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Students' tasks in learning and teaching mathematics.

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Introduction

The final sentence of the abstract for this session reads: "It is intended that the session will take a realistic stance in setting the discussion of tasks in the context of the reality of classrooms experienced by teachers." A moment's reflection on this 'intention' raises some fundamental questions, including:

- What is the reality of classrooms experienced by teachers?
- Is there a single 'identifiable reality' to which reference may be made?
- What does this English person, who is working in Norway, know about the reality of mathematics classrooms in Sweden?

I will take these questions in reverse order.

The fact is, this English person has no first-hand knowledge of mathematics classrooms in Sweden. The little I know comes from working with colleagues from Sweden and students from Sweden who join the doctoral courses where I work. As I engage in discussion with people who are deeply aware of the regularities of teaching and learning mathematics in Swedish classrooms I recognise that my English experience may be different in kind, however, the challenges of teaching and learning are very similar perhaps varying only in degree and balance. My intention is to address a reality that I hope each one of you will recognise, albeit that 'reality' is the product of each person's individual experience.

This leads naturally to the middle question, if the 'reality' of the classroom is a mental construct arising from individual experience, is it reasonable to imply the existence of an identifiable reality? Well, yes and no! Cross-cultural/national studies of teaching, learning and pupil performance[1], suggest that it is possible to identify national characteristics of teaching and classroom practices, and even regional differences in pupils' profiles of competencies in mathematics. Analyses of TIMSS and PISA results demonstrate a very close profile of students' competencies in Norway and Sweden, and a less pronounced but nevertheless noticeable Nordic profile including Denmark and Iceland. Further the analysis reveals the Nordic profile to be similar to that of the English speaking countries. Analysis of the TIMSS video data revealed remarkable homogeneity of classroom practices within countries compared to differences across countries. Furthermore, a study in the United States revealed that US teachers and Japanese teachers have quite different mental 'scripts' of mathematics lessons. Thus, the evidence appears to suggest that it is not unreasonable to claim that there are some common characteristics of, say, mathematics classrooms in Sweden, although I do not mean to imply there is no variation within a country. The latter is demonstrated clearly in Jo Boaler's work over a decade ago[2] that drew attention to the remarkable difference in mathematics teaching within two schools in England.

I turn now to address the first question: What is this presumed reality of a classroom that I hope will be recognised by mathematics teachers in Sweden. I believe I can do no better than to use the words of Paul Ernest:

> … each student, teacher, classroom, school and country has a life history with antecedent and concurrent events and experiences … the buzzing booming rough-and-tumble complexity of the mathematics classroom[3].

In using the expression 'reality of classrooms experienced by teachers' my point is to acknowledge the complexity of the situation within which teachers work, rather than some idealised and sanitized experimental test-bed constructed for an academic discourse. The researchers I know, who are concerned with the development of mathematical tasks, and I include especially those I cite in this presentation, are well aware of this complexity and the challenges faced by teachers in managing their classrooms as arenas for learning mathematics.

The reality of classrooms

Research in the UK has revealed that teachers aim to establish classrooms that are characterised by 'normal desirable states of pupil activity'.[4] The teachers' concern is principally focused on what the students in their classes are *doing*. Other research in the UK has exposed that students have expectations about teaching and their teachers.[5] Pupils go to school to work, they expect the work to 'count', teachers should be able to teach, make pupils work, and keep control. But even when the teacher's 'normal desirable state' coincides with the students' expectations it does not follow that it is an effective arena for learning mathematics, as my own study of a year ten mathematics classroom revealed. [see literature 3].
Researchers in the US have drawn attention to the importance of establishing classroom norms that are conducive to learning mathematics.[6] They identify norms at three levels; social norms of explanation, justification and argumentation that might characterise classrooms in any subject; sociomathematical norms which include understanding about what is mathematically acceptable, different, sophisticated, efficient, elegant, etc.; and mathematical norms which relate to those mathematical objects and processes that can be accepted without further explanation. A Norwegian mathematics educator has argued that it is essential for students to have a 'proper metaconcept of mathematics'.[7] In other words, students need to have an awareness of the nature of mathematics, and what the subject is about, that supports their development of competencies in the subject. The process of teaching is further complicated because, as a French mathematics educator observes, it is necessary for students to engage in tasks in which the didactical markers are removed, so they become 'adidactial situations'.[8] That is, the resolution of the task must rely on the students' mathematical thinking and not on any clue that arises from the regularities of classroom practices that could lead the student to the correct answer.
The complex reality of teaching mathematics, outlined above entails developing norms (social, socio-mathematical, mathematical), and students' metaconcepts, and challenging students' expectations. This takes time. It cannot be imposed by decree; it emerges through a long term project in which the teacher works to adapt the class. I believe that if a teacher is to succeed with a class it is essential to start where the class is. It is up to the teacher to first adjust to the expectations of the students and then work on the students' expectations so they adapt to the target norms of the teacher. It requires, first and foremost, the students' confidence in the teacher: the students recognise *this is a 'good' teacher; this teacher helps me to learn*; where 'good' and what counts as 'learning' are defined within the students' expectations.[9] It is only when such confidence has been achieved that it is possible to work on students' expectations. There are plenty of tasks around that can be used to inspire students' confidence at the outset; they can be found in most mathematics textbooks.

A practice based on theory

Developments in teaching must be based upon a clear rationale that is understood by the teacher. It follows that new approaches, tasks and activities should be supported by a theoretical foundation of learning and teaching mathematics that informs and guides teaching activities, and subsequently guides evaluation. In our mathematics teaching developmental research that my colleagues and I pursue the guiding theoretical principle is that of *inquiry*. The routine implementation of an inquiry cycle of plan-act-observe-reflect-feedback-plan- is taken as fundamental to learning mathematics, teaching mathematics, and developing the teaching of mathematics. Students' (and teachers') inquiry into mathematics is our goal, and inquiry is the approach that we take to reach the goal. The intention is to support students' learning through the introduction of tasks that stimulate inquiry, and to work on the classroom norms so that the tasks are interpreted as mathematical inquiries, rather than a hunt for didactical clues. This is difficult because it places great demands on teachers; one of my colleagues has described the process as double innovation[10], because it includes both new tasks and new approaches to managing those tasks.

I return to the reality of the mathematics classroom. There is a curriculum, there are examinations, there is a school schedule or time-table, and there are school rules or conventions to adopt; these are external demands and constraints that must be considered. Within the classroom there are resources that offer opportunities for working mathematically and constraints that appear to limit opportunities. There is the complexity of mathematics (facts, skills, concepts, strategies, personal qualities) and there is recognition that teaching needs to include a variety of approaches if the complexity of learning mathematics, especially in the context of the external demands are to be accommodated (exposition, discussion, practice, practical activity, problem solving, investigational work, everyday applications). These different approaches are used for a variety of purposes. It is essential that the purpose for using an approach is clear and that the approach is suited to the purpose, that is, what is to be learned. Then one can consider tasks that are fit for purpose within each approach.

Evaluation of tasks

It is not possible within a short presentation to include a wide variety of task types, especially after including an extensive essay to qualify the intention of the presentation. However, there exists a rich literature on tasks and task design and it is worth engaging seriously with this.[11] My purpose here is to share and illustrate a classification of task demand that resonated strongly with my own beliefs about teaching and learning mathematics, the essence of which is outlined in the foregoing.

The tasks given to students can challenge them in a number of ways. The student might be challenged to make sense of the way the task is presented because of the complexity of vocabulary and expression. If the learning purpose is about comprehension of text this might be an appropriate task, but it might not make sense if the purpose is to engage in mathematical thinking or learn some mathematics. The task might be designed to develop students' precision and accuracy in working mathematically and possibly challenge the student's patience. However, if the ultimate purpose of the task is that students learn mathematics then I believe it is necessary to consider the cognitive (mathematical) challenge of the task. Researchers in the US, working in the area of mathematics teaching development, have produced a classification system that they use to evaluate mathematical tasks used in classrooms.[12] These researchers consider the cognitive demand of 4 types of task, two types of lower-level demand tasks (memorization tasks, and procedures without connections tasks) and two types of higher-level demand tasks (procedures with connections tasks, and doing mathematics tasks). Each type of task is described by characterising features, for example, doing mathematics tasks 'require complex and nonalgorithmic thinking', 'require students to understand', 'demand self-monitoring or self-regulation of one's own cognitive processes', require considerable cognitive effort', etc. The researchers are also conscious that the implementation of tasks could lead to a reduction of the cognitive demand and explain the circumstances in which this may occur. The research confirms the assertions made above regarding classroom norms, students' metaconcept of mathematics and their willingness to accept adidactical situations.

In summary, the points I want to make are that it is essential that we, teachers of mathematics, consider the nature of the demands in the tasks we give to students. The challenge of the task must be consistent with the learning goals for which the task is being used. If the intention of the task is for students to learn some mathematics then the task should provide an appropriate challenge for the student to engage consciously with that mathematics. However, well-chosen or well-designed tasks by themselves are not sufficient, they need to be implemented in a classroom where the reality is characterised by appropriate social and socio-mathematical norms, and where the students have a proper metaconcept of mathematics.

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