English Translation of
the Japanese Mathematics Curricula
in the Course of Study

March, 2008
Grades 1-9

Akihiko Takahashi
Tad Watanabe
Makoto Yoshida

This is an unofficial translation of the mathematics curricula (Grades 1-9) in the Course of Study, published by the Ministry of Education, Culture, Sports, Science, and Technology-Japan in March, 2008.

The Course of Study is in the public domain and is available on the Ministry's Web site at: http://www.mext.go.jp/a_menu/shotou/new-cs/youryou/index.htm

CONTENTS

STANDARD NUMBERS OF CLASS PERIODS
REQUIRED FOR MATHEMATICS

Revised by the Ministry of Education, Culture, Sports, Science, and Technology Japan on April 1, 2008, and to be put in force beginning on April 1, 2011 for the elementary schools and on April 1, 2012 for the lower-secondary schools.

1. Time allocation for mathematics in the elementary schools

Grade	Age (years old)	Standard number of class periods per year for mathematics	Standard total number of class periods per year	Standard number of school weeks per year	Standard length of class period
1	6	136 (4 per week)	850	more than 34 weeks	45 minutes
2	7	175 (5 per week)	910	more than 35 weeks	45 minutes
3	8	175 (5 per week)	945	more than 35 weeks	45 minutes
4	9	175 (5 per week)	980	more than 35 weeks	45 minutes
5	10	175 (5 per week)	980	more than 35 weeks	45 minutes
6	11	175 (5 per week)	980	more than 35 weeks	45 minutes

2. Time allocation for mathematics in the lower-secondary schools

Grade	Age (years old)	Standard number of class periods per year for mathematics	Standard total number of class periods per year	Standard number of school weeks per year	Standard length of class period
1	12	140 (4 per week)	1015	more than 35 weeks	50 minutes
2	13	105 (3 per week)	1015	more than 35 weeks	50 minutes
3	14	140 (4 per week)	1015	more than 35 weeks	50 minutes

MATHEMATICS FOR ELEMENTARY SCHOOL

I. OBJECTIVES

Through mathematical activities students will (1) acquire basic and fundamental knowledge and skills about numbers, quantities and geometric figures, (2) cultivate their ability to consider phenomena from their daily lives with foresight to generate and organize logical thinking steps to follow through, and to represent those phenomena, (3) recognize the joy of mathematical activities and the merit of mathematical manipulation, and (4) foster a disposition to willingly make use of mathematics in daily life and studies.

II. OBJECTIVES AND CONTENT IN EACH GRADE

[Grade 1]

1. Objectives

   (1) Through activities such as those involving concrete objects, students will enrich their number sense. They will understand the meaning and the representations of numbers. They will also understand the meaning of addition and subtraction, investigate ways to carry out calculations and develop the ability to use them appropriately.

   (2) Through activities such as those involving concrete objects, students will enhance their experiences that will form the foundation for understanding the ideas of quantities and their measurements as well as enrich their measurement sense.

   (3) Through activities using concrete objects, students will accumulate experiences that will form the foundation for understanding geometry as well as enrich their sense of geometry.

   (4) Through activities using concrete objects, students will be able to represent numbers, quantities, and their relationships by using words, mathematical expressions, and diagrams, and will be able to interpret such representations.

2. Content

A. Numbers and Calculations

(1) Through activities such as the counting of objects, students will understand the meaning of numbers and be able to use numbers appropriately.
   a. To compare numbers of objects in groups by making one-to-one correspondence between objects.
   b. To correctly count or represent the number and order of objects.
   c. To develop a sequence of numbers and to represent numbers on a number line by thinking about the order and the size of those numbers.
   d. To consider a number in relation to other numbers in ways such as regarding it as a sum or difference of other numbers
   e. To understand how to represent 2-digit numbers.
   f. To understand how to represent 3-digit numbers in simple cases.
   g. To consider numbers using ten as a unit.

(2) Students will understand the meaning of addition and subtraction, and will be able to use those operations appropriately.
   a. To become aware of situations in which addition and subtraction may be applied.
   b. To investigate the ways to calculate the sum of two 1-digit numbers and the difference, which is the inverse, so that the students can add and subtract accurately and reliably.
   c. To think about the ways to calculate simple cases of sums and differences involving 2-digit numbers.

B. Quantities and Measurements

(1) Students will enhance their experiences that will form the foundation of understanding quantities and their measurements through activities such as comparison of quantities.
   a. To compare length, area, and volume.
   b. To compare quantities by using familiar objects as units.

(2) Students will be able to read clock times in their daily lives.

C. Geometric Figures

(1) Students will enrich their experiences that form the foundation for understanding the concepts of geometric figures through observing the shapes of objects from students' everyday lives and composing shapes using such objects.
   a. To recognize the shapes of objects or to notice their features.
   b. To represent the position of an object by correctly using such words concerning directions and positions as "front and rear, " "right and left," and "above and below."

D. Quantitative Relations

(1) Students will be able to represent the cases in which addition and subtraction may be applied by using mathematical expressions and will be able to interpret such expressions.

(2) Students will be able to represent numbers of objects by using pictures and diagrams and will be able to interpret such representations.

[Mathematical Activities]

(1) For the contents of "A: Number and Operations," "B: Quantities and Measurement," "C: Geometric Figures," and "D: Quantitative Relations," instruction should incorporate mathematical activities such as those listed below.
   a. Counting concrete objects by categorizing or making equal size groups, and representing the results of counting in an organized manner.
   b. Representing the meaning of mathematical operations and the processes of calculation by using concrete objects, words, mathematical expressions, and diagrams.
   c. Comparing objects from the surroundings in terms of their lengths, areas, and volumes either directly or by using another object as a reference.
   d. Identifying various geometric figures from their surroundings, and composing and decomposing geometric figures using concrete objects.
   e. Writing mathematical expressions to represent quantities in concrete situations and interpreting mathematical expressions in context.

[Terms / Symbols]
>   ones place
>   tens place
>   +
>   −
>   =

[Grade 2]

1. Objectives

    (1) Through activities such as those involving concrete objects, students will enrich their number sense. They will deepen their understanding of the meaning and the representations of numbers, addition and subtraction operations, and applications. Students will understand the meaning of multiplication, investigate ways to carry out calculations and develop the ability to use them appropriately.

    (2) Through activities such as those involving concrete objects, students will understand the process of measuring and the units for length and volume. They will further enrich their measurement sense.

    (3) Through activities using concrete objects, students will be able to understand geometric figures such as triangles and quadrilaterals, and enrich their sense of geometry.

    (4) Through activities using concrete objects, students will be able to represent numbers, quantities and their relationships by using words, numbers and mathematical expressions as well as diagrams, tables and graphs, and will be able to interpret such representations.

2. Content

    A. Numbers and Calculations

    (1) Students will extend their abilities to use numbers by fostering an understanding of the meaning and representation of numbers.
    a. To count objects by rearranging them into groups of the same size or by classifying them.
    b. To understand the base-10 numeration system so that they can write and compare numbers up to 4-digit numbers.
    c. To understand relative size of numbers by regarding 10 or 100 as a unit.
    d. To consider a number in relation to other numbers in ways such as regarding it as a product of other numbers.
    e. To become aware of simple fractions such as $\frac{1}{2}$ and $\frac{1}{4}$.

    (2) Students will deepen their understanding of addition and subtraction and further develop their ability to use those operations.
    a. To understand that the addition and subtraction of 2-digit numbers are based on the basic addition and subtraction of single-digit numbers by exploring the ways to calculate the sums of 2-digit numbers and the differences, which are their inverse, and to be able to add and subtract accurately and reliably; to understand how to add and subtract using algorithms.
    b. To explore ways to add and subtract simple 3-digit numbers.
    c. To investigate properties of addition and subtraction and to utilize those properties as they explore the ways to calculate and check their answers.

(3) Students will understand the meaning of multiplication and be able to use it appropriately.
   a. To become aware of situations in which multiplication may be applied.
   b. To investigate simple properties of multiplication and to utilize those properties as they study basic multiplication facts and check the answers of calculations.
   c. To learn the multiplication table and to be able to correctly multiply 1-digit numbers.
   d. To explore the ways to calculate the simple multiplication of a 2-digit number and a 1-digit number.

B. Quantities and Measurements

(1) Students will understand the meaning and units of linear measurement, and they will be able to measure length.
   a. To become aware of units of length - millimeter (*mm*), centimeter (*cm*) and meter (*m*).

(2) Students will understand the meaning and units of volume, and they will be able to measure volume.
   a. To become aware of units of volume - milliliter (*ml*), deciliter (*dl*), and liter (*l*).

(3) Students will understand elapsed time and be able to use it appropriately.
   a. To become aware of days, hours, and minutes, and to understand the relationships among them.

C. Geometric Figures

(1) Students will be able to understand geometric figures by paying attention to the elements that compose the geometric figures through observing shapes of and composing shapes with objects.
   a. To recognize triangles and quadrilaterals.
   b. To recognize squares, rectangles, and right triangles.
   c. To recognize objects that have the shape of a box.

D. Quantitative Relations

(1) Students will be able to understand the mutual relationship between addition and subtraction.

(2) Students will be able to represent the cases in which multiplication may be applied by using mathematical expressions and will be able to interpret those expressions.

(3) Students will be able to organize numbers and quantities from their everyday lives, represent them by using simple tables and graphs, and interpret such representations.

[Mathematical Activities]

(1) For the contents of "A: Number and Operations," "B: Quantities and Measurement," "C: Geometric Figures," and "D: Quantitative Relations," instruction should incorporate mathematical activities such as those listed below.
   a. Identifying situations in everyday life where whole numbers are being used.
   b. Identifying patterns and properties of multiplication through the development and study of the multiplication table.
   c. Estimating the length and volume of objects in their surroundings, as well as measuring objects using standard units.
   d. Drawing and making squares, rectangles, and right triangles, as well as using those figures in tiling.
   e. Representing and explaining the relationship between addition and subtraction.

[Terms / Symbols]

     unit

     straight line

     right angle

     vertex

     side

     face

     ×

     <

     >

3. Remarks Concerning Content

  (1) As for the content A-(1), ten thousand (*man* in Japanese) should be included.

  (2) As for the content A-(2) and D-(1), ( ) and □ may be used when necessary.

  (3) As for the content A-(2)-c, commutative and associative properties should be taught.

  (4) As for the content A-(3)-b, the pattern of an increase in product when the multiplier increases by one and the commutative property of multiplication should be taught.

[Grade 3]

1. Objectives

  (1) Students will be able to use addition and subtraction appropriately. They will further deepen their understanding of multiplication and use it appropriately. They will also understand the meaning of division, investigating ways to carry out calculations as well as developing the ability to use them appropriately. Furthermore, they will understand the meaning and the representations of decimal numbers and fractions.

  (2) Students will understand the process of measuring and the units for length, weight, and elapsed time.

  (3) Students will be able to understand geometric figures such as isosceles triangles and equilateral triangles by paying attention to the elements that compose geometric figures.

  (4) Students will be able to represent and interpret numbers, quantities, and their relationships by using words, numbers, and mathematical expressions as well as diagrams, tables, and graphs.

2. Content

  A. Numbers and Calculations

  (1) Further extend students' ability to use numbers by deepening their understanding of the written numeration system.

    a. To become aware of the units in the period of ten-thousands ("*man*" in Japanese)

    b. To become aware of numbers that are 10 and 100 times as much/many as or $\frac{1}{10}$ of another number, and to be able to represent those numbers.

    c. To deepen their understanding of relative size of numbers

(2) Students will be able to add and subtract accurately and reliably and will further enhance their ability to use those operations appropriately.

    a. To explore ways to add and subtract 3- and 4-digit numbers and to understand that those calculations are based on addition and subtraction of 1- and 2-digit numbers; to understand how to add and subtract using algorithms.

    b. To add and subtract accurately and reliably, and to use those operations in different situations appropriately.

    c. To explore properties of addition and subtraction and use them when considering ways of adding, subtracting, or checking answers.

(3) Students will extend their understanding of multiplication. They will be able to multiply accurately and reliably and use the operation in different situations appropriately.

    a. To understand that multiplication of 2- and 3-digit numbers by 1- or 2-digit numbers is based on basic multiplication facts by exploring ways to complete the calculations; to understand how to multiply using an algorithm.

    b. To be able to multiply accurately and reliably, and to be able to use multiplication appropriately.

    c. To explore properties of multiplication and use them when considering ways of multiplying or checking answers.

(4) Students will understand the meaning of division and be able to use it appropriately.

    a. To become aware of different situations in which division is used; to become aware of remainders.

    b. To understand the relationships of division to multiplication and subtraction.

    c. To calculate accurately and reliably when both the divisors and the quotients are 1-digit numbers.

    d. To explore the simple case of division with 1-digit divisors and 2-digit quotients.

(5) Students will understand the meaning and representation of decimal numbers.

    a. To use decimal numbers in expressing quantities that are less than 1; to understand the place value of tenths and be able to write decimal numbers to the tenths place.

    b. To understand the meaning of addition and subtraction of decimal numbers through the tenths place; to explore ways to add and subtract decimal numbers and be able to calculate sums and differences through the tenths place.

(6) Students will understand the meaning and representation of fractions.

    a. To understand that fractions are used to express an amount obtained as a result of equal partitioning and are used to express quantities less than 1; to understand fraction notation.

    b. To become aware that a fraction can be considered as a collection of unit fractions.

    c. To explore addition and subtraction of fractions in simple cases and understand their meaning.

(7) Students will become aware of how numbers are represented on the abacus ("*soroban*" in Japanese) and use it to calculate simple addition and subtraction.

    a. To become aware of how to represent numbers on the abacus.

    b. To become aware of how to add and subtract using the abacus.

B. Quantities and Measurements

(1) Students will further deepen their understanding of length. Moreover, they will understand the meaning and units of weight measurement and will be able to measure weights of objects.
    a. To become aware of the linear unit - kilometer (*km*).
    b. To become aware of the units of weight - gram (*g*) and kilogram (*kg*).

(2) Students will be able to estimate length and weight. Moreover, they will be able to select appropriate units and instruments to measure length and weight.

(3) Students will understand elapsed time.
    a. To become aware of the unit of time - second.
    b. To be able to determine clock time and elapsed time, which are relevant in their daily lives.

C. Geometric Figures

(1) Students will be able to understand geometric figures by paying attention to the elements that compose them through observing and composing geometric figures.
    a. To recognize isosceles triangles and equilateral triangles.
    b. To recognize angles.
    c. To recognize circles and spheres; to recognize the center, the radius, and the diameters of a circle.

D. Quantitative Relations

(1) Students will be able to represent the cases in which division may be applied by using mathematical expressions and will be able to interpret such expressions.

(2) Students will understand mathematical expressions that represent relationships of quantities and will be able to use them.
    a. To represent quantitative relationships using mathematical expressions and to recognize relationships among mathematical expressions and diagrams.
    b. To represent quantitative relationships in mathematical expressions by using □ and to investigate mathematical expressions by substituting numbers for the □.

(3) Students will be able to gather, sort, and organize data, clearly represent the data in tables and graphs, and interpret tables and graphs.
    a. To become aware of how to interpret and draw bar graphs.

[Mathematical Activities]

(1) For the contents of "A: Number and Operations," "B: Quantities and Measurement," "C: Geometric Figures," and "D: Quantitative Relations," instruction should incorporate mathematical activities such as those listed below.
    a. Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams.
    b. Representing and comparing decimal numbers and fractions using concrete objects, diagrams, and number lines.
    c. Investigating the relationships among the units within each attribute (length, volume, and weight).
    d. Constructing isosceles and equilateral triangles using compass and ruler.
    e. Organizing data by dates, locations, etc., and representing data in tables.

[Terms / Symbols]

      sign of equality

      sign of inequality

      decimal point

      place of $\frac{1}{10}$ (tenth)

      number line

      denominator

      numerator

      ÷

3. Remarks Concerning Content

    (1) As for the content A-(1), one hundred million (*oku* in Japanese) should be taught.

    (2) As for the content A-(2), (3) students should be able to calculate simple calculations mentally.

    (3) As for the content A-(2)-c, commutative and associative properties should be taught.

    (4) As for the content A-(3), calculations that involve 0 in multiplier or/and multiplicand should be taught.

    (5) As for the content A-(3)-c, commutative, associative, and distributive properties should be taught.

    (6) As for the content A-(5) and (6), decimal numbers such as 0.1 should be taught in relation to fractions such as $\frac{1}{10}$ by using number lines.

    (7) As for the content B-(1), the unit ton (*t*) should also be included.

[Grade 4]

1. Objectives

    (1) Students will deepen their understanding of division and use it appropriately. Students will further deepen their understanding of the meaning and the representations of decimal numbers and fractions, as well as addition and subtraction of decimal numbers and fractions, investigating ways to carry out calculations. Moreover, students will understand approximated numbers and use them appropriately.

    (2) Students will understand the process of measuring and the units of measurement for area and will be able to determine the area of geometric figures. They will also understand the process of measuring angles and the unit of measurement for angles.

    (3) Students will be able to understand plane figures, such as parallelograms and rhombuses, and solid figures, such as rectangular prisms, by paying attention to the elements that compose the geometric figures and the relationships of those elements.

    (4) Students will be able to represent and investigate numbers, quantities, and their relationships by using words, numbers, and mathematical expressions as well as diagrams, tables, and graphs.

2. Content

   A. Numbers and Calculations

   (1) Students will deepen their understanding of the base-10 numeration system.
      a. To understand the units in the periods of "hundred million" ("*oku*" in Japanese) and trillion ("*cho*" in Japanese); to summarize the base-10 numeration system.

   (2) Students will understand approximated numbers and use them appropriately.
      a. To become aware of situations where approximate numbers are appropriate.
      b. To become aware of rounding as a way to approximate numbers.
      c. To estimate the results of computations as appropriate.

   (3) Students will deepen their understanding of division of whole numbers and will be able to calculate accurately and reliably. They will extend their ability to utilize division in different situations appropriately.
      a. To understand that division of 2- and 3-digit numbers by 1- and 2-digit numbers is based on basic division facts, by exploring ways to complete the calculations; to understand how to divide using an algorithm.
      b. To be able to divide accurately and reliably and to be able to use division appropriately.
      c. To explore the relationship among dividends, divisors, quotients and remainders, and to summarize this relationship in the following expression: Dividend = divisor × quotient + remainder
      d. To explore properties of division and use these properties to think about ways of dividing or checking answers.

   (4) Students will extend their ability to use whole number computations in appropriate situations by further developing their computation skills.

   (5) Students will deepen their understanding of addition and subtraction of decimal numbers. Moreover, they will understand the meaning of multiplying and dividing decimal numbers and use the operations in appropriate situations.
      a. To become aware of the fact that written decimal numbers are structured in the same manner as whole numbers and to deepen their understanding of relative sizes of numbers.
      b. To explore ways to calculate the sum and the difference of decimal numbers and be able to calculate accurately.
      c. To explore ways to multiply and divide decimal numbers by whole numbers and be able to multiply and divide decimal numbers by whole numbers accurately and reliably.

   (6) Students will deepen their understanding of fractions. They will also understand the meaning of addition and subtraction of fractions with like denominators; and they will be able to use addition and subtraction of fractions in appropriate situations.
      a. To notice that some fractions represent the same numbers in simple cases.
      b. To explore ways to add and subtract fractions with like denominators; to be able to add and subtract fractions with like denominators.

   (7) Students will be able to add and subtract using the abacus.

B. Quantities and Measurements

(1) Students will understand the meaning and the units for measuring area, and they will be able to determine area by calculation.
   a. To become aware of the units of area - square centimeter (cm^2), square meter (m^2), and square kilometer (km^2)
   b. To investigate how to determine the area of rectangles and squares.

(2) Students will understand the meaning and the unit for measuring angles, and they will be able to measure angles.
   a. To conceive the size of an angle as the amount of turn.
   b. To become aware of the unit of angle measurement - degrees (˚).

C. Geometric Figures

(1) Students will deepen their understanding of geometric figures by paying attention to the elements that compose them and their positional relationships, through observing and composing geometric figures.
   a. To understand relationships such as parallelism and perpendicularity of lines.
   b. To recognize parallelograms, trapezoids and rhombuses.

(2) Students will be able to understand solid figures through observing and composing solid figures.
   a. To recognize cubes and rectangular prisms.
   b. To understand parallelism and perpendicularity of lines and planes in connection with a rectangular prism.

(3) To understand how to represent the position of an object in space.

D. Quantitative Relations

(1) Students will be able to represent and investigate the relationship between two quantities as they vary simultaneously.
   a. To represent how the quantities vary on a broken-line graph and to interpret the features of their variation.

(2) Students will understand mathematical expressions that represent quantitative relationships and be able to use them.
   a. To understand mathematical expressions that contain some of the four basic operations and parentheses ( ), and to be able to calculate them.
   b. To understand the idea of a formula and to use formulas.
   c. To represent numbers and quantities in mathematical expressions by using □ and △, and to investigate the relationships by substituting numbers for □ and △.

(3) Students will deepen their understanding of the properties of the four basic operations.
   a. To summarize commutative, associative, and distributive properties.

(4) Students will be able to gather, sort, and organize data according to their purposes, to clearly represent data in tables and graphs, and to investigate features of data.
   a. To investigate data by sorting and organizing from two viewpoints.
   b. To become aware of how to interpret and draw broken-line graphs.

[Mathematical Activities]

(1) For the contents of "A: Number and Operations," "B: Quantities and Measurement," "C: Geometric Figures," and "D: Quantitative Relations," instruction should incorporate mathematical activities such as those listed below.
   a. Making appropriate estimates for computation, and evaluating the appropriateness of the results of computation.
   b. Investigating and explaining ways to determine the area of figures composed of rectangles using concrete objects, words, numbers, mathematical expressions, and diagrams.
   c. Measuring the area of objects in their surroundings.
   d. Investigating properties of geometric figures by tiling the plane with parallelograms, rhombuses, and trapezoids.
   e. Identifying quantities in their surroundings that vary simultaneously, and investigating their relationships by representing them in tables and graphs.

[Terms / Symbols]
   sum
   difference
   product
   quotient
   more than or equal to
   less than or equal to
   less than
   proper fraction
   improper fraction
   mixed number
   parallel
   perpendicular
   diagonal line
   plane

3. Remarks Concerning Content

   (1) As for the content A-(1), the cases in which large numbers are represented by putting commas after every three digits should be taught.

   (2) As for the content A-(2)-c, (3) and (4) students should be able to complete simple calculations mentally. Furthermore, students should be able to use mental calculations while carrying out algorithms and estimating.

   (3) As for the content A-(3)-d, the property of division that the quotient will remain the same when both the dividend and the divisor are multiplied or divided by the same number should be taught.

   (4) As for the content A-(5) -c, include cases in which a whole number is divided by a whole number and the quotient is a decimal.

   (5) As for the content B-(1)-a, include the units acre (*a*) and hectare (*ha*).

   (6) As for the content C-(2), drawing sketches and nets of shapes should be taught.

   (7) As for the content D-(4), ways to check for omissions and duplications when investigating data should be taught.

[Grade 5]

1. Objectives

    (1) Students will deepen their understanding of the properties of whole numbers. They will also deepen their understanding of the meaning of multiplication and division of decimal numbers as well as addition and subtraction of fractions, investigating ways to carry out calculations, and developing the ability to use operations and properties appropriately.

    (2) Students will be able to determine the area of polygons, such as triangles and parallelograms, and the volume of rectangular prisms. Students will understand the average of measured quantities and the ratio of two unlike quantities.

    (3) Students will deepen their understanding of plane figures and be able to understand solid figures, such as prisms.

    (4) Students will consider quantitative relationships and investigate their features by using circle graphs and percentage bar graphs.

2. Content

    A. Numbers and Calculations

    (1) Students will deepen their understanding of properties of whole numbers
        a. To become aware of the fact that whole numbers may be classified as even and odd numbers
        b. To become aware of factors and multiples

    (2) Students will deepen their understanding of the base-10 numeration system for whole numbers and decimal numbers, and be able to use this understanding effectively in computation and for other purposes.
        a. To determine numbers that are 10, 100, $\frac{1}{10}$, and $\frac{1}{100}$ times as much as the original number

    (3) Students will deepen their understanding of multiplication and division of decimal numbers, and be able to use them appropriately.
        a. To understand the meaning of multiplying or dividing by decimal numbers through exploring the ways to multiply or divide decimal numbers by whole numbers.
        b. To explore ways to multiply and divide by decimal numbers, and be able to calculate accurately; to understand the size of remainders.
        c. To understand that the same properties of multiplication and division for whole numbers will apply to decimal multiplication and division.

    (4) Students will deepen their understanding of fractions. They will understand the meaning of addition and subtraction of fractions with unlike denominators and be able to calculate accurately.
        a. To be able to write decimal numbers as fractions and vice versa.
        b. To understand that the result of dividing a whole number by another whole number may be expressed as a fraction.
        c. To understand that when the numerator and the denominator are multiplied or divided by the same number, the size of a fraction remains the same.
        d. To consider the size relationship of fractions and summarize the ways to compare fractions.
        e. To explore ways to add and subtract fractions with unlike denominators and be able to calculate accurately.

f.   To understand the meaning of multiplying and dividing fractions by whole numbers; to explore ways to multiply and divide fractions by whole numbers and be able to calculate accurately.

B. Quantities and Measurements

(1) Students will be able to calculate the area of various figures.
a.   To explore ways to calculate the area of triangles, parallelograms, rhombuses, and trapezoids.

(2) Students will understand the meaning and recognize the units for measuring volume; and they will be able to calculate volume.
a.   To become aware of the units of volume - cubic centimeter (cm^3) and cubic meter (m^3).
b.   To explore ways to determine the volume of cubes and rectangular prisms.

(3) Students will understand the measurements of various quantities.
a.   To become aware of average measured quantities.

(4) Students will understand how to represent and compare quantities that are obtained as a ratio of two unlike quantities.
a.   To become aware of per-unit quantities.

C. Geometric Figures

(1) Students will deepen their understanding of geometric figures by observing and composing geometric figures.
a.   To recognize polygons and regular polygons.
b.   To understand congruence of geometric figures.
c.   To investigate and construct geometric figures by finding the properties of fundamental figures.
d.   To understand the ratio of the circumference of a circle to its diameter.

(2) Students will understand solid figures through observing and composing solid figures.
a.   To recognize prisms and cylinders

D. Quantitative Relations

(1) Students will use tables to consider the relationships between two quantities as the quantities vary simultaneously.
a.   To recognize proportional relationships in simple cases.

(2) Students will deepen their understanding of mathematical expressions that represent quantitative relationships. They will be able to recognize the correspondence between two quantities and the aspect of variation in a quantitative relationship that can be represented by a simple mathematical expression.

(3) Students will understand percentage.

(4) Students will be able to gather, sort, and organize data according to their purposes, to represent data in circle graphs and percentage bar graphs, and to investigate the features of data.

[Mathematical Activities]

(1) For the contents of "A: Number and Operations," "B: Quantities and Measurement," "C: Geometric Figures," and "D: Quantitative Relations," instruction should incorporate mathematical activities such as those listed below.

    a. Investigating and explaining the meaning and the ways of calculating with decimal numbers, using words, numbers, mathematical expressions, diagrams, and number lines.

    b. Investigating and explaining the ways of determining the area of triangles, parallelograms, rhombuses, and trapezoids, using concrete materials, words, numbers, mathematical expressions and diagrams.

    c. Drawing and making congruent figures.

    d. Determining inductively the fact that the sum of angles in a triangle is 180 degrees and explaining their thinking. Determining deductively the fact that the sum of angles in a quadrilateral is 360 degrees and explaining their thinking.

    e. Selecting and using appropriate tables and graphs to display data purposefully.

[Terms / Symbols]
    greatest common divisor
    least common multiple
    finding a common denominator
    simplifying a fraction
    base
    lateral surface
    proportionality
    %

3. Remarks Concerning Content

    (1) As for the content A-(1) -b, the greatest common divisor and the least common multiple should be taught in concrete settings; avoid putting too much weight on finding these procedurally. Also, prime numbers should be discussed in the process of investigating divisors.

    (2) As for the content C-(1) -d, 3.14 is used as the ratio of the circumference of a circle to its diameter (pi).

    (3) As for the content C-(2)-a, drawing sketches and nets of shapes should be taught.

    (4) As for the content D-(3), representing ratios by using *buai* (the ratio based on 10 percent in Japanese) should be taught.

[Grade 6]

1. Objectives

    (1) Students will deepen their understanding of the meaning of multiplication and division of fractions, investigate the ways to carry out the calculations, and use them appropriately.

    (2) Students will be able to determine the area of circles as well as the volume of prisms. They will also understand and be able to determine speed.

    (3) Students will understand scale drawings, symmetric figures, and deepen their understanding of geometric figures.

    (4) Students will understand ratio and direct proportion, use their understanding to consider quantitative relationships, and represent the relationships in mathematical expressions by using letters. Furthermore, students will investigate the distribution of data and be able to consider them statistically.

2. Content

A. Numbers and Calculations

(1) Students will understand multiplication and division of fractions, and be able to use them appropriately.
   a. To understand the meaning of multiplying and dividing by fractions based on multiplication and division by whole numbers and decimal numbers.
   b. To explore ways to calculate multiplication and division of fractions, and to be able to calculate accurately.
   c. To understand that the same properties of operations that are true with whole numbers still apply when numbers become fractions.

(2) Students will extend their ability to use computations with decimal numbers and fractions in appropriate situations by further consolidating their computation skills.

B. Quantities and Measurements

(1) Students will be able to estimate the area of shapes in their surroundings by approximating them with familiar figures.

(2) Students will be able to determine the area of figures by calculation.
   a. To explore ways to determine the area of circles.

(3) Students will be able to determine the volume of figures by calculation.
   a. To explore ways to determine the volume of prisms and cylinders.

(4) Students will understand and determine speed.

(5) Students will understand the system of the metric units.

C. Geometric Figures

(1) Students will deepen their understanding of geometric figures through observing and composing geometric figures.
   a. To understand scale drawings.
   b. To understand symmetric figures.

D. Quantitative Relations

(1) Students will understand ratio.

(2) Students will be able to consider the relationship between two quantities varying simultaneously.
   a. To understand proportional relationships and to investigate their characteristics by using mathematical expressions, tables, and graphs.
   b. To solve problems by using proportional relationships.
   c. To become aware of inversely proportional relationships.

(3) Students will deepen their understanding of mathematical expressions that represent quantitative relationships and be able to use them.
   a. To represent numbers and quantities in mathematical expressions by using letters such as a, x, etc. instead of using words, $\square$ and $\triangle$, and to investigate the relationships by substituting numbers for the letters.

(4) Students will investigate the distribution of data, and be able to consider and represent the data statistically.

a. To become aware of the average of data.
b. To become aware of the table and the graph to represent frequency distribution.

(5) Students will be able to systematically analyze all possible outcomes for actual events.

[Mathematical Activities]

(1) For the contents of "A: Number and Operations," "B: Quantities and Measurement," "C: Geometric Figures," and "D: Quantitative Relations," instruction should incorporate mathematical activities such as those listed below.
  a. Investigating and explaining the ways of carrying out fraction computations using words, numbers, mathematical expressions, diagrams and number lines.
  b. Identifying units used in their surroundings and exploring how they relate to the units they studied previously.
  c. Identifying scaled drawings and symmetric figures in their surroundings.
  d. Identifying quantities in their surroundings that are proportional, and solving problems using proportionality.

[Terms / Symbols]
  line symmetry
  point symmetry
  :

3. Remarks Concerning Content

(1) As for the content A-(1), looking at division as multiplication by using reciprocal numbers, and combining multiplication and division involving several numbers (including whole numbers and decimal numbers) in a calculation of fractions should be taught.

(2) As for the content B-(2)-a, 3.14 is used as the ratio of the circumference of a circle to its diameter (pi).

III. THE CONSTRUCTION OF TEACHING PLANS AND REMARKS CONCERNING CONTENT IN EACH GRADE

1. When constructing teaching plans, the following points should be considered :

(1) The content of each grade should be taught continuously in the subsequent grades, as necessary. In order to master and maintain basic skills concerning numbers, quantities, and geometric figures, opportunities for practice should be provided intentionally as the occasion demands. Furthermore, teaching through appropriate repetition may be necessary to maintain content coherence throughout the grades.

(2) The content in each grade should be taught in relation to other domains, A, B, C, and D.

(3) Mathematical activities play important roles in learning mathematics, such as: securely acquiring basic and fundamental knowledge and skills; fostering the abilities of logical thinking, reasoning, and representing; and experiencing the joy and the meaningfulness of learning mathematics. Therefore, the content of each grade should be taught through mathematical activities.

(4) Based on the aim of moral education (Chapter 1, I (2) and III (1)), the content of moral education must be taught in mathematics appropriately by considering its relationship to the teaching in moral education class.

2. As to the content of II, the following points should be considered:

   (1) Teachers should help students to develop a rich sense of numbers, quantities, and geometric figures; to be able to grasp rough size and approximate shape to make adequate decisions; and to be able to think about how to process efficiently.

   (2) In order to foster students' abilities of logical thinking, reasoning, and representing, teachers should include learning activities, such as: thinking and explaining using words, numbers, mathematical expressions, diagrams, tables, and graphs, and exchanging ideas with each other by representing these ideas throughout the content at each grade level.

   (3) "Terms and Symbols" in each grade level are given to clarify the depth and breadth of the content in the specific grade level. The terms and symbols should be taught in conjunction with the appropriate contents; and students should understand the merit of using terms and symbols in their reasoning and representations.

   (4) Teachers should help students to not only master calculations by using standard algorithms but also to be able to estimate the result of calculations and make adequate decisions about methods and results of calculations. Furthermore, when teaching the contents of (A) in lower grades, teachers should help students adequately use "*soroban*" and other concrete manipulatives to deepen students' understanding of the meanings of numbers and calculations.

   (5) In order to develop a rich sense of numbers, quantities, and geometric figures, and to foster students' ability to use tables and graphs to represent ideas, teachers should consider using computers whenever appropriate or necessary.

MATHEMATICS FOR LOWER SECONDARY SCHOOL

I. OBJECTIVES

Through mathematical activities, students will (1) deepen their understanding of principles and rules about numbers, quantities, and geometric figures, (2) develop fluency for mathematical representations and procedures, and (3) foster the disposition to utilize their mathematical understanding, representations and procedures in reasoning and making judgments.

II. OBJECTIVES AND CONTENTS IN EACH GRADE

[Lower Secondary Grade 1 (Grade 7)]

1. Objectives

    (1) Students will deepen their understanding of the concepts of numbers by expanding the types of numbers to include positive and negative numbers. In addition, students will understand the necessity for and the meaning of letter symbols and equations they will cultivate the ability to represent patterns and relationships of numbers and quantities generally and concisely, and to process these symbols, equations, and relationships accordingly. They will also develop the ability to utilize linear equations with one variable.

    (2) Through observation, manipulation and experimentation with plane and solid figures, students will develop further their ability to intuitively observe and reason about geometric patterns and relationships, as well as develop their ability to logically reason and represent those patterns and relationships.

    (3) Through investigations of actual phenomena, students will deepen their understanding of proportional and inversely proportional relationships.

    (4) Students will gather and organize data purposefully, and develop the ability to interpret any trends of the data.

2. Contents

    ### A. Numbers and Mathematical Expressions

    (1) Students will understand positive and negative numbers through their experiences in everyday situations. They will develop the fluency to calculate with positive and negative numbers, as well as develop their ability to reason about and represent positive and negative numbers.

       a. To understand the need for and the meaning of positive and negative numbers.
       b. To understand the meaning of the four arithmetic operations with positive and negative numbers by relating them to what they studied in the elementary grades.
       c. To be able to calculate with positive and negative numbers.
       d. To represent and process real-life situations using positive and negative numbers.

    (2) Students will be able to represent relationships and rules involving numbers and quantities using mathematical expressions with letters. Students will also foster their ability to interpret mathematical expressions and to operate with mathematical expressions.

       a. To understand the need for and meaning of using letters in mathematical expressions.
       b. To be able to multiply and divide mathematical expressions with letters.

    c. To be able to add and subtract simple linear expressions.

    d. To understand that relationships and rules involving numbers and quantities may be represented in mathematical expressions with letters, and to be able to write and interpret mathematical expressions with letters.

(3) Students will understand equations and be able to use linear equations in their inquiries.

    a. To understand the need for and the meaning of equations, and to understand the meaning of letters in equations and solutions of equations.

    b. To understand that equations may be solved by using the properties of equality.

    c. To be able to solve simple linear equations, and to use linear equations in real-life situations.

[Terms and Symbols]

    natural numbers
    positive and negative signs
    absolute values
    terms
    coefficients
    transposing terms
    $\leqq$
    $\geqq$

B. Geometric Figures

(1) Students will deepen their understanding of plane figures by constructing and exploring relationships among plane figures and, thus, gaining foresight through observation, manipulation and experimentation. Also, students will cultivate their ability to explore and represent plane figures logically.

    a. To understand the basic construction, such as angle bisector, perpendicular bisector of a segment, and perpendicular line; to be able to utilize geometric construction in concrete situations.

    b. To understand translation, reflection and rotation; to explore relationships between two geometric figures.

(2) Students will deepen their understanding of solid figures through observation, manipulation and experimentation. Moreover, students will further their ability to measure geometric figures.

    a. To become aware of spatial relationships among lines and planes in space.

    b. To perceive solid figures as being composed of motions of plane figures along a line or a plane; to represent solid figures on a plane; and to identify properties of solid figures from their representations in a plane.

    c. To be able to calculate the arc length of a sector and its area; to be able to calculate the surface area and the volume of pyramids and spheres.

[Terms and Symbols]
     arc
     secant
     solids of revolution
     skew
     π
     //
     ⊥
     ∠
     △

C. Functions

(1) Students will deepen their understanding of direct and inverse proportional relationships by examining correspondences and variations of two quantities in real-life situations; students will foster their ability to identify, represent, and examine functional relationships.
    a. To understand the meaning of functional relationships.
    b. To understand the meaning of direct and inverse proportional relationships.
    c. To understand the meaning of the coordinates.
    d. To understand the characteristics of direct and inverse proportional relationships and be able to represent the relationships using tables, mathematical expressions and graphs.
    e. To be able to identify direct or inverse proportional relationships in concrete situations and be able to explain them.

[Terms and Symbols]
     functions
     variables
     domain

D. Data Handling

(1) Students will gather data purposefully and organize the data in tables and graphs, utilizing tools such as computers; students will be able to interpret the trends by examining the representative values and the measures of dispersion.
    a. To understand the need for and the meaning of histograms and representative values.
    b. To identify and explain trends by using representative values and histograms.

[Terms and Symbols]
     mean
     median
     mode
     relative frequency
     range
     rank

[Mathematical activities]

(1) As students study the contents of the strands ("A. Numbers and Operations," "B. Geometric Figures," "C. Functions," and "D. Data Handling") and their mutual relationships, opportunities to engage in mathematical activities, such as the following, should be provided.

    a. Identifying and extending properties of numbers and geometric figures based on previously learned mathematics.

    b. Applying mathematics in everyday and community situations.

    c. Communicating and explaining ideas clearly and logically by using mathematical representations.

3. Remarks Concerning Content

(1) In relation to "A Numbers and Mathematical Expressions" Item (1), sets of numbers and the possibility of the four arithmetic operations should be discussed.

(2) In relation to the content A-(2)-e, the idea of representing the size relationships using inequalities should be discussed.

(3) In relation to the content A-(3)-c, solving simple proportions should be discussed.

(4) In relation to the content B-(1)-a, the idea that a tangent line to a circle is perpendicular to a radius at the point of tangency should be discussed.

(5) As for the content B-(2)-b, sketches, nets, and perspective drawings should be discussed.

(6) In relation to the content D-(1), errors, approximate values, and scientific notation $a \times 10^n$ should be discussed.

[Lower Secondary Grade 2 (Grade 8)]

1. Objectives

(1) Students will develop the ability to calculate and modify mathematical expressions with letters; and they will understand and foster their ability to use systems of linear equations with 2 variables.

(2) Students will deepen their understanding of properties of basic plane figures through such activities as observation, manipulation and experimentation while understanding the need for and meaning of mathematical arguments; students will develop their ability for logical inquiry and representations.

(3) Students will understand linear functions through exploration of concrete phenomena while developing their ability to identify, represent and utilize functional relationships in inquiries.

(4) Students will understand and foster the ability to utilize probability through investigations of uncertain phenomena.

2. Contents

A. Numbers and Mathematical Expressions

(1) Students will be able to identify relationships of quantities in concrete phenomena and will be able to represent and interpret these relationships in mathematical expressions with letters. Students will be fluent with the basic arithmetic of literal expressions.

    a. To be able to add and subtract polynomials; to be able to multiply and divide monomials.

    b. To understand that mathematical expressions with letters may be used to capture and explain relationships among quantities.

    c. To be able to modify mathematical expressions with letters purposefully.

(2) Students will understand systems of linear equations in 2 variables, and they will be able to use them in inquiry.
    a. To understand the meaning of linear equations with 2 variables and their solutions.
    b. To understand the need for and the meaning of systems of linear equations with 2 variables, and their solutions.
    c. To be able to solve simple systems of linear equations with 2 variables, and to be able to use them in concrete situations.

[Terms and Symbols]
    like terms

B. Geometric Figures

(1) Students will be able to identify properties of basic plane figures through activities such as observation, manipulation and experimentation, and verification using the properties of parallel lines.
    a. To understand properties of parallel lines and angles, and use those properties to verify and explain other geometric properties.
    b. To understand that properties of angles in polygons may be derived from the properties of parallel lines and those of angles in triangles.

(2) Students will understand congruence of geometric figures, thus deepening their perspective of geometric figures. Students will develop the ability for logical analysis and representation by verifying properties of geometric figures using ideas such as congruence conditions of triangles.
    a. To understand the meaning of congruence of geometric figures, and to understand the congruence conditions of triangles.
    b. To understand the need for and the meaning of proofs, and to understand the methods of proof.
    c. To logically verify basic properties of triangles and parallelograms using the congruence conditions of triangles, and to identify new properties by reading proofs.

[Terms and Symbols]
    vertical angles
    internal angles
    external angles
    definition
    proof
    converse
    $\equiv$

C. Functions

(1) Students will understand linear functions by analyzing the change and correspondences of two quantities identified in concrete situations, while developing their ability to represent and analyze functional relationships.
    a. To know that some phenomena may be characterized by linear functions.
    b. To understand and relate representations of linear functions, i.e., tables, mathematical expressions, and graphs.

    c. To perceive linear equations with 2 variables as representations of functions.

    d. To grasp and explain concrete phenomena using linear functions.

[Terms and Symbols]

    rate of change

    slope

D. Data Handling

(1) Students will understand probability by observing and experimenting with uncertain phenomena, and they will be able to use probability in analysis and representation.

    a. To understand the need for and the meaning of probability, and to be able to determine probability in simple cases.

    b. To grasp and explain uncertain phenomena.

[Mathematical Activities]

(1) As students study the contents of the strands ("A. Numbers and Operations," "B. Geometric Figures," "C. Functions," and "D. Data Handling") and their mutual relationships, opportunities to engage in mathematical activities such as the following should be provided.

    a. Identifying and extending properties of numbers and geometric figures based on previously learned mathematics

    b. Applying mathematics in everyday and community situations.

    c. Communicating and explaining ideas clearly and logically by using mathematical representations

3. Remarks Concerning Content

(1) In relationship to the content B-(2)-c, the fact that squares, rhombuses, and rectangles are special cases of parallelograms should be discussed.

[Lower Secondary Grade 3 (Grade 9)]

1. Objectives

(1) Students will deepen their understanding of numbers by understanding square roots of numbers. Furthermore, students will extend their ability to manipulate and calculate mathematical expressions, and they will also understand and develop the ability to use quadratic equations.

(2) Students will understand similarity of geometric figures, the relationship between the inscribed angle and the central angle (Central Angle Theorem), and the Pythagorean Theorem through observation, manipulation and experimentation. Students will extend their ability to use those understanding to explore properties, to analysis of properties of geometric figures, and to measure. Students will further develop their ability to represent and analyze geometric figures logically and with foresight.

(3) Students will understand functions in the form of $y = ax^2$ through explorations of concrete phenomena, while extending their ability to identify, represent and analyze functional relationships.

(4) Students will develop the ability to interpret the trends in a population by selecting and analyzing trends in samples taken from the population.

2. Content

A. Numbers and Mathematical Expressions

(1) Students will understand the square roots of positive whole numbers and use them in representations and explorations.
 a. To understand the need for and the meaning of square roots.
 b. To be able to carry out simple computations involving square roots.
 c. To be able to use square roots to represent and process concrete situations.

(2) Students will be able to expand and factor simple polynomials as well as modifying and interpreting mathematical expressions according to purposes.
 a. To be able to multiply polynomials and monomials, and to be able to divide polynomials by monomials.
 b. To be able to multiply simple linear expressions, and to be able to expand and factor using the following formulas:

$(a+b)^2=a^2+2ab+b^2$

$(a-b)^2=a^2-2ab+b^2$

$(a+b)(a-b)=a^2-b^2$

$(x+b)(x+b)=x^2+(a+b)+ab$

 c. To be able to capture and explain relationships of numbers and quantitative relationships.

(3) Students will understand quadratic equations and be able to use them in explorations.
 a. To understand the need for and the meaning of quadratic equations and their solutions.
 b. To solve quadratic equations by factoring and by completing squares.
 c. To learn the quadratic formula and use the formula to solve quadratic equations.
 d. To use quadratic equations in concrete situations.

[Terms and Symbols]
　　radical sign
　　rational number
　　irrational number
　　factor
　　$\sqrt{}$

B. Geometric Figures

(1) Students will be able to verify properties of geometric figures by using similarity conditions of triangles, while extending their ability to represent and analyze situations logically. Students will be able to use properties of similar figures in explorations.
 a. To understand the meaning of similarity of plane figures and the similarity conditions for triangles.
 b. To logically verify basic properties of geometric figures by utilizing properties such as similarity conditions of triangles.
 c. To identify and verify properties related to parallel lines and ratios of segments.
 d. To understand the meaning of similarity of solids, and to understand the relationships between the scale factor and the ratio of area and the ratio of volume of similar figures.
 e. To be able to use properties of similar figures in concrete situations.

(2) Students will identify and understand the relationship between the inscribed angle and the central angle through activities such as observation, manipulation and experimentation, and will be able to use the relationship in explorations.
    a.   To understand the meaning of the relationship between the inscribed angle and the central angle, and to understand that this relationship can be proved.
    b.   To be able to use the relationship between the inscribed angle and the central angle in concrete situations.

(3) Students will identify and understand the Pythagorean Theorem through activities such as observation, manipulation and experimentation, and will be able to use the theorem in explorations.
    a.   To understand the meaning of the Pythagorean Theorem, and to understand that the theorem may be proved.
    b.   To be able to use the Pythagorean Theorem in concrete situations.

[Terms and Symbols]

∽

C. Functions

(1) Students will understand functions in the form of $y = ax^2$ by analyzing the change and correspondences of two quantities identified in concrete situations, while developing their ability to represent and analyze functional relationships.
    a.   To become aware that some phenomena may be considered using the function of the form, $y = ax^2$.
    b.   To understand and relate representations of functions of the form $y = ax^2$, i.e., tables, mathematical expressions, and graphs.
    c.   To grasp and explain concrete phenomena using functions of the form $y = ax^2$.
    d.   To understand that functional relationships may be found in various phenomena.

D. Data Handling

(1) Students will understand that they can infer the trends in a population by analyzing the trends in samples, perhaps utilizing computers in the process.
    a.   To understand the need for and the meaning of sampling.
    b.   To infer and explain trends in a population by using simple sampling.

[Terms and Symbols]
    census

[Mathematical Activities]

(1) As students study the contents of the strands ("A. Numbers and Operations," "B. Geometric Figures," "C. Functions," and "D. Data Handling") and their mutual relationships, opportunities to engage in mathematical activities such as the following should be provided.
    a.   Identifying and extending properties of numbers and geometric figures based on previously learned mathematics
    b.   Applying mathematics in everyday and community situations.
    c.   Communicating and explaining ideas clearly and logically by using mathematical representations

3. Remarks Concerning Content

   (1) In relationship to the content A-(2), quadratic equations with real number solutions should be discussed.

   (2) As for the content A-(3), quadratic equations with real number solutions should be discussed.

   (3) As for the content A-(3)-b, quadratic equations of the form $ax^2 = b$, where a and b are rational numbers, and $x^2+px+q=0$, where p and q are integers, should be discussed. When factoring to solve quadratic equations, primarily those equations to which the formulas in (A)-2-b may be applied should be discussed. When completing squares to solve quadratic equations, focus primarily on the equations with even numbers as coefficients of x terms.

   (4) In relationship to the content B-(2), the converse of the Central Angle Theorem should be discussed.

III. THE CONSTRUCTION OF TEACHING PLANS AND REMARKS CONCERNING CONTENT IN EACH GRADE

1. As for construction of the teaching plans, teachers should consider the following factors.

   (1) It is acceptable to treat lightly some of the content of a particular grade and discuss the content more extensively in a later grade as long as the change does not negatively influence the achievement of grade level objectives. It is also possible to discuss some topics from later grades in an earlier grade as long as such changes do not go far beyond the earlier grade level objectives.

   (2) To ensure students' learning when new content is introduced, teachers should consider providing opportunities for re-learning by intentionally discussing relevant topics studied previously.

   (3) Based on the aim of moral education (Chapter 1 I (2) and III (1)), the contents of moral education must be taught in mathematics, appropriately, by considering its relationship with the teaching in moral education class.

2. As to the content of II, the following points should be considered:

   (1) "Terms and Symbols" in each grade level are given to clarify the depth and breadth of the content in each specific grade level. Terms and symbols should be taught closely connected to the appropriate contents; and students should understand the merit of using those terms and symbols to communicate their reasoning and representations.

   (2) In teaching the contents discussed above, teachers should consider using abacuses, calculators, computers, and information networks, as necessary and/or appropriate. In particular, teachers should consider using these tools when teaching topics that are related to numerical computations or when teaching through activities such as observation, manipulation, and experimentation.

3. As for mathematical activities, teachers should consider the following factors.

   (1) Opportunities should be provided for students to experience and understand the purposes and necessity for studying mathematics, while enjoying mathematical activities.

   (2) Opportunities should be provided for students to identify a problem on their own, plan a possible solution strategy, carry out the plan, evaluate the results, and improve their strategy.

   (3) Opportunities should be provided for students to share the results of mathematical activities with each other, perhaps through reports that reflect and summarize the activities.

4. The purpose of project-based learning is to facilitate students' engagement in mathematical activities by solving problems that integrate content across the strands in mathematics and/or by connecting mathematical content to other subject areas and/or everyday phenomena, so that students develop their abilities to think logically, make appropriate judgments, and create/use effective representations.