increasingly used to undertake:
- visualisations
- repetitive calculations
- complex symbolic calculations
- numerical calculations
- documentation and presentation of results

3.4 Interaction with other subjects

Mathematics will be treated as a separate subject during the foundation level teaching, but can be part of an interaction with other subjects within the pupil's area of study, with a focus on e.g. geometry and trigonometry.
If mathematics is a compulsory subject for the pupil, the extension material should be chosen so that the mathematical content, and the skills developed from working with the material, supplement the aims of the other compulsory subjects. The mathematics course should complement other technical and scientific subjects or other subject areas that are part of the pupil's education or of special interest to the individual pupil. If the pupil is taking mathematics along with a science, a joint approach to teaching should be planned, where the concept of models is central.
If mathematics is an elective subject, the extension material can be chosen such that the content of the course, and the skills developed by working with the material, supplement the aims of the compulsory subjects.

4. Evaluation

4.1 Continuous evaluation

The pupil shall receive feedback throughout the course of the year about their level of attainment for both their written and oral work. This assessment should be based upon the individual pupil's expected progress.

Assessments can be based upon:
- written examinations and tests
- smaller written reports
- project reports
- oral presentations

4.2 Examination types

A centrally set written examination and an oral examination will be held.

The written examination
The written exam will be based upon support material, which will be handed out at the start of the preparation time and on an examination paper handed out at the start of the examination. The examination paper will consist of questions which are based upon the core material outlined in paragraph 2.2 and upon the support material. The examination will last 5 hours. 10 teaching hours, split between two days will be provided as preparation time, hereunder guidance from a teacher. During the examination, the examinee has access to all forms for aids and IT-tools, with the exception of contact with the outside world.

The oral examination
The oral examination will be based upon the project reports written during the course, c.f. paragraph 3.2.
30 minutes will be allotted for the examination, with 30 minutes preparation time, where all types of learning aids may be used, except contact with the outside world.
The examinee will choose a question unseen. Each question will consist of 2-3 sub-questions, where one will be based upon one of the reports from the course. One of the other sub-questions may deal with a subject area that was not necessarily dealt with in the selected report.
The proposals for the reports, along with the questions for the oral examination, will be sent to the external examiner before the examination is due to be held.

4.3 Assessment criteria

When assigning marks, the emphasis should be upon to what degree the examinee has met the aims outlined in paragraph 2.1. An emphasis should be given to whether the examinee:
- shows familiarity with a mathematical way of thinking and reasoning
- can switch between the different ways of representing a mathematical concept
- can independently use mathematical theories and methods to solve theoretical and practical problems, as well as substantiate their solutions
- has achieved enough mathematical insight to be able to use mathematical theories and methods in technical and scientific subjects

- can set up and use mathematical models, as well as assess the results, hereunder assess the models range of uses
- can use IT-aids and mathematics programmes to both calculations and substantiation/documentation
- can express themselves in and switch between the symbolic language of mathematics and everyday written or spoken Danish
- shows an ability to take a broad view of mathematics, an ability to generalise and an understanding of how to argue their point of view

For both the written and oral examination a grade will be given based on a overall assessment.
The grade for the written examination is based solely upon the 5 hour written examination
The grade for the oral examination is based solely upon the pupil's oral presentation.

Appendix D

Current HTX curriculum - Mathematics B

1. Identity and purpose

1.1 Identity
Mathematics as a subject consists of both theoretical mathematics as well as applied mathematics, including both purely mathematical and interdisciplinary applications.
Mathematics is of great importance in a democratic society, where knowledge of mathematical methodology is a prerequisite for understanding and participating in the political decision making process. The subject's practical dimension is given extra emphasis in this course, via the use of mathematical theories and models to describe analyse and evaluate technical, scientific and social topics and relations.

1.2 Purpose
Using mathematical and practical problems as their starting point, the pupil will gain skills that give the individual a mathematical qualification, both on paper and in reality. The subject aims to contribute to the development of the pupil's personal qualifications, including their ability to structure their work and think logically.
Working with mathematical material aims to help the pupils to obtain mathematical abilities that will allow an individual pupil to understand, analyse, assess and make decisions in social, professional and educational contexts.

2. Mathematical aims and content

2.1 Mathematical aims
Pupils should be capable of:

- writing formulae and functions based upon a non-mathematical description of a problem containing related variables, as well as solving these mathematical problems and interpreting the results
- setting up, solving and interpreting simple geometrical problems using both classical Euclidean and coordinate geometry
- using vectors in 1 dimension to solve problems in mathematics and technical and scientific subjects
- calculating, interpreting and using functions for both the derivative and simple antiderivatives, hereunder different interpretations of definite and indefinite integrals
- recognising and knowing a mathematical way of thinking and should be able to perform simple mathematical reasoning and present simple proofs
- be able to change between the different ways of representing a single mathematical concept using mathematical theories and methods to formulate, mathematize, analyse and solve practical problems, as well as validate and substantiate their solutions, analysing concrete theoretical and practical problems, primarily within the fields of technology and science,
- using CAS tools and mathematics programmes for calculation and documentation/substantiation
- expressing themselves in, and changing between, the symbolic language of mathematics and everyday written or spoken Danish,

2.2 Core Material

The core material is:
- the order of operations for algebraic expressions, equation reduction, equation solving both analytically and graphically, rules for calculating with exponents, roots and natural numbers
- definition of cosine, sine and tangent with the help of a unit circle, where the angle is given in degrees
- basic Euclidean geometry and trigonometry in right angled and other generalised triangles in conjunction with 2 and3-d shapes (prism, cylinder, cone, truncated cone, pyramid, truncated pyramid, sphere, section of a sphere, spherical cap). Calculation of the surface area and volume of the afore mentioned shapes
- analytical description of lines, parabolas and circle in appropriate coordinate systems
geometric and analytical vector calculations in 1 dimension, hereunder: vector coordinates, scalar product, projecting one vector onto another, breakdown into components, equations for lines, distances and angles in 1 dimension
- the concept of functions, the characteristics of polynomial and exponent functions, hereunder their graphical representation, domains ranges, origins, monotonic functions and local maxima and minima
- determination of compound and inverse functions
- the concepts of continuity and differentiability, along with the definition of the derivative as a (growth) rate
- calculation of the derivative for the afore mentioned functions, as well as the sum rule, difference rule, product and quotient rule in differentiation, the combination of two functions and the inverse function
- determination of rules of regression for functions when using IT
- the relationship between differentiation and optimization
- integration of polynomial and exponent functions, hereunder the use of integration to calculate area
- the sum and difference rule of integration plus integration of a function multiplied by a constant
- calculation and symbol manipulation using IT
- the use of IT tools to formulate an answer using correct mathematical notation

2.3 Extension Material

The extension material should account for 20% of the total amount of mathematics teaching, and should be selected such that it:
- contributes to the achievement of the mathematical aims
- involves mathematical theory that is in extension of the core material
- expands the pupil's grasp of the concept of functions by working with different types of functions, e.g. trigonometric functions, logarithmic functions and exponential functions/exponential relationships
- supports the pupil in developing an understanding that mathematics can be used in the context of many different subjects. This should be done by choosing areas within the subject that help to fulfil the aims for other subjects that the pupil is taking and where an interdisciplinary approach is natural.
- puts topics from the core material into perspective and expands the mathematical aims that are based on that material

3. Organisation

3.1 Pedagogic principles

The mathematics course will consist of an approach that switches between theory and application, specifically its use in solving technical or scientific problems.
With the use of both inductive and deductive educational approaches, the pupil will work with a theory that is to be used to solve a given problem. The distinctive feature of mathematics is the creation of mathematical proofs based upon axioms, and therefore an important part of the teaching is helping the pupil to works with mathematical logic and deduction. At the same time, it is important that the pupil, through mathematical activities, learns that an experimental approach to mathematics strengthens their understanding of the theoretical material. The use of CAS tool plays an important role in this. By increasing the amount of independent study and

thought and by working with some of the material at different levels of abstraction, the pupil's mathematical skills will be improved.
The pupil should also think of mathematics as a subject that can be used to solve problems in other subjects, specifically practical problems from technical subjects and more theoretical problems from scientific subjects. Via the use of an indicative approach to their work and problem solving tools taken from mathematics, the pupil should work with analysis, setting up models to obtain solutions and evaluating the achieved results both in mathematics and in other technical and scientific subjects.

3.2 Learning Formats

Classwork will be based upon practical problems, where mathematics is used as a tool to analysis and mathematize. Teaching will be both topic and project orientated and the pupils will switch between working as an individual and in groups. The projects and the work involved will be organised such that the pupil gets progressively more chances to show a broad view of the subject and their independence.
Oral presentation skills will also be targeted as will the written dimension of the subject, where the focus will, as time passes, increasingly be on mathematizing problems and a natural use of diverse learning aids. It is important that the pupil documents their work.

Written work
The aim of the written work is:
- to work upon mathematical problems and thus contribute towards the pupil's knowledge of the material
- to practise written communication, hereunder the correct use of mathematical language and symbols
- to give the pupil an opportunity to document the in mathematical qualifications
- to provide a basis for the teacher's assessment of the pupil's level of attainment and the pupil's own evaluation of their attainment level.
- to train a systematic approach and give the pupil the opportunity to gain an broader view of the subject

The written work can take the form of tests, group projects or individual work.
The tasks given to the pupils should be formulated such that they can be given to the pupils on several occasions, with the focus being on a different aspect of the task each time.
Furthermore the pupils should complete a number of project reports that, together, cover both the core and extension material. These projects should be larger, less prescribed tasks, where the pupil themselves has to think about some of the prerequisites for the task and its possible content.

3.3 IT

The individual pupil will be introduced to the different uses of mathematics programmes

During the course of the teaching, CAS and other mathematics programmes can be used to undertake:
- visualisations
- repetitive calculations
- symbolic calculations
- numerical calculations
- documentation and presentation of results

3.4 Interaction with other subjects

Mathematics will be treated as a separate subject during the foundation level teaching, but can be part of an interaction with other subjects within the pupil's area of study, with a focus on e.g. geometry and trigonometry.
Mathematics will be part of an interdisciplinary approach along with technical-scientific subjects.

4. Evaluation

4.1 Continuous evaluation

The pupil shall receive feedback throughout the course of the year about their level of attainment for both their written and oral work. This assessment should be based upon the individual pupil's expected progress.

Assessments can be based upon:
- written examinations and tests
- smaller written reports
- project reports
- oral presentations
- active participation in the class

4.2 Examination types

The assessment will consist of a project report and an oral examination based on said report, c.f. paragraph 3.2. The content of the report will be within the boundaries of a centrally decided theme or topic.
Directly after the period of time given for preparing the project report has run out, the school will send a copy of the report to the external examiner. The internal and external examiner will discuss which topics from the report the pupil will have to give an account of in the oral examination, before this examination starts.
For the oral part of the examination 30 minutes will be allotted with 30 minutes preparation time, where all types of learning aids may be used, except contact with the outside world.
The examinee will choose a question unseen. This question will be based upon on of the project reports written during the course of the year.
The examination consists partly of the examinee answering the chosen question and partly of the examinee presenting a project and answering any requests for elaboration from the examiner.
The examinees presentation of the project may only account for 2/3 of the allotted examination time, including answering any questions from the examiner.
The proposals for the reports, along with the questions for the oral examination, will be sent to the external examiner before the examination is due to be held.

4.3 Assessment criteria

When assigning marks, the emphasis should be upon to what degree the examinee has met the aims outlined in paragraph 2.1. An emphasis should be given to whether the examinee:
- set up and use mathematical models and evaluate the results
- can produce and structure clear documentation
- can use CAS and mathematics programmes to both calculations and substantiation/documentation
- can account for a mathematical way of thinking and perform simple acts of mathematical reasoning
- can switch between the different ways of representing a mathematical concept
- can use mathematical theories and methods to solve problems based upon both theoretical and practical situations
- can express themselves in and switch between the symbolic language of mathematics and everyday written or spoken Danish
- shows an ability to take a broad view of mathematics

A single grade will be given based on a overall assessment of the project and the oral examination, hereunder the answer given to the chosen question.

Appendix E

Current STX curriculum – Mathematics A

1. Identity and purpose

1.1 Identity

The building blocks of mathematics are abstraction and logical thinking. This subject includes a long list of modelling and problem solving techniques. Mathematics is indispensable in many professions, e.g. in science and technology, in medicine and ecology, in economics and social sciences and as the basis for political decision making. Mathematics is also important in our every day lives. The widespread use of mathematics is due to the abstract nature of this subject and is a reflection of the fact that many, widly different phenomena, act identically. When hypotheses and theories are formulated in the language of mathematics, new insights are often gained. Mathematics has accompanied cultural/social developments from the time of our earliest civilisations and mankind's first thoughts about the concept of numbers and forms. As a scientific subject, mathematics has developed via a continuous interaction between its application and the creation of new theories.

1.2 Purpose

One of the aims of the teaching is to give the pupils knowledge of some of the important parts of mathematics' interactions with culture, science and technology. In addition, the aim is to give the pupils an insight into how mathematics can contribute to understanding, formulating and solving problems within a number of different subjects, as well as an insight into mathematical reasoning. Through these insights the pupils should become better able to assess other people's use of mathematics and be competent to complete a higher education course where mathematics is required.

2. Mathematical aims and content

2.1 Mathematical aims

Pupils should be capable of:
- working with formulae, including being able to translate between the symbolic language of mathematics and natural language, and independently using symbolic language to describe how variables are related and to solve problems with a mathematical content
- using simple models based on statistics or probability theory to describe a given set of data or phenomena from another subject area. They should also be capable of posing questions as a result of the use of such models, and have an idea of what answers might be expected, as well as being able to clearly formulate the conclusions.
- using functions and their derivatives in setting up mathematical models based upon data or knowledge from other subject areas. They should also be able to have an opinion about the idealisations and range of such models, be able to analyse given mathematical models, and undertake simulations and extrapolations
- using different interpretations of antiderivatives and different methods of solving differential equations
- creating geometrical models and solving geometrical problems using trigonometry. They should also be capable of giving an analytical description of geometric shapes in coordinate systems and of using these to answer theoretical and practical questions
- explaining mathematical reasoning and proofs, as well as giving an account of how mathematical theories are built using deductive logic

- demonstrating knowledge of the application of mathematics in selected subjects, including knowledge of its use when working with a more complex problem
- demonstrating a knowledge of mathematics' evolution in interaction with society's social, cultural and historical development.
- using IT tools to solve mathematical problems they are presented with.

2.2 Core Material

The core material is:
- the order of operations for algebraic expressions, complex exponentiation, rational and irrational numbers, equation solving using analytical and graphic methods and the use of IT-tools
- descriptive functions for direct and inverse proportionality, as well as exponentiation and polynomial and exponential relationships between variables
- simple statistic models for handling data, graphical presentation of statistical material, empirical statistical descriptors, how representative samples are
- trigonometric functions in similar triangles and trigonometric equations for any triangle, vectors in two and three dimensions defined by given coordinates, the use of vector-based coordinate geometry to set up and solve problems within plane and solid geometry
- the concept $f(x)$, characteristic elements of the following elementary functions: linear, polynomial, exponential, exponentiation, logarithmic, cosine, sine; characteristic qualities of these function's graphical representations, use of regression
- the definition and interpretation of the derivative, hereunder growth rate and differentials, the derivatives for elementary functions and the rules for the differentiation of $f + g$, $f - g$, $k \cdot f$, $f \cdot g$, $\frac{f}{g}$, proof of selected derivatives
- monotonic functions, maxim, minima and optimization along with the connection between these concepts and the derivative
- antiderivatives for elementary functions, definite and indefinite integrals, the rules for the integration of $f + g$, $f - g$, $k \cdot f$ as well as integration by substitution, proof for the relationship between integrals and antiderivatives, the volume of a solid of revolution
- first order linear differential equations and logistic differential equations, qualitative analysis of given differential equations along with drawing up simple differential equations.
- fundamental properties of mathematical models, modelling

2.3 Extension Material

Pupils will not be able fulfil the aims for mathematics A if they only study the core material. The extension material in this subject, including its interactions with other subjects, aim to give perspective and added depth to the core material, expand the pupil's horizons within the subject and allow space for local interests and show consideration for the individual school.
In order for pupils to live up to all the mathematical aims given, the extension material, which accounts for approx. 1/3 of teaching should, amongst other things, include:
- reasoning and presentation of proofs within calculus as well as a project involving deductive reasoning within a chosen topic
- differential equation models, including setting them up, using them and solving differential equations
- the use of at least two types of models based on statistics or probability theory, the collection and processing of data in order to investigate a proposed hypothesis
- a course on the history of maths

3. Organisation

3.1 Pedagogic principles

The teaching of mathematics is organised to enable the individual pupil to reach the aims given here. The pupil's independent handling of mathematical problems is central to the teaching of mathematics.
Via an experimental approach to mathematical topics and problems, the pupil's grasp of mathematical concepts and innovative abilities will be developed. This can happen, for example, via preparing lessons on inductive logic, so pupils have the opportunity to independently formulate theories based on specific examples,

The experimental part of mathematics cannot stand alone. Therefore certain topics should be presented in such a way that pupils gain a clear understanding of the deductive reasoning on which mathematical theory is built.
The individual pupil's understanding of mathematics will be developed though oral presentations. Emphasis will be placed on teaching the practical applications of mathematics, and pupils will be shown how the same mathematical methods can be applied to very different phenomena.
Lessons will be organised to allow for a progression in both the methods used for problem solving and the mathematical content during the course. At the same time, to help pupils retain basic skills and knowledge, these will regularly be reviewed.
CAS programmes will be used, not just to carry out the more complicated symbolic calculations, but also to support the pupils in learning mathematical skills and concepts.

3.2 Learning Formats

A significant part of the course will take the form of project or topic based tasks, that cover various parts of the core and extension material or it will take the form of problems that allow an interdisciplinary approach. For each large section of the course, there will be set mathematical aims, the format the work will be presented in will be considered and the pupils will deliver a written piece of work that can document their results or conclusions concerning the interdisciplinary problem presented.
Part of the coursework will be carried out in groups, with the intention of developing the pupils' grasp of mathematical concepts via peer discussions within the group.
Oral presentation skills are consciously emphasised, including the independent study and presentation of given mathematical texts.
In the classroom, emphasis will be placed on problem solving as a vital technique for supporting the acquisition of mathematical concepts, methods and skills. Problem solving and calculation will occur both in lessons and as homework. Pupils will also produce large, written reports after working on specific projects or topics.

3.3 IT

Lessons should be organised so that pocket calculators, IT and mathematical programmes are important learning aids in the pupils' work with acquiring concepts and solving problems. Time to train the pupils in the use of these aids to perform calculations, to rearrange formulas, to handle statistic data, to gain an overview of graphs in order to solve equations, for differential and integral calculus and to solve differential equations should be included in the lesson plans. In addition the use of pocket calculators, IT and mathematical programmes should be included when planning work with the experimental approach to topics and problem solving.

3.4 Interactions with other subjects

When mathematics is included in a pupil's group of chosen subjects, an interdisciplinary approach, which includes a more widespread use of mathematics, should be taken. The aim of this is to help the pupil achieve a deeper insight into the descriptive power of mathematics and the importance of weighing and discussing the assumptions of a given mathematical description and the reliability of the results given by said description.
Courses should be prepared that have the development of the pupils' knowledge of mathematics interplay with culture, science and technology as their main goal. This should happen via cooperation with other subject areas or by using the pupils' knowledge of these subjects.

4. Evaluation

4.1 Continuous evaluation

Both the teaching and what the pupils have gotten out of the lessons will be evaluated throughout the course.
For each large project or topic tackled, how the pupils will be evaluated shall be clearly described. Parts of the course covering large areas of the core material will normally finish with a test to evaluate whether the mathematical aims have been met.
After each large project or topic, the teacher and pupils will undertake an evaluation of the teaching, the format used and the progress towards fulfilling the mathematical aims.

Throughout the entire period of senior high school, the pupils will work with their written skills and will regularly turn in written papers. These will be corrected and commented upon, based upon the assessment criteria given in paragraph 4.3.

4.2 Examination types

A written and oral examination will be held.

The written examination

5 hours will be allotted for the written examination. It will consist of questions from within the core material and its aim is to evaluate the relevant aims outlined in paragraph 2.1. Aids will not be allowed for the first part of the examination, which will last for 1 hour, after which the answers will be turned in. During the second part of the examination, all types of aids may be used, with the exception of communication with the outside world. The questions in this part of the examination will be posed assuming that the examinee has access to CAS programmes which can perform symbolic manipulation, cf. paragraph 3.3

The oral examination

For each class, the school can choose from one of the three examination types given below:
Type a) An oral examination consisting of one large general question divided into smaller, more concrete sub-questions. The questions presented in the examination are released in good time before the examination and are formed such that they, as a unit, make it possible to assess whether the mathematical aims described in paragraph 2.1 have been met. The questions, and a description of the teaching carried out throughout the course, will be sent to the external examiner, who has to approve the questions before the examination is held.
30 minutes per examinee will be allotted for the examination, with 30 minutes of preparation time.
The examination is divided into two parts.
The first part will consist of the examinee's presentation of their answers to both the questions they have picked out, and any requests for elaboration from the examiner.
The second part takes the form of a conversation between the examiner and examinee based upon the chosen question.
Type b) An oral examination based upon reports written in conjunction with teaching. The individual examinee's reports must fulfil the aims given in paragraph 2.1. The examination questions are formed such that there is a title, followed by specific sub-questions that relate to the reports. The questions and a descriptive list of the reports and of the teaching carried out throughout the course will be sent to the external examiner, who will approve the questions before the examination is held.
30 minutes will be allotted per examinee for this examination, with 30 minutes of preparation time.
The examination is divided into two parts.
The first part will consist of the examinee's presentation of their report and its mathematical aims plus their answers to any requests for elaboration from the examiner.
The second part takes the form of a conversation between the examiner and examinee based upon the report and its subject area.
Type c) An oral examination where the subject matter tested is the material taught throughout the year. The examination questions will be released in good time before the examination dates and shall, as a whole, cover the aims and mathematical content of the course. A significant proportion of the examination questions should be posed in such a way that it is possible to bring in knowledge from projects and topics covered during the year and the respective pupil's reports on these topics.
Each individual question should be posed so it has a title, which gives the overall subject of the examination, followed by specific sub questions.
30 minutes will be allotted per examinee for this examination, with 30 minutes of preparation time.
The examination is divided into two parts.
The first part will consist of the examinee's presentation of their answer to the chosen question plus their answers to any requests for elaboration from the examiner.

The second part takes the form of a conversation between the examiner, examinee and external examiner based upon the question's overall subject area.

4.3 Assessment criteria

Marking the examination is an assessment of to what extent the examinee's presentation lived up to the aims given in paragraph 2.1
In this assessment an emphasis will be placed upon whether the examinee:
1) *has basic mathematical skills, hereunder:*
- can use symbolic mathematical terms and mathematical concepts
- has knowledge of mathematical methods and can use them correctly
- is capable of using IT- tools appropriately

2) *can use mathematics to solve problems presented, hereunder:*
- can choose appropriate methods to solve the presented problem
- can present a mathematical problem or approach by solving a given problem in a clear, understandable fashion.
- can explain mathematical models that are presented and discuss their range of uses

3) *has a good grasp of mathematics and an ability to set it in perspective, hereunder:*
- can put the development of mathematics into perspective
- has a good grasp of an area, where mathematics is used in conjunction with other subjects, along with an ability to reflect upon the use of mathematics in other subjects.
- can switch between the theoretical and practical sides of the subject in conjunction with modelling and problem solving tasks.
- can demonstrate an insight into the characteristics of mathematical reasoning.

In both the oral and written examinations, a grade will be awarded based upon an a assessment of the total examination. When examination type b) is chosen, only the performance in the actual oral examination will be considered.

Appendix F

Current STX curriculum – Mathematics B

1. Identity and purpose

1.1 Identity
The building blocks of mathematics are abstraction and logical thinking. This subject includes a long list of modelling and problem solving techniques. Mathematics is indispensable in many professions, e.g. in science and technology, in medicine and ecology, in economics and social sciences and as the basis for political decision making. Mathematics is also important in our every day lives. The widespread use of mathematics is due to the abstract nature of this subject and is a reflection of the fact that many, wildly different phenomena, act identically. When hypotheses and theories are formulated in the language of mathematics, new insights are often gained. Mathematics has accompanied cultural/social developments from the time of our earliest civilisations and mankind's first thoughts about the concept of numbers and forms. As a scientific subject, mathematics has developed via a continuous interaction between its application and the creation of new theories.

1.2 Purpose
One of the aims of the teaching is to give the pupils knowledge of some of the important parts of mathematics' interactions with culture, science and technology. In addition, the aim is to give the pupils an insight into how mathematics can contribute to understanding, formulating and solving problems within a number of different subjects, as well as an insight into mathematical reasoning. Through these insights the pupils should become better able to assess other people's use of mathematics and be competent to complete a higher education course where mathematics is required.

2. Mathematical aims and content

2.1 Mathematical aims
Pupils should be capable of:
- working with formulae, including being able to translate between the symbolic language of mathematics and natural language, and independently using symbolic language to describe how variables are related and to solve simple problems with a mathematical content
- using simple models based on statistics or probability theory to describe a given set of data or phenomena from another subject area. They should also be capable of posing questions as a result of the use of such models, and have an idea of what answers might be expected, as well as being able to clearly formulate the conclusions.
- using simple functions in setting up mathematical models based upon data . They should also be able to undertake simulations and extrapolations and have an opinion about the idealisations and range of the models
- using the derivative and antiderivative for simple functions and interpret different representation of these
- giving an account of available geometrical models and solving geometrical problems
- carrying out simple mathematical reasoning and complete simple proofs
- demonstrating knowledge of the application of mathematics in selected subjects, including knowledge of its use when working with a more complex problem
- demonstrating a knowledge of mathematics' evolution in interaction with society's social, cultural and historical development

- using IT tools to solve mathematical problems they are presented with, hereunder handling more complex formulae and determining the derivative and antiderivative for more complex functions.

2.2 Core Material

The core material is:
- the order of operations for algebraic expressions, complex exponentiation, equation solving using analytical and graphic methods and the use of IT-tools
- descriptive functions for direct and inverse proportionality, as well as exponentiation and polynomial and exponential relationships between variables
- simple statistic models for handling data, graphical presentation of statistical material, empirical statistical descriptors, how representative samples are
- trigonometric functions in similar triangles and trigonometric equations for any given triangle
- the concept *f(x)*, characteristic elements of the following elementary functions: linear, polynomial, exponential, exponentiation, logarithmic; characteristic qualities of these function's graphical representations, use of regression
- the definition and interpretation of the derivative, hereunder growth rate and differentials, the derivatives for elementary functions and the differentiation of $f + g$, $f - g$, $k \cdot f$, $f \cdot g$, $f \quad g$, proof of selected derivatives
- monotonic functions, maxim, minima and optimization along with the connection between these concepts and the derivative
- antiderivatives for elementary functions, definite and indefinite integrals, the use of integration to calculate the area enclosed by two curves for non-negative functions
- fundamental properties of mathematical models, modelling

2.3 Extension Material

Pupils will not be able fulfil the aims for mathematics B if they only study the core material. The extension material in this subject, including its interactions with other subjects, aim to give perspective and added depth to the core material, expand the pupil's horizons within the subject and allow space for local interests and show consideration for the individual school.
In order for pupils to live up to all the mathematical aims given, the extension material, which accounts for approx. 1/3 of teaching should, amongst other things, include:
- reasoning and presentation of proofs within differential calculus and other chosen topics
- mathematical models, including setting them up, using differentiation and the derivative
- the use of at least two types of models based on statistics or probability theory, the collection and processing of data in order to investigate a proposed hypothesis
- a course on the history of maths

3. Organisation

3.1 Pedagogic principles

The teaching of mathematics is organised to enable the individual pupil to reach the aims given here. The pupil's independent handling of mathematical problems is central to the teaching of mathematics.
Via an experimental approach to mathematical topics and problems, the pupil's grasp of mathematical concepts and innovative abilities will be developed. This can happen, for example, via preparing lessons on inductive logic, so pupils have the opportunity to independently formulate theories based on specific examples,
The experimental part of mathematics cannot stand alone. Therefore certain topics should be presented in such a way that pupils gain a clear understanding of the deductive reasoning on which mathematical theory is built.
The individual pupil's understanding of mathematics will be developed though oral presentations. Emphasis will be placed on teaching the practical applications of mathematics, and pupils will be shown how the same mathematical methods can be applied to very different phenomena.
Lessons will be organised to allow for a progression in both the methods used for problem solving and the mathematical content during the course. At the same time, to help pupils retain basic skills and knowledge, these will regularly be reviewed.

CAS programmes will be used, not just to carry out the more complicated symbolic calculations, but also to support the pupils in learning mathematical skills and concepts.

3.2 Learning Formats

A significant part of the course will take the form of project or topic based tasks, that cover various parts of the core and extension material or it will take the form of problems that allow an interdisciplinary approach. For each large section of the course, there will be set mathematical aims, the format the work will be presented in will be considered and the pupils will deliver a written piece of work that can document their results or conclusions concerning the interdisciplinary problem presented.
Part of the coursework will be carried out in groups, with the intention of developing the pupils' grasp of mathematical concepts via peer discussions within the group.
Oral presentation skills are consciously emphasised, including the independent study and presentation of given mathematical texts.
In the classroom, emphasis will be placed on problem solving as a vital technique for supporting the acquisition of mathematical concepts, methods and skills. Problem solving and calculation will occur both in lessons and as homework. Pupils will also produce large, written reports after working on specific projects or topics.

3.3 IT

Lessons should be organised so that pocket calculators, IT and mathematical programmes are important learning aids in the pupils' work with acquiring concepts and solving problems. Time to train the pupils in the use of these aids to perform calculations, to rearrange formulas, to handle statistic data, to gain an overview of graphs, to solve equations and for differential and integral calculus should be included in the lesson plans. In addition the use of pocket calculators, IT and mathematical programmes should be included when planning work with the experimental approach to topics and problem solving.

3.4 Interactions with other subjects

When mathematics is included in a pupil's group of chosen subjects, an interdisciplinary approach, which includes a more widespread use of mathematics, should be taken. The aim of this is to help the pupil achieve a deeper insight into the descriptive power of mathematics and the importance of weighing and discussing the assumptions of a given mathematical description and the reliability of the results given by said description.
Courses should be prepared that have the development of the pupils' knowledge of mathematics interplay with culture, science and technology as their main goal. This should happen via cooperation with other subject areas or by using the pupils' knowledge of these subjects.

4. Evaluation

4.1 Continuous evaluation

Both the teaching and what the pupils have gotten out of the lessons will be evaluated throughout the course.
For each large project or topic tackled, how the pupils will be evaluated shall be clearly described. Parts of the course covering large areas of the core material will normally finish with a test to evaluate whether the mathematical aims have been met.
After each large project or topic, the teacher and pupils will undertake an evaluation of the teaching, the format used and the progress towards fulfilling the mathematical aims.
Throughout the entire period of senior high school, the pupils will work with their written skills and will regularly turn in written papers. These will be corrected and commented upon, based upon the assessment criteria given in paragraph 4.3.

4.2 Examination types

A written and oral examination will be held.

The written examination

4 hours will be allotted for the written examination. It will consist of questions from within the core material and its aim is to evaluate the relevant aims outlined in paragraph 2.1. Aids will not

be allowed for the first part of the examination, which will last for 1 hour, after which the answers will be turned in. During the second part of the examination, all types of aids may be used, with the exception of communication with the outside world. The questions in this part of the examination will be posed assuming that the examinee has access to CAS programmes which can perform symbolic manipulation, cf. paragraph 3.3

The oral examination
For each class, the school can choose from one of the three examination types given below:
Type a) An oral examination consisting of one large general question divided into smaller, more concrete sub-questions. The questions presented in the examination are released in good time before the examination and are formed such that they, as a unit, make it possible to assess whether the mathematical aims described in paragraph 2.1 have been met. The questions, and a description of the teaching carried out throughout the course, will be sent to the external examiner, who has to approve the questions before the examination is held.
30 minutes per examinee will be allotted for the examination, with 30 minutes of preparation time.
The examination is divided into two parts.
The first part will consist of the examinee's presentation of their answers to both the questions they have picked out, and any requests for elaboration from the examiner.
The second part takes the form of a conversation between the examiner and examinee based upon the chosen question.
Type b) An oral examination based upon reports written in conjunction with teaching. The individual examinee's reports must fulfil the aims given in paragraph 2.1. The examination questions are formed such that there is a title, followed by specific sub-questions that relate to the reports. The questions and a descriptive list of the reports and of the teaching carried out throughout the course will be sent to the external examiner, who will approve the questions before the examination is held.
30 minutes will be allotted per examinee for this examination, with 30 minutes of preparation time.
The examination is divided into two parts.
The first part will consist of the examinee's presentation of their report and its mathematical aims plus their answers to any requests for elaboration from the examiner.
The second part takes the form of a conversation between the examiner and examinee based upon the report and its subject area.
Type c) An oral examination where the subject matter tested is the material taught throughout the year. The examination questions will be released in good time before the examination dates and shall, as a whole, cover the aims and mathematical content of the course. A significant proportion of the examination questions should be posed in such a way that it is possible to bring in knowledge from projects and topics covered during the year and the respective pupil's reports on these topics.
Each individual question should be posed so it has a title, which gives the overall subject of the examination, followed by specific sub questions.
30 minutes will be allotted per examinee for this examination, with 30 minutes of preparation time.
The examination is divided into two parts.
The first part will consist of the examinee's presentation of their answer to the chosen question plus their answers to any requests for elaboration from the examiner.
The second part takes the form of a conversation between the examiner, examinee and external examiner based upon the question's overall subject area.

4.3 Assessment criteria
Marking the examination is an assessment of to what extent the examinee's presentation lived up to the aims given in paragraph 2.1
In this assessment an emphasis will be placed upon whether the examinee:
1) *has basic mathematical skills, hereunder:*
- can use symbolic mathematical terms and mathematical concepts
- has knowledge of mathematical methods and can use them correctly
- is capable of using IT- tools appropriately

2) *can use mathematics to solve problems presented, hereunder:*
- can choose appropriate methods to solve the presented problem
- can present a mathematical problem or approach by solving a given problem in a clear, understandable fashion.
- can explain mathematical models that are presented and discuss their range of uses
3) *has a good grasp of mathematics and an ability to set it in perspective, hereunder:*
- can put the development of mathematics into perspective
- has a good grasp of an area, where mathematics is used in conjunction with other subjects, along with an ability to reflect upon the use of mathematics in other subjects.
- can switch between the theoretical and practical sides of the subject in conjunction with modelling and problem solving tasks.
- can demonstrate an insight into the characteristics of mathematical reasoning.

In both the oral and written examinations, a grade will be awarded based upon an assessment of the total examination. When examination type b) is chosen, only the performance in the actual oral examination will be considered.