A Framework for Teaching Mathematics through Problem Solving in Lower Secondary School

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Problem solving plays an important role in mathematics at all grade levels. However, knowing how to incorporate problem solving meaningfully into the mathematics curriculum is not at all obvious. Fortunately, a considerable amount of research on teaching and learning mathematical problem solving has been conducted during the past 40 years or so and, taken collectively, this body of work provides useful suggestions for both teachers and curriculum writers. Using the problem-solving research literature as a guide, my colleagues – Randall Charles and Diana Lambdin -- and I have developed a framework for a structured approach to teaching mathematics through problem solving at the lower secondary school level. The framework serves as the basis for structured instruction consisting of a carefully crafted sequence of five problem types in each of 30 problem sets for each grade level. The five types of problems are -- strategy, estimation, algebraic thinking, multiple-step, and concept. (A sample of each type of problem follows the description of the framework.) Throughout the program, students are aided by an emphasis on a three-phase problem-solving process: *Read and Understand*, *Plan and Solve*, and *Look Back and Check*. In this short paper, I describe an instructional program that makes use of the framework.

The framework

As we have continued to refine our ideas about the role of problem solving in mathematics instruction, we have come to realize that problems and problem solving should play a much more central role than merely to motivate students and provide them with opportunities to apply the math they have learned. This realization has resulted from both our own research and that of many others. Specifically, the program is built on three important research-based principles. First and foremost, research tells us that the problems and exercises must have important mathematics concepts and processes associated with them. Second, the problems must be accessible and engaging to the students, building on what they know and can do. Third, the teacher must ensure that the classroom environment supports students learning in this way and encourages them to think deeply about their solution methods and those of their classmates. The framework

for lower secondary school is made up of four components: (1) consistent lesson structure and teaching strategy, (2) continual practice and review, (3) emphasis on problem-solving strategies, representations, algebraic thinking, and estimation skills, and (4) journal writing.

Consistent lesson structure and teaching strategy

Classroom instruction consists of a carefully crafted sequence of five problem types in each of the 30 problem sets per grade level: *strategy, estimation, one-step*, *multiple-step*, and *concept*. These problem sets provide enough material for nearly every day of the entire school year. During instruction teachers make consistent use of a three-phase process: *Read and Understand, Plan and Solve, and Look Back and Check*. This problem-solving process is based on the work of master problem solver, George Polya, and has been used successfully in a variety of commercially developed mathematics programs for middle grades students (e.g., the "Launch–Explore–Summarize" strategy used in the *Connected Mathematics Program*). A set of 10 specific *teaching actions* is used to guide the teacher throughout the three-phase process (see a list of these 10 actions at the end of this paper).

Continual practice and review

In any good instructional program, students are expected to learn basic skills and key algorithms, in addition to engaging in explorations of worthwhile problems. Moreover, gaining proficiency in using important problem-solving skills and thinking strategies also requires practice and review; it is not enough simply to introduce students to key strategies, it is important to revisit these strategies and skills on a regular basis. In our program, key mathematics concepts and skills are embedded in the problem sets and important problem-solving strategies are introduced and revisited in a cyclic manner.

Instruction on problem-solving strategies, representations, algebraic thinking, and estimation

In the program the first problem in every set is a strategy problem. The *problem-solving strategies* emphasized in our lower secondary program include: write an equation, work backwards, make a simpler problem, make a diagram, and make a graph.

The term *representation* refers both to process and to product. As a process it refers to the act of creating in one's mind a mental image or images of a mathematical idea. As a product it refers to some physical form of that idea, such as a picture, a diagram, a graph, or an object. Why is the idea of representation so important? Simply stated, the more ways

a student can think about a mathematical concept, the better that student will understand the underlying mathematical idea. For example, diagrams are given particular emphasis in the development of one-step problems. Such diagrams show how the quantities in a problem are related to each other and why a particular operation or operations can be used to solve the problem. The consistent use of diagrams to show the meanings of operations not only improves students' abilities to solve one-step word problems, but it also provides a solid foundation for the formal study of algebra.

In light of the growing importance placed on algebra in the curriculum, the *algebraic thinking* problems in the program develop concepts such as patterns, interpreting graphs, interpreting the language of equations, and more. This work with algebra is in addition to learning how to write equations to represent information given in a problem statement.

Traditionally, *estimation* has been thought of as a supplemental skill. However, research indicates that most of the mathematics used in everyday living relies far more on mental computation and estimation than on the use of computational algorithms. In our program, the second problem in every set is an estimation problem. In addition, the use of estimation and estimation strategies (such as rounding, use of compatible numbers, and so on) is embedded in the problem-solving process throughout the program. In particular, students use estimation to determine if their answer is reasonable (in the *Look Back and Check* problem-solving phase).

Journal writing

In our program every set of problem includes at least one *Math Journal* question. These open-ended questions provide students with opportunities to explain their reasoning and to justify their thought processes and strategies. Such writing encourages students to show their work while promoting mathematical communication.

Instilling healthy attitudes and beliefs

An important goal of any good mathematics program should be to promote positive attitudes and helpful beliefs about doing mathematics. In particular, the student pages in our program frequently provide user-friendly "Think" and "Remember" hints to offer practical suggestions designed to promote student perseverance and success.

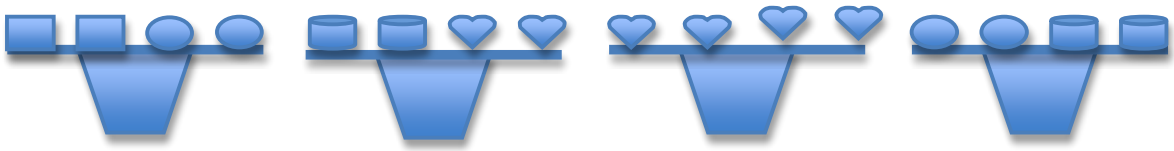
Sample problems

Strategy problem (strategy: Make a graph): Central Cellular charges $15.00 per month plus $0.10 for every minute. *Omega Cellular* charges a flat $0.20 for every minute. Which phone company has the best rates?

Estimation problem: Video Views produces music videos. This year their promotion budget was 1.2 million

euros. The company needs to reduce its promotion budget by 5.8%. By about how much will they reduce the promotion budget? (Describe how you arrived at this estimate.)

Algebraic thinking problem (systems of equations): Look at the scales. The number below each scale is the total weight of the objects on the scale. There are cans, boxes, cones, and balls. Items that are alike have the same weight. Find the weight of each object.



32 12 8 26

Multiple-step problem: A Christmas tree farm bought 187 bundles of trees. Each bundle contains 24 trees. The farm planted the same number of trees on each of 8 hectares. How many trees did the farm plant on each hectare?

Concept problem (concept: mean of a set of data): Replace two numbers in the data set shown without changing the mean of the set. (Do not replace any numbers with 0.): 121 123 125 127 129 131 133

Teaching actions for problem solving

The framework identifies a set of 10 important actions teachers should use when engaging students in problem-solving activities. These actions correspond to the three phases of the problem-solving process.

Read and Understand

- Have students read the problem—discuss words or phrases they may not understand.
- Use whole-class discussion to focus on the importance of understanding the problem.
- Lead a whole-class discussion of possible approaches for solving the problem. *(optional)*

Plan and Solve

- Observe and question students to determine where they are in the solution process.
- Provide hints if needed (Use only focusing hints at first.)
- Provide problem extensions as appropriate.
- Require students who obtain a solution to be sure to “answer the question.”

Look Back and Check

- Have students show and discuss their solutions.
- Relate the problem to previously solved problems and have students solve and discuss extensions.
- Discuss special features (e.g., pictures, unfamiliar terms).

Reference

Charles, R. I., Lester, F. K., & Lambdin, D.V. (2005). *Problem Solving Experiences: Making Sense of Mathematics.* Parsippany, NJ: Dale Seymour Publications.