1. Matte som naturvetenskap: Mathematics as a Natural Science, MANYology

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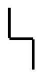
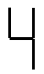
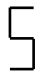
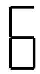
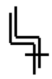
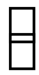
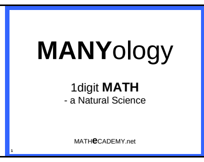
*using free PYRAMID**e**DUCATION. He has written a thesis on Postmodern Mathematics.*

***10** is a cognitive, cognitive bomb.   **S**ave the world with 1digit Math.   **C**ount & **A**dd in **T**ime & **S**pace.*

Rediscovering Mathematics as a Natural Science, MANYology
Let us rediscover mathematics as a natural science grounded in the natural fact Many.
What do we do when we meet Many? Two things. First we Count, then we Add. And we do so where we live, in Time and Space. So in this CATS-approach to mathematics, Count&Add in Time&Space, mathematics is learned, not through books, but through Counting & Adding.

Numbering Many by Using 1.order, 2.order and 3.order Counting
There are 3 ways of counting Many: 1.order-counting, 2.order-counting and 3.order-counting.
* 1.order-counting rearranges sticks in icons containing the degree of Many it describes: four sticks in the 4-icon, five sticks in the 5-icon, etc. 1.order-counting stops at ten.
* 2.order-counting uses bundling and stacking in icon-bundles e.g. in 5s, but not in tens.
* 3.order-counting uses bundles of ten, the only number with a name but without an icon.

									
1	2	3	4	5	6	7	8	9	Double-click PowerPointPres.

Re-counting 7 1s in 3s, in 5s and in 2s
As an example of 2.order-counting, let us re-count 7 1s in 3s, 5s and 2s.

I I I I I I I -> III III) I) -> ▌▌) I) -> 2) 1) -> 2.1 3s

I I I I I I I -> IIIII) II) -> ▌) II) -> 1) 2) -> 1.2 5s

I I I I I I I -> II II II) I) -> ▌▌▌) I) -> 3)1) -> 3.1 2s -> II I) I) -> I) I) I) -> 1) 1) 1) -> 11.1 2s

Counting 7 1s in 3s, 2 times we take away 3-bundles to be placed in a left bundle-cup, either as actual bundles, or as sticks counting bundles by being placed in the left bundle-cup. The unbundled stick is placed in a right single-cup. Thus the counting result is 2.1 3s, using a decimal point to separate the bundles to the left from then unbundled to the right; and including the unit 3s. Likewise counting 7 1s in 5s gives 1.2 5s.

Counting 7 1s in 2s gives 3.1 2s. However, in the bundle-cup we also have a bundle of bundles that can be moved to a new cup to the left, counting the bundles of bundles. Thus counting 7 1s in 2s gives 11.1 2s.

Counting 3 8s in tens gives 2.4 tens, only this time we have no icon for ten: 3 8s = 2.4 tens.

In all cases, counting means bundling in a chosen bundle-size, and counting always produces decimal numbers carrying units. So natural numbers are decimal numbers carrying units.

10 as a Cognitive Bomb
Is this what the Book says? No, it says: ‘We only count in tens, and we do not write 2.4 tens, only 24, which we call a natural number. So we throw away the unit tens and misplace the decimal point one to the right.’ Thus the Book writes natural numbers in an unnatural way.

By allowing only counting in tens, ten becomes 10, the follower of nine, and with eleven as its follower. However, 10 is an unnatural way of writing 1.0 bundle, 10 just means 1 bundle. So when counting in 7s, seven becomes 10, the follower of six, and with eight as its follower. And the counting sequence becomes: 5, 6, bundle, b1, b2 etc., or 5, 6, 10, 11, 12 etc.

1digit Mathematics
By introducing 2digit numbers, 10 and other unnatural numbers create learning problems. Consequently we ask: Can mathematics be learned with 1digit numbers alone? Surprisingly enough the answer is: Yes, the core of mathematics can be learned as 1digit Math.

Adding OnTop or NextTo
Adding 3.2 4s and 2.3 5s can take place OnTop or NextTo. OnTop means changing units from 4s to 5s (or from 5s to 4s) by re-counting. NextTo means adding by uniting bundle-sizes. Changing units from 4s to 5s, or from kgs to $ is called proportionality, normally learned in middle school; and adding in combined bundle-size is called integration, normally learned late in high school if ever. But using 1digit mathematics, both are learned in grade 1.
Thus 3.2 4s and 2.3 5s can be added OnTop as 6.3 4s or as 5.2 5s, or NextTo as 3.0 9s.

Predicting Results using Calculations and Calculators
Furthermore, re-counting 3.2 4s in 5s can be predicted by a calculation on a calculator.

Since ‘take away 2’ can be iconized as ‘-2’ and ‘take away 2s’ can be iconized as ‘/2’, ‘from 8 take away 2’ can be iconized as ‘8 = (8-2)+2’; and ‘from 8 take away 2s’ can be iconized as ‘8 = (8/2)*2’. This provides two formulas for predicting re-counting:

‘The re-stack formula’ T = (T-b) + b & ‘The re-count formula’ T = (T/b) * b

So to re-count 3.2 4s in 5s, first we enter ‘(3*4+2*1)/5’ giving 2.8 5s. Next, to see if we can trust the .8 we take away the 2 5s. Entering ‘(3*4+2*1)-2*5’ gives 4, so the re-counting result can be predicted to be 2.4 5s. To test this prediction we perform the actual re-counting: First we de-bundle the 3.2 4s in 1s, then we the re-bundle the 1s in 5s:

3.2 4s -> 3)2) -> IIII IIII IIII II -> I I I I I I I I I I I I I I -> IIIII IIIII IIII -> 2)4) -> 2.4 5s

So the prediction holds. From now on we don’t need to re-count by de-bundling and re-bundling since we can predict the result on a calculator thus becoming a number-predictor.

Proportionality as Shifting Units or Re-counting
The re-count formula can be used in all cases involving changing units. Thus with a given per-number 4kg/5$, the questions 6kg = ?$ and 8$ = ?kg’ can be answered by re-counting:

6kg = (6/4)*4kg = (6/4)*5$ = 7.5$ and 8$ = (8/5)*5$ = (8/5)*4kg = 6.4kg

Trigonometry uses re-counting a =(a/c)*c =sinA*c, b =(b/c)*c =cosA*c, a =(a/b)*b = tanA*b. So does calculus: dy = (dy/dx)*dx = y’*dx. The core of Physics and Chemistry is re-counting.

Solving 1digit Equations
The equation x+5 = 9 is solved by re-stacking 9 as 9 = (9-5) + 5 = 4+5, i.e. by x = 9-5.
The equation x*2 = 8 is solved by re-counting 8 in 2s as 8 = (8/2)*2 = 4*2, i.e. by x = 8/2.
The equation x*2+3 = 9 is solved by re-stacking and re-counting i.e. by x = (9-3)/2.

Counting Using Cup-writing
When the units are the same we can add OnTop using cup-writing:

3.2 4s+2.3 5s = 2.4 5s+2.3 5s = 5.7 5s = 5)7) = 5+1) 7-5) = 6)2) = 1) 6-5) 2) = 1)1)2) = 11.2 5s

The 7 1s is re-counted to 1.2 5s by transferring 5 1s as 1 5s from the single- to the bundle-cup. The 6 5s is re-counted to 1.1 5*5s by transferring the 5 5s as 1 5*5 from the bundle-cup to the bundles of bundles-cup, thus giving the total of 1 bundle of 5 5s and 1 bundle of 5s and 2 1s.

Adding or Removing Extra Cups
With 2.3 5s, what happens if we add an extra cup to the right?

2.3 5s = 2)3)   - (adding a cup to the right) ->   2)3)) = 23.0 5s.

Apparently adding an extra cup to the right means that the 3 1s becomes 3 5s, and that the 2 5s becomes 2 5*5s, so that 2.3 5s becomes 23 5s. I.e. moving the decimal point 1 place to the right means multiplying with the bundle-number. Likewise, removing 1 cup from the right moves the decimal point 1 place to the left, which means dividing with the bundle-number:

23.0 5s = 2)3))   - (removing a cup from the right) ->   2)3) = 2.3 5s.

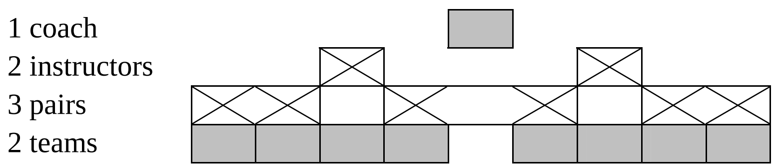
CATS: A Natural Way to Learn Mathematics as a Natural Science
1digit Math respects the Piaget 'through the hands to the head'-principle of natural learning: to grasp with the head, first grasp with the hand. This natural way to learn mathematics by counting and adding turns traditional mathematics upside down. Natural numbers turns out to be decimal numbers including units; the operation order turns out to be the opposite /, *, -, +. Adding stacks becomes the root of proportionality and integration to be introduced in grade 1. Multi-digit numbers become cognitive bombs to be postponed until mathematics has been learned using 1digit numbers alone.

Teacher Training Using PYRAMIDeDUCATION
This CATS-approach rooting mathematics as a natural science cannot be learned at traditional universities. Hence a web-based academy www.MATH**e**CADEMY.net has been created to learn the CATS-approach to mathematics as a natural science. The MATH**e**CADEMY.net offers free teacher education as well as free master degrees to teachers having learned traditional mathematics but wanting to learn mathematics as a natural science investigating the natural fact Many. Also any university can freely franchise the MATH**e**CADEMY.net.

Guided by a coach, the learners work in groups of 8 using PYRAMID**e**DUCATION: the 8 learners are organized in 2 teams of 4 learners choosing 3 pairs and 2 instructors by turn.

The coach coaches the instructors instructing the rest of their team. Each pair works together to solve Count&Add problems and routine assignments; and to carry out an educational task to be reported in an essay rich on observations of examples of cognition, both re-cognition and new cognition, i.e. both assimilation and accommodation in the Piaget sense.

The coach assists the instructors in correcting the Count&Add assignments. In a pair a learner corrects the other learner's routine-assignment. A pair is the opponent on the essay of another pair. Each learner pays for the education by coaching a new group of 8 learners, which is easy since the learners learn, not by books but by CATS, Counting and Adding in Time and Space.



8 learners and 1 coach

The learning activities fall into 2x4 parts:
* Count&Add in Time&Space 1 for primary school: C1, A1, T1 and S1
* Count&Add in Time&Space 2 for secondary school: C2, A2, T2 and S2.

The study units are accessible on the website MATH**e**CADEMY.net. They are activity-based and short and free. Their content is given in the summary below.

Literature
Tarp, A. (2004, 2008). ICME papers on Natural Mathematics from K-12, MATHeCADEMY.net

Zybartas, S. & Tarp, A. (2005). One Digit Mathematics. *Pedagogika* (78/2005), Vilnius, Lithuania.

Summary of the MATHeCADEMY.net Study Units, Count & Add in Time & Space

	QUESTIONS	ANSWERS
C O U N T **1**	How to count Many? How to re-count 8 in 3s: T = 8 = ? 3s How to re-count 6kg in \$: T = 6kg = ?\$ How to re-count 5\$ in kg: T = 5\$ = ?kg How to count Many in standard bundles?	By bundling and stacking, the total T predicted by T = (T/b)*b. T= 8 = ?*3 = ?3s, T = 8= (8/3)*3 = 2*3 + 2 = 2.2*3 = 2 2/3*3. If 4kg = 2\$ then 6kg = (6/4)*4kg = (6/4)*2\$ = 3\$ and 5\$ = (5/2)*2\$ = (5/2)*4kg = 10kg. Bundling bundles gives a multiple stack, a stock or polynomial: T = 423 = 4BundleBundle + 2Bundle + 3 = 4tenten2ten3 = 4*B^2 + 2*B + 3
C O U N T **2**	How to count possibilities? How to predict unpredictable numbers?	By using the numbers in Pascal's triangle. We 'post-dict' that the average number is 8.2 with the deviation 2.3. We 'pre-dict' that the next number, with 95% probability, will fall in the confidence interval 8.2 ± 4.6 (average ± 2*deviation).
A D D **1**	How to add stacks concretely? T = 27 + 16= 2ten7+1ten6 = 3ten13 = ? How to add stacks abstractly?	By re-stacking overloads predicted by the restack-equation T= (T-b)+b. T = 27 + 16 = 2 ten 7+1 ten 6 = 3 ten 13 = 3 ten 1 ten 3 = 4 ten 3 = 43. Vertical calculation uses carrying. Horizontal calculation uses FOIL.
A D D **2**	What is a fold-number? What is a prime-number? What is a per-number? How to add per-numbers?	Fold-numbers can be folded: 10=2fold5. Prime-numbers cannot be folded: 5=1fold5. Per-numbers occur as shifting units when double-counting. The \$/day-number p is multiplied with the day-number b before added to the total \$-number T: T2 = T1 + p*b
T I M E **1**	How to reverse counting & adding? Counting ? 3s and adding 2 gave 14. Can all calculations be reversed?	By calculating backwards, i.e. by moving a number to the other side of the equation sign and reversing its calculation sign. x*3+2=14 is reversed to x*3 = 14-2 and to x = (14-2)/3. Yes. x+a = b is reversed to x= b-a, x*a = b is reversed to x = b/a, x^a = b is reversed to x = a√b, a^x = b is reversed to x = logb/loga.
T I M E **2**	How to predict the terminal number when the change is constant? How to predict the terminal number when the change is variable, but predictable?	By using constant change-equations: If Ko = 30 and ΔK/n = a = 2, then K7 = Ko+a*n = 30+2*7 = 44. If Ko = 30 and ΔK/K = r = 2%, then K7= Ko*(1+r)^n= 30*1.02^7= 34.46. By solving a variable change-equation: If Ko = 30 and dK/dx = K', then ΔK = K-Ko = ∫K'dx.
S P A C E **1**	How to count plane and spatial properties of stacks and boxes? How to count round forms?	By using a ruler, a protractor and a triangular shape. By the 3 Greek Pythagoras: mini, midi & maxi. By the 3 Arabic recount-equations: sinA = a/c, cosA = b/c, tanA = a/b. By using π to transform polygons to circles where π = n*sin(180/n) for n big.
S P A C E **2**	How to predict the position of points and of lines? How to use the new calculation technology?	By using a coordinate-system: If Po(x,y) = (3,4) and if Δy/Δx = 2 then P1(8,y) = P1(x+Δx,y+Δy) = P1((8-3)+3,4+2*(8-3)) = (8,14). Computers can calculate a set of numbers (vectors) and a set of vectors (matrices).
Q L	What is QL, quantitative literature? Does quantitative literature also have the 3 different genres: fact, fiction and fiddle?	Quantitative literature tells about multiplicity in time and space. Yes, the word and the number language share genres: Fact is a since-so calculation or a room-calculation. Fiction is if-then calculation or a rate-calculation. Fiction is so-what calculation or a risk-calculation.