Helping Children Develop Mathematical Habits of Mind

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What should the fundamental goals for elementary school mathematics include? A random person in the street might reply that school mathematics should focus on skills and procedures such as computing, graphing, and solving equations. However, mathematics teachers know that mathematics is not just about skills and procedures. Learning about mathematical concepts— such as numeration and place value, additive and multiplicative thinking, equivalence, and similarity (to name just a few)—is also an obvious goal of school mathematics. However, even mastering procedures and learning concepts is not enough; students must also learn to solve problems.

An influential report by the US National Research Council broadened this list of goals for school mathematics by offering the image of mathematical proficiency as a sturdy rope constructed by braiding together five different components or strands (Kilpatrick, Swafford, and Swindell, 2001). The report's authors identified the strands of mathematical proficiency as procedural fluency, conceptual understanding, adaptive reasoning, strategic competence, and productive disposition. The first two of these strands need no explanation, but the other three require some elaboration. Adaptive reasoning was defined as "capacity for logical thought, reflection, explanation, and justification" and strategic competence was defined as "ability to formulate, represent, and solve mathematical problems" (p. 115). In other words, these two strands of the mathematical proficiency braid involve ways of thinking about mathematics. The report defined productive disposition as "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p. 115). In other words, this strand is about an individual's motivations, attitudes, and beliefs—clearly related to how mathematics relates to him/herself.

Another way to think about these three strands of mathematical proficiency—adaptive reasoning, strategic competence, and productive disposition—is to consider them together as what some authors have labeled "mathematical habits of mind" (Goldenberg, Shteingold, and Feurzeig, 2003). When we recognize that "mathematics is both a body of facts accumulated over the millennia and a body of ways of thinking that have enabled people to discover or invent these facts, . . . [then it becomes clear that] teaching these ways of thinking, or mathematical habits of mind, is a vital part of mathematics instruction at every level (Goldenberg et al., p. 16.).

Writing in a book chapter entitled "Mathematical Habits of Mind for Young Children," Goldenberg et al. noted that there are many habits of mind, but they focused their attention on just five communication-related habits of mind: (1) Thinking about word meanings, (2) Justifying claims and proving conjectures, (3) Distinguishing between agreement and logical necessity, (4) Analyzing answers, problems, and methods, and (5) Seeking and using heuristics to solve problems. (During my presentation at the Matematikbiennalen 2010, I will offer examples of these five habits of mind, and more.)

Teachers find that a habits-of-mind approach to mathematics instruction is closely related to teaching mathematics through problem solving—an approach that necessitates that students make sense of what they are learning. As a result, to Goldenberg et al.'s list of habits of mind for young children, I would emphasize attention to a sixth habit of mind that is more overarching: Seeking to make sense of all aspects of school mathematics—concepts, procedures, heuristics, and problems. We can think about a model of learning mathematics in which understanding is represented by an increasingly connected and complex web of mathematical knowledge. According to this model, students develop understanding of a topic or concept when they figure out how each new idea is related to other things they already know (Brownell, 1947; Hiebert and Carpenter, 1992; Schroeder and Lester, 1989). Understanding grows as a student's personal web of connections becomes more and more complex. Indeed, the web might be imagined as a "hammock-like structure in which knots are joined to other knots in an intricate webbing. Even if one knot comes undone, the structure does not collapse, but still bears weight—as opposed to what might happen if each individual rope was strung only from one point to another with no interweaving" (Russell, 1999, p. 4).

Finally, a seventh habit of mind that I have found quite productive—both in teaching children and in working with mathematics teachers (both novice and experienced)—is seeking to identify and connect different representations of mathematical ideas. For example, when elementary school children are working on a story problem, they should be encouraged to think about how they could write a number sentence to represent the information, as well as draw a picture or create a graph depicting the information. By doing these things, they are using the power of different representations to build a stronger web of mathematical understanding. Similarly, when children are trying to make sense of a new concept (such as division of fractions), their understanding can be much enhanced if they are in the habit of thinking about how they could use manipulative materials or pictures (sets of objects, area models, or a number line) to illustrate the idea of fraction division for various division number sentences or what sort of story problem could be represented by those same number sentences.

Some teachers may be concerned about adding habits of mind to the already long list of goals for school mathematics. But teachers in the elementary grades often have the advantage of teaching multiple subjects (language arts, science, and social studies, as well as mathematics), which means they may have opportunities to encourage students in using creative, logical, and relational thinking throughout the school day. Finding ways for mathematics instruction to promote habits of mind that "support, extend, and refine common sense—not replace it or even supplement it with a set of clever 'magical' computational tricks and methods—helps develop students' critical thinking and their skills both in mathematics and other areas" (Goldenberg et al., 2003, p. 29).

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