

A Framework for Teaching Mathematics through Problem Solving in Lower Secondary School

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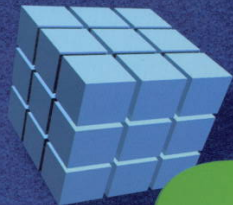
&

National Center for Mathematics Education,
Gothenburg, Sweden

Grade
8

PROBLEM SOLVING EXPERIENCES

MAKING SENSE
OF MATHEMATICS



5.2

$\frac{3}{4}$

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A good math program improves students' abilities to solve problems by —

- developing their ability use problem-solving strategies,
- deepening their understanding of important math concepts,
- developing their ability to estimate quantities and measurements,
- developing their ability to select operations to solve problems,
- improving their ability to communicate what they know,
- encouraging the development of positive attitudes and dispositions toward mathematics.

Aspects of the Framework

- Tasks
- Teacher's role
- Frequency
- Time
- Classroom environment
- Philosophy

Aspects of the Framework

- **Tasks**
- **Teacher's role**
- **Frequency – every day**
- **Time – it depends!**
- **Classroom environment – crucial!**
- **Philosophy – a later discussion**

Types of tasks

- **Strategy problems**
- **Concept problems**
- **Multi-step problems**
- **Algebraic thinking problems**
- **Estimation problems**

Richard works for a locker installation company. After he installs the lockers, he must number the lockers consecutively beginning with 1. The numbers are put on one digit at a time. Richard used 492 digits in all.

How many lockers did he install?

What problem-solving strategies are most likely to be helpful?

What important mathematics concepts are included?

121, 123, 125, 127, 129, 131, 133

Replace two of the numbers in the list above without changing the mean.

Is this a problem?

What kind of problem is it?

A different kind of problem

At a school dance, only some of the students danced. Bengt noticed that for one dance $\frac{2}{3}$ of the boys were dancing with $\frac{3}{5}$ of the girls.

What fraction of the students were dancing?

Deepen number sense & problem-solving flexibility

1. Use drawings or objects
2. Find common numerators
3. Guess and check
4. Write an equation

1. Use drawings or objects

B B B W W W W W
B B B W W W W W
B B B



Total: 9 boys & 10 girls, or 19 students. 6 dancing couples, so 12/19 are dancing..

2. Use common numerators (# of dancing boys & girls is the same, so numerators must be the same)

$$2/3 = 4/6 = 6/9$$

$$3/5 = 6/10$$



12/19 are dancing.

3. **Guess & check** (Pick a “nice” number first)

Suppose there are 60 dancing couples. Then,

$$\left. \begin{array}{l} 2/3b = 60 \text{ and } b = 90 \\ 3/5g = 60 \text{ and } g = 100 \end{array} \right\} 120/190 = 12/19, \text{ so } \underline{12/19} \text{ are dancing.}$$

4. **Write an equation**

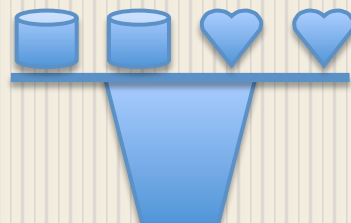
$2/3b = 3/5g$, so $b = (9/10)g$. Now just substitute whole numbers for g (or b) that give whole numbers for men. If $g = 10$, then $b = 9$, and $2/3b = 2/3(9) = 6$ and $3/5g = 3/5(10) = 6$. So, 12 dancing people out of a total of 19 -- 12/19 are dancing.

Look at the scales. The number below each scale is the total weight of the objects on the scale. There are cans, boxes, heart shapes, and balls. Items that are alike have the same weight.

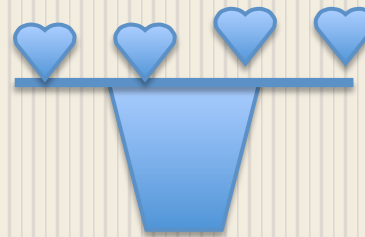
- **Find the weight of each object.**



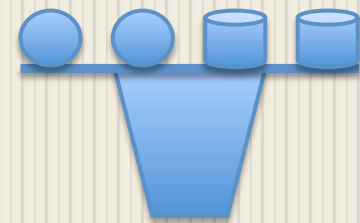
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12

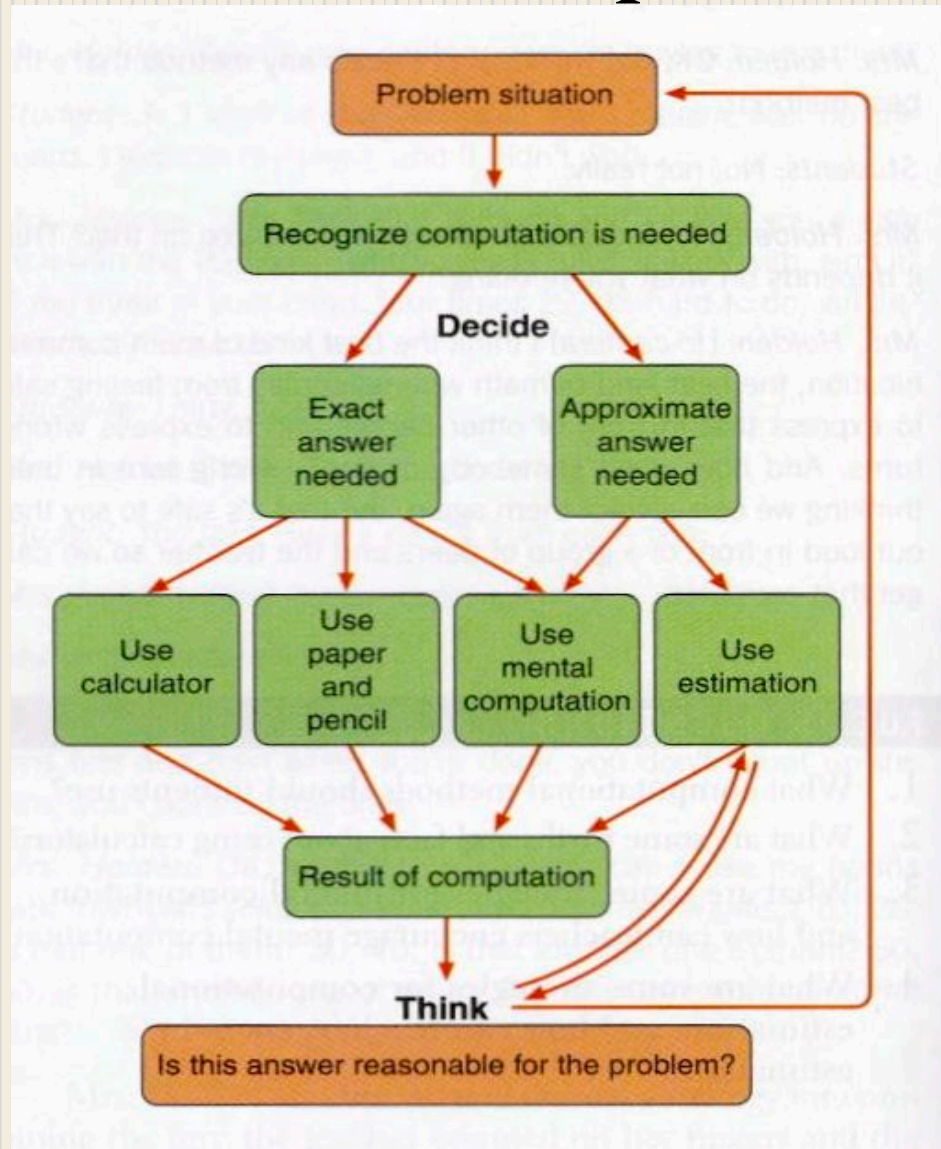


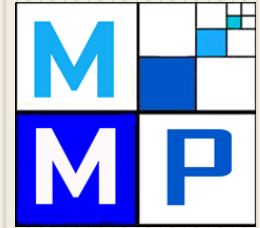
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26

Computation decision points

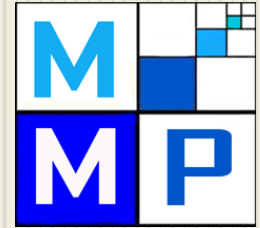




What if the estimate was 6000?

139 x 43

- **Where did this estimate come from?**
- **Was it a good approach?**
- **How should it be adjusted?**
- **Why might someone select 150 instead of 140?**

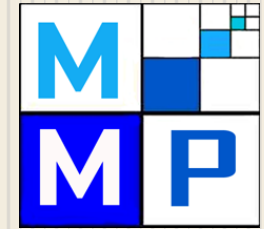


$$3,482 \div 7$$

Think multiplication

In which place value would your answer land?

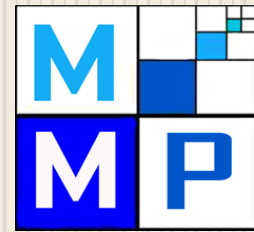
0.1 1 10 100 1000



$$5,210 + 298 \approx 5,400 \quad 5,500 \quad 7,000 \quad 8,000$$

$$59 \times 11 \approx 60 \quad 500 \quad 600 \quad 6,000$$

$$268 \div 9.9 \approx 25 \quad 250 \quad 2.5 \quad 2,500$$



Is the Answer Over or Under?

Problem

- $37 + 75$

- $476 - 117$

- $349 \div 45$

- 17×38

Over/Under

100

300

10

800

How much time?

Strategy problem	15 to 20 minutes
Concept problem	15 to 20 minutes
Algebraic thinking problem	5 to 10 minutes
Estimation problem	3 to 5 minutes
Multi-step problem	5 to 10 minutes

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- **Teacher's role**
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- **Philosophy – a later discussion**

The teacher's roles: Guide, Model, & Monitor

- Teacher actions: Before, During and After
- Selecting appropriate tasks
- Grouping students appropriately
- Observing & listening to students
- Encouraging communication and reflection

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91 Good Pay

Solve. Look for a pattern.

Wiley Willie applied for a new job. He offered to work for \$0.01 the first week, for \$0.02 the second week, for \$0.04 the third week, for \$0.08 the fourth week, and so on. The boss decided that this was a good deal and hired Willie. But, the boss is in for a surprise. How much will Willie have earned altogether after 20 weeks? (*Hint: A calculator might be helpful.*)



You can **LOOK FOR A PATTERN** to solve this problem because

- Something repeats or changes
- What repeats or changes does so in the same way all the time

Answer _____

Convince Me Explain how you solved the problem.

Math Journal Describe the pattern in Willie's total earnings from one week to the next.

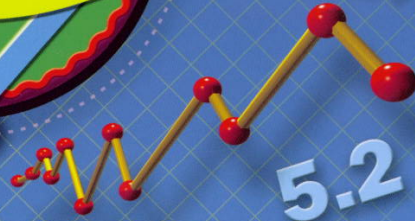
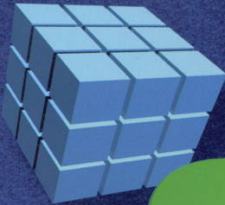
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How to contact me –

My email address:

lester@indiana.edu

I'll be back from time to time!

Visit the NCM website --

<http://ncm.gu.se>

92 Tourist Treasures

Estimate. Use front-end estimation.
Circle the letter of the correct answer.

A store in Boston sells a "Welcome to Boston" package. The package contains a T-shirt (\$12.99), a map (\$3.49), a hat (\$8.84), and a pen (\$1.28). About how much do the items in the welcome package cost altogether?

- A. about \$10.00
- B. about \$35.00
- C. about \$28.00
- D. about 21 packages

Math Journal The store wants to make a 25% profit on each package. For about how much should the storeowner sell the package?

REMEMBER

Using front-end digits is one way to estimate. Add, subtract, multiply, or divide the front-end digits. Then, adjust the estimate by looking at the digits to the right.

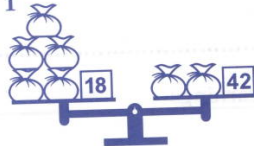
93 Are You Losing Your Marbles?

The pans in each picture are balanced. Read the balancing rules. Write the number of marbles in each bag.

Balancing Rules

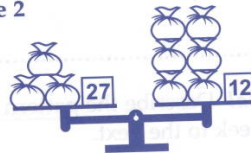
- A balance must have the same number of marbles on each side.
- On each balance, all bags must have the same number of marbles.

Puzzle 1



1 bag = ___ marbles

Puzzle 2



1 bag = ___ marbles

★ THINK

Could you remove marbles from each side and still have balanced pans?
Could you remove bags from each side?

94 I Need My Space

Write the Hidden Questions. Then solve.

A new school is being built. Each classroom must be large enough so that 30 students could each have 200 ft^3 of space. The floor of each classroom is 20 ft by 30 ft. How many feet high must the ceiling be so that each student has enough space?

Hidden Question 1 _____

Answer _____

Hidden Question 2 _____

Answer _____

Final Question How many feet high must the ceiling be?

Answer _____

Math Journal How are volume and area different? How are they related?

★ THINK

To solve multiple-step problems, you need to

1. Answer the Hidden Question(s)
2. Use your answer(s) to the Hidden Question(s) to answer the Final Question

TEST TIP

Draw a picture to help you solve the problem.

95 Lots of Lockers

Richard works for a locker installation company. After he installs all the lockers, he must number the lockers consecutively beginning with 1. The numbers are put up 1 digit at a time. Richard used 492 digits in all. How many lockers did he install?

Answer _____